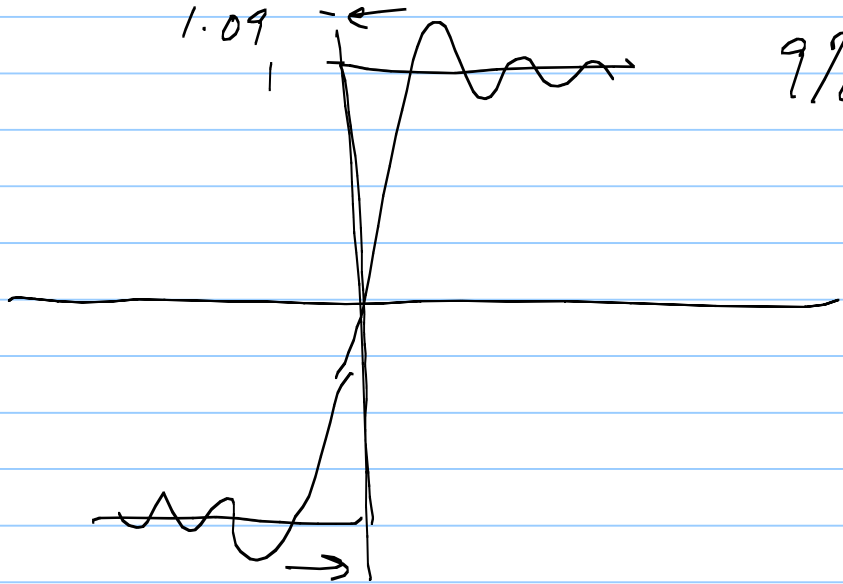
Note Title 06-09-2011

EC 5142 Sep. 6 2011



9% overshoot

Gibbs phenomenon

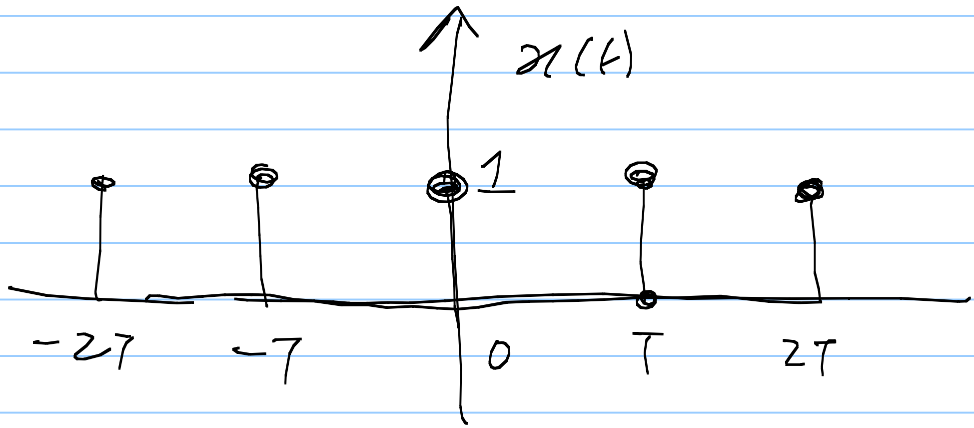
The Mathematics of Circuit Analysis _ E. A. Guillemin pp. 485 - 496

