EC5142 19 Oct 2011

Note Title 19-10-2011

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{l=0}^{M} b_l\, x[n-l]$$

$$Y(z)\underbrace{\left[1+\sum_{k=1}^{N} a_k z^{-k}\right]}_{A(z)} = X(z)\underbrace{\left[\sum_{l=0}^{M} b_l z^{-l}\right]}_{B(z)} \qquad a_0 = 1$$

$$\frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)} = H(z) \leftarrow \text{Transfer function}$$

$$\updownarrow$$

$$h[n] \leftarrow \text{impulse response}$$

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} \leftarrow \text{Frequency response}$$

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Roots of $B(z)$ are <u>zeros</u> of the system

" " $A(z)$ " <u>poles</u> "

BIBO stable: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

Recall $|H(z_0)| < \infty \Rightarrow \sum_{n=-\infty}^{\infty} |h[n]| \, |z_0|^{-n} < \infty$

$\Rightarrow z_0 \in \text{RoC}$

Consider a BIBO stable system:

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

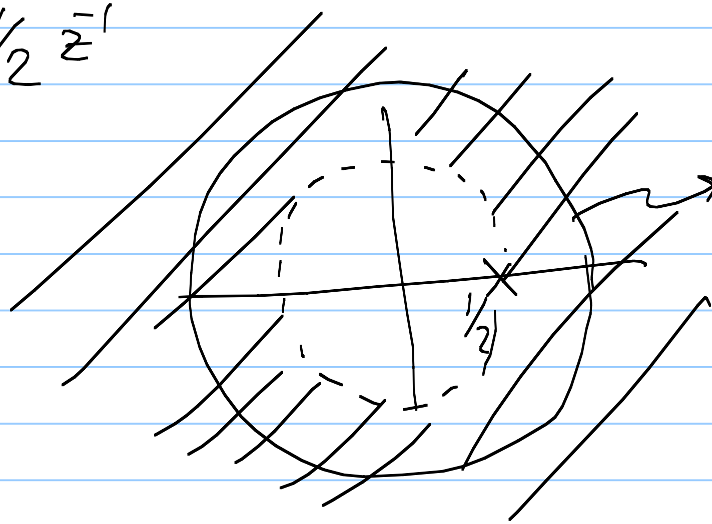
$$\sum_{n=-\infty}^{\infty} |h[n]| \, |z|^{-n} < \infty$$

$$\sum_{n=-\infty}^{\infty} |h[n]| \, |e^{j\omega}|^{-n} < \infty$$

$\Rightarrow \quad e^{jw} \in \text{RoC}$

<u>Eg</u> $h(n) = \left(\frac{1}{2}\right)^n u(n) \longleftrightarrow \dfrac{1}{1-\frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{2}$

$\Rightarrow \displaystyle\sum_{n=-\infty}^{\infty} |h(n)| < \infty$




$|z| = 1$

$2^n u(n) \longleftrightarrow \dfrac{1}{1-2z^{-1}} \qquad |z| > 2$

not stable

$$-2^n u[-n-1] \longleftrightarrow \frac{1}{1-2z^{-1}} \qquad |z| < 2$$

$e^{j\omega} \in ROC \Rightarrow$ stable

$$-(1/2)^n u(-n-1) \longleftrightarrow \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |z| < 1/2$$

not stable

$$a^{|n|} \longleftrightarrow \frac{1-a^2}{1+a^2-a(z+z^{-1})} \qquad |a| < |z| < 1/|a|$$