$$x_N(t) = \sum_{k=-N}^{N}{}' \, a_k e^{jk\Omega_0 t}$$

$$x(t) - x_N(t) = e_N(t) \not\to 0 \text{ as } N \to \infty$$

$$\frac{1}{T}\int_T |e_N(t)|^2 dt \to 0 \text{ as } N \to \infty$$

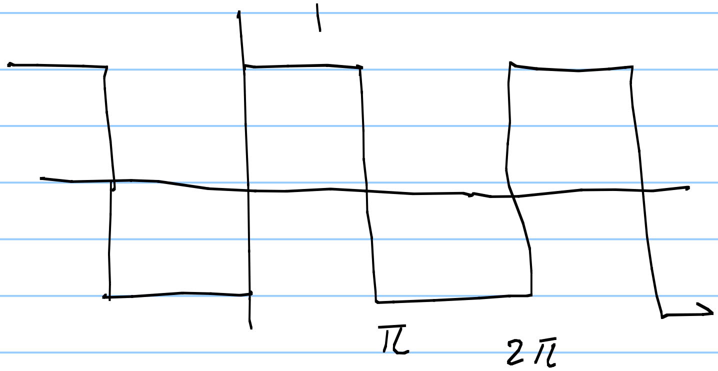
$\Rightarrow$ convergence is in the mean-square sense



$a_k = 0$

Lookup Dirichlet's conditions

$\longleftrightarrow$ $a_1 = \frac{1}{2j}$; $a_{-1} = a_1^*$



$\longleftrightarrow$ $$\frac{4}{\pi}\left\{ \sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \cdots \right\}$$

"On Bandwidth" – David Slepian

Proc. IEEE 1977?

$$x[n] = x[n+N]$$

$$e^{jk\Omega_0 n}$$

$$\Omega_0 = \frac{2\pi}{N}$$

$$= e^{jk\frac{2\pi}{N}n}$$

$$\phi_k[n] = e^{jk\frac{2\pi}{N}n}$$

$$\boxed{x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}}$$

$$= \phi_{k+N}[n]$$

$$\langle x[n], \phi_l[n] \rangle = \sum_{k=0}^{N-1} a_k e^{j\frac{k2\pi}{N}n} \cdot e^{-jl\frac{2\pi}{N}n}$$

$$\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi l k}{n}} = N\, a_l$$

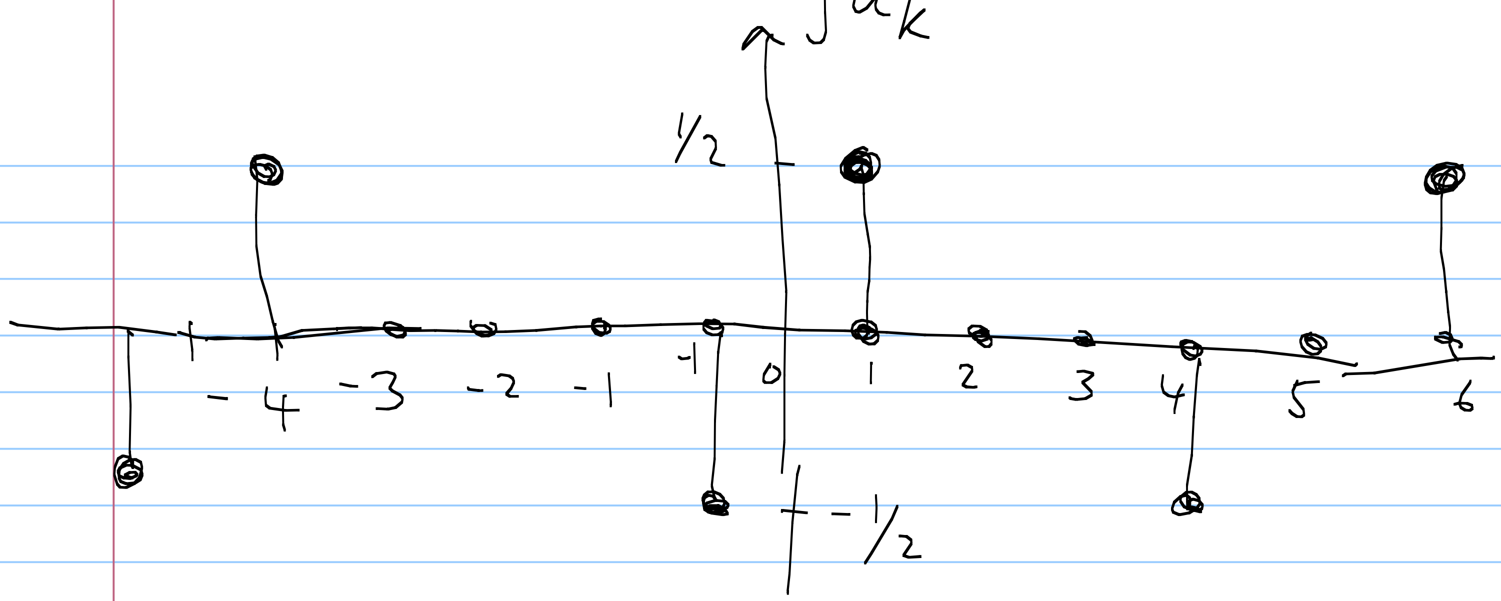
$$a_l = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j\, l \frac{2\pi}{N} n}$$