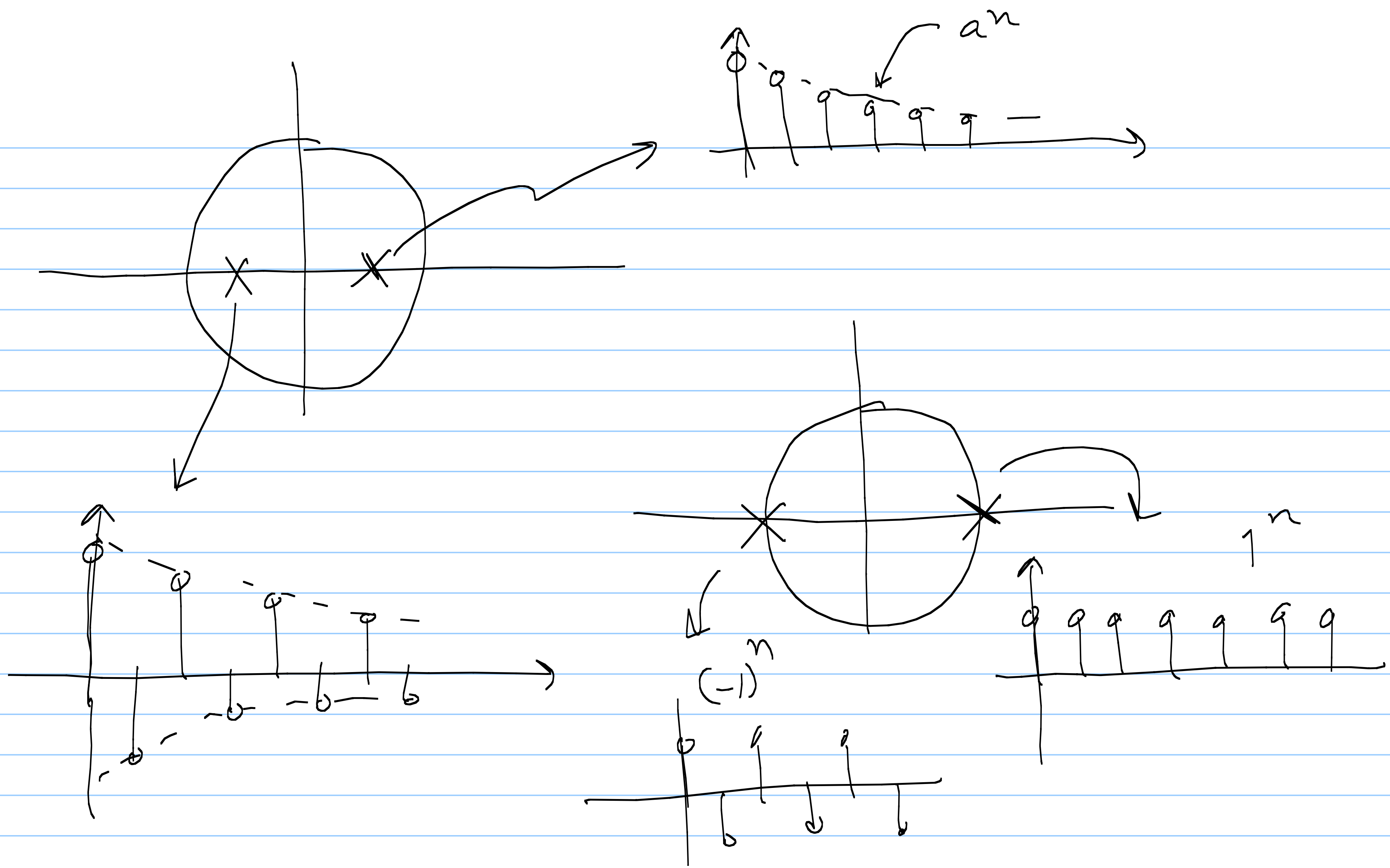
Causality implies that RoC is a region outside a certain circle

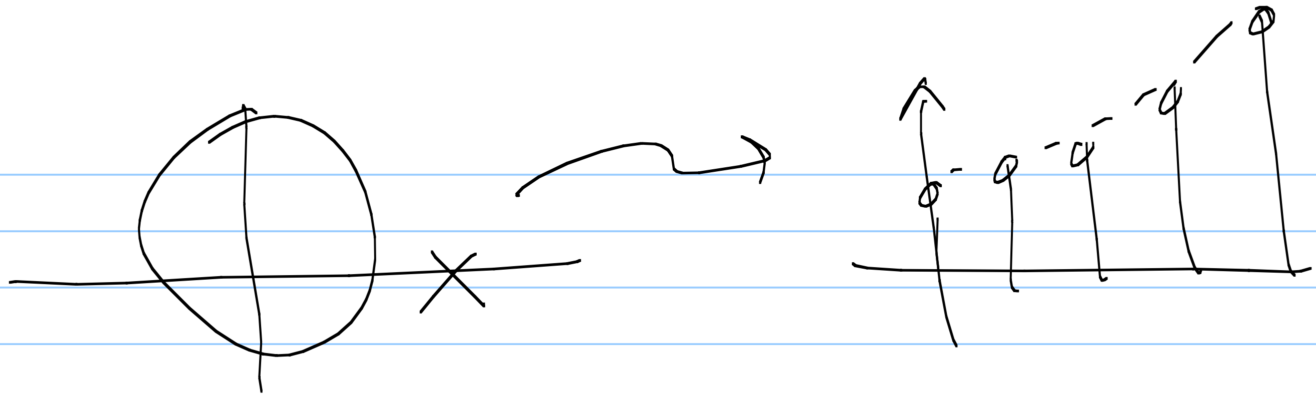
For rational TF, ROC is outside the circle corresponding to largest radius pole

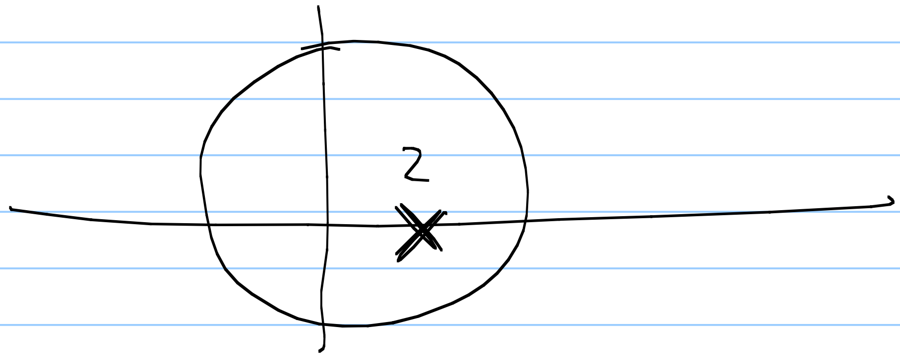
For stability, $e^{j\omega} \in$ RoC.

$\Rightarrow$ all poles must be <u>strictly</u> <u>inside</u> the <u>unit circle</u>

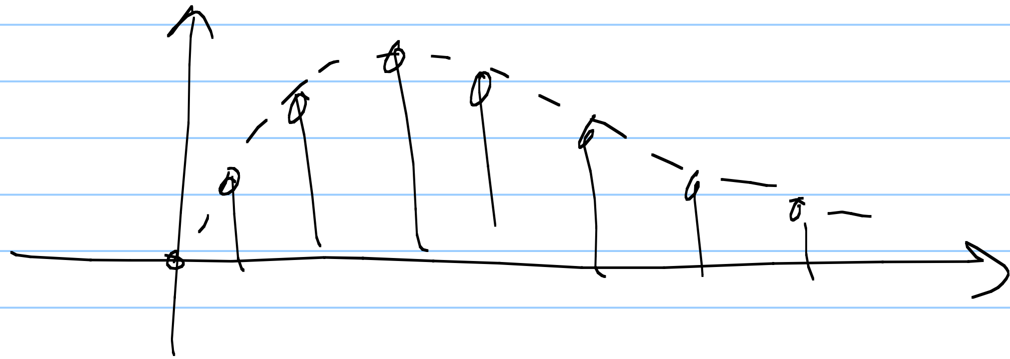
$h[n]$ is finite $\Leftrightarrow$ $H(z)$ except for 0 and/or $\infty$