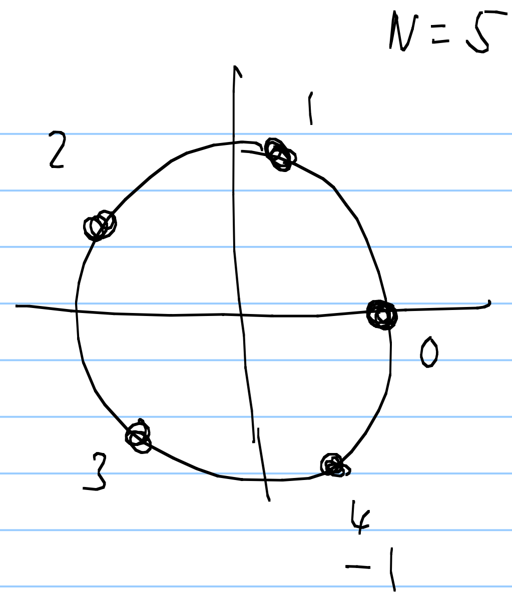
$$= a_{l+N}$$

<u>Eg</u>

$$\sin \frac{2\pi}{N} n \longleftrightarrow a_1 = \frac{1}{2j} = a_{-1}^{*} \Rightarrow a_{-1} = \frac{-1}{2j}$$

$N = 5$



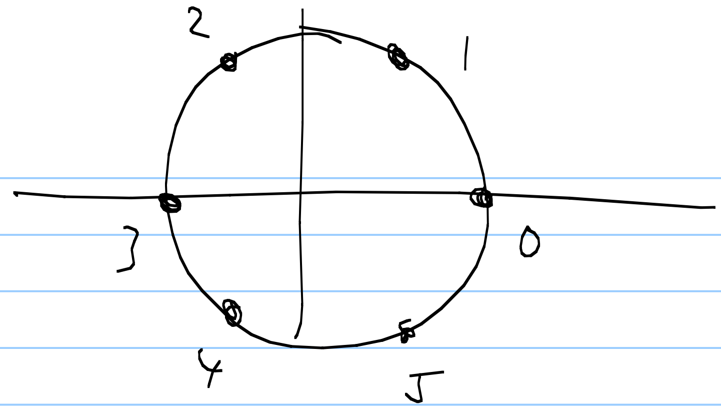


Eg 1

$$\sin 3 \frac{2\pi}{5} n = \sin \left(1 - 2/5\right) \frac{2\pi}{5} n$$

$$= - \sin 2 \frac{2\pi}{5} n$$

$N = 6$



Properties

(1) Linearity

(2) $x[n-n_0] \xleftrightarrow{\text{DTFS}} a_k \, e^{-jk\frac{2\pi}{N}n_0}$

(3) $x[n]\, y[n] \longleftrightarrow a_k \circledast_N b_k$

(4) $x[n] \circledast_N y[n] \longleftrightarrow N \cdot a_k \cdot b_k$

(5) $e^{jl\frac{2\pi}{N}n} x[n] \longleftrightarrow a_{k-l}$

(6) $\frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |a_k|^2$

Think about time scaling.