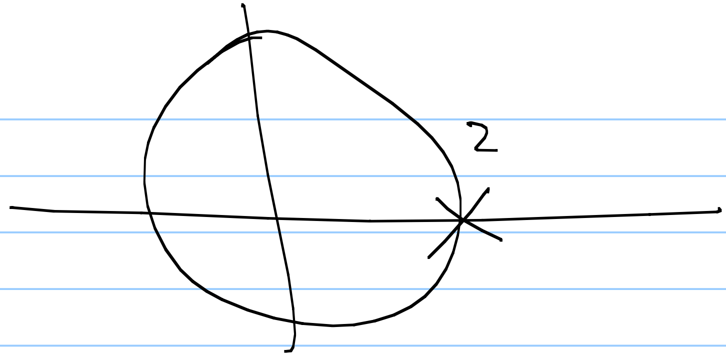

a^n
(-1)^n
1^n






$n a^n u[n]$

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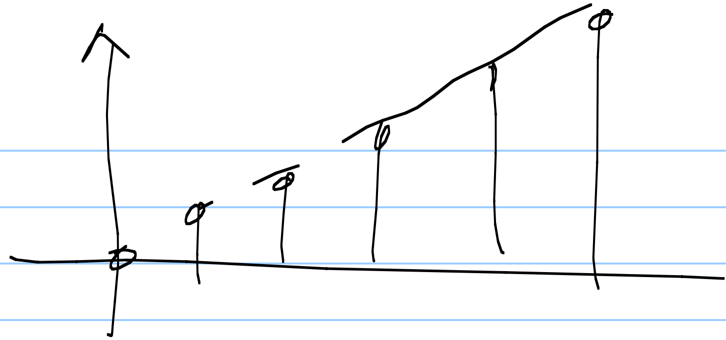
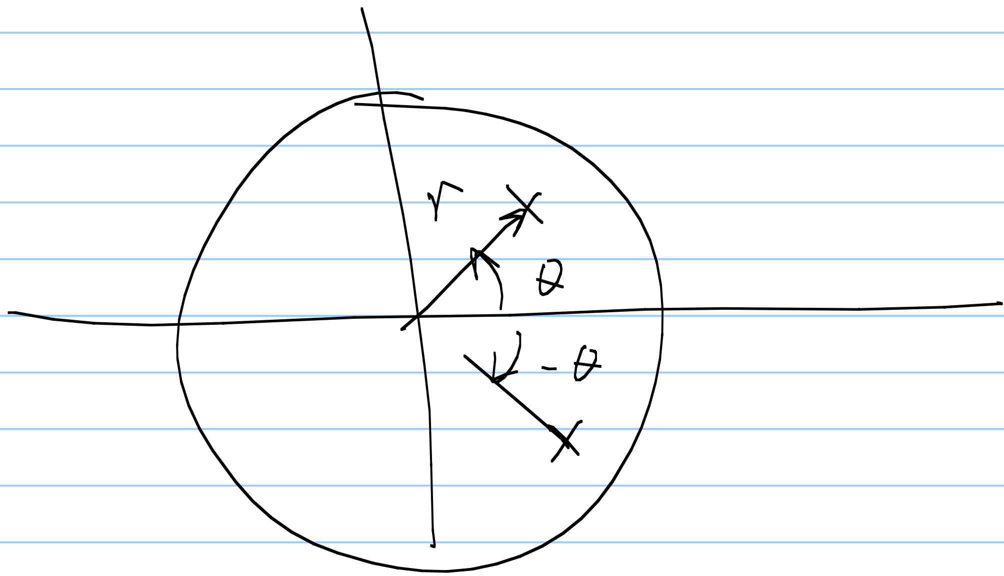
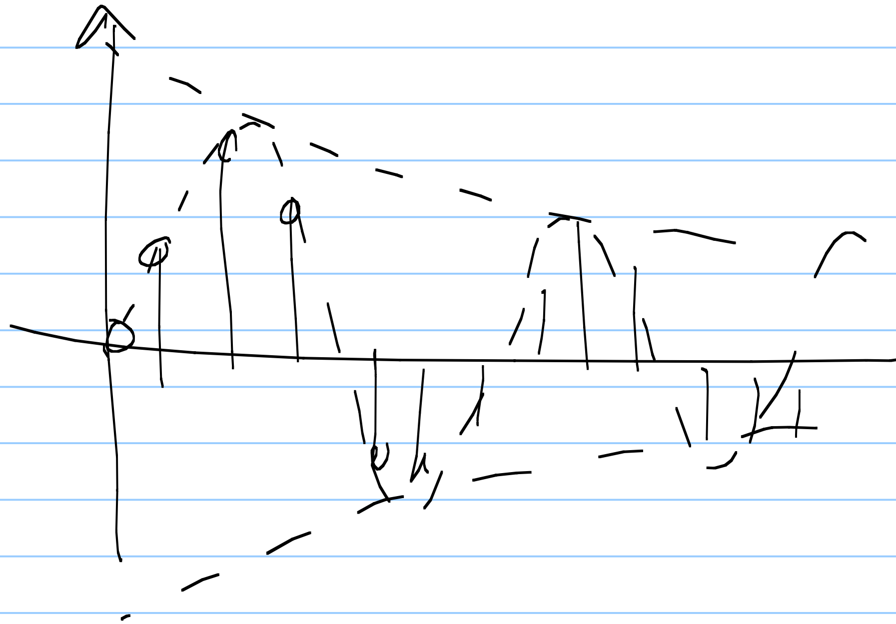



r
θ
-θ



If uncancelled nontrivial poles exist,

then $h(n)$ will contain terms like $a^n u(n)$

or $n a^n u(n)$, etc., which last (theoretically)

for $\infty$ duration $\Rightarrow$ IIR systems
(infinite impulse response)

Otherwise, $h(n)$ is finite: FIR

finite impulse response

$$H(z)\Big|_{z=e^{j\omega}} = H(e^{j\omega})$$

$$H(z) = \frac{B(z)}{A(z)} = K \frac{\prod_{\ell=1}^{M} (1 - z_\ell z^{-1})}{\prod_{k=1}^{N} (1 - p_k z^{-1})}$$

$$= K \, z^{N-M} \frac{\prod_{\ell=1}^{M} (z - z_\ell)}{\prod_{k=1}^{N} (z - p_k)}$$

trivial poles/zeros

$$H(e^{j\omega}) = K\, e^{j\omega(N-M)} \frac{\prod_{l=1}^{M} (e^{j\omega} - z_l)}{\prod_{k=1}^{N} (e^{j\omega} - p_k)}$$

$$|H(e^{j\omega})| = |K| \frac{\prod_{l=1}^{M} |e^{j\omega} - z_l|}{\prod_{k=1}^{N} |e^{j\omega} - p_k|}$$