EC 5142 Oct. 13 2011

Note Title 13-10-2011

Convolution in the time domain:

$$y[n] = x[n] * h[n]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} x[m] h[n-m] \right] z^{-n}$$

$$\overset{?}{=} \sum_{m=-\infty}^{\infty} x[m] \left[\sum_{n=-\infty}^{\infty} h[n-m] z^{-n} \right]$$

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$$= \sum_{m=-\infty}^{\infty} x[m] \, H(z) \, z^{-m}$$

$$= H(z) X(z) \qquad RoC \supseteq RoC_x \cap RoC_h$$

$$a^n u[n] * (-b^n u[-n-1]) \longleftrightarrow \frac{1}{(1-az^{-1})(1-bz^{-1})}$$

$$\downarrow \qquad \downarrow$$

$$\frac{1}{1-az^{-1}} \qquad \frac{1}{1-bz^{-1}} \qquad |a| < |z| < |b|$$

$|z| > |a| \qquad |z| < |b|$

$X(\theta) \equiv X(e^{j\theta})$

Multiplication in the time Domain

$$x[n]\,h[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\theta)H(\omega-\theta)\,d\theta$$

$$\sum_{n=-\infty}^{\infty} x[n]\,h[n]\,e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\theta)e^{j\theta n}\,d\theta\right]h[n]\,e^{-j\omega n}$$

$$\stackrel{?}{=} \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta) \left[\sum_{n=-\infty}^{\infty} h[n] e^{-j(\omega-\theta)n} \right] d\theta$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta) H(\omega-\theta) d\theta$$ circular convolution

What about corresponding z-transform property?

$$x[n]\, h[n] \stackrel{z}{\longleftrightarrow} ?$$

Will turn out to be complex convolution

[Complex Variables & Laplace Transform for Engineers
— Wilbur LePage, Dover]

<u>Initial Value Theorem</u> $x[n] = 0 \quad n < 0$

$$X(z) = x[0] + x[1]z^{-1} + x[2]z^{-2} + \dots$$

$$\lim_{z \to \infty} X(z) = x[0]$$

Final Value Theorem Let $v[n]$ be right sided,

i.e. $v[n] = 0 \quad n < M$

Let $x[n] = v[n] - v[n-1]$

$$X(z) = (1 - z^{-1}) V(z)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=M}^{\infty} \left(v[n] - v[n-1] \right) z^{-n}$$

$$\lim_{z \to 1} X(z) = \sum_{n=M}^{\infty}{}' \left(v[n] - v[n-1] \right)$$

$$= \lim_{N \to \infty} \sum_{n=M}^{N}{}' \left(v[n] - v[n-1] \right)$$

$$= \lim_{N \to \infty} \Big[v[M] - v[M-1] + v[M+1] - v[M] + \dots + v[N-1] + v[N] \Big]$$

$$= \lim_{N \to \infty} \left[-\underset{\substack{\downarrow \\ 0}}{v[N-1]} + v[N] \right]$$

$$= v[\infty]$$

$$= \lim_{z \to 1} X(z)$$

$$= \lim_{z \to 1} (1 - z^{-1}) V(z)$$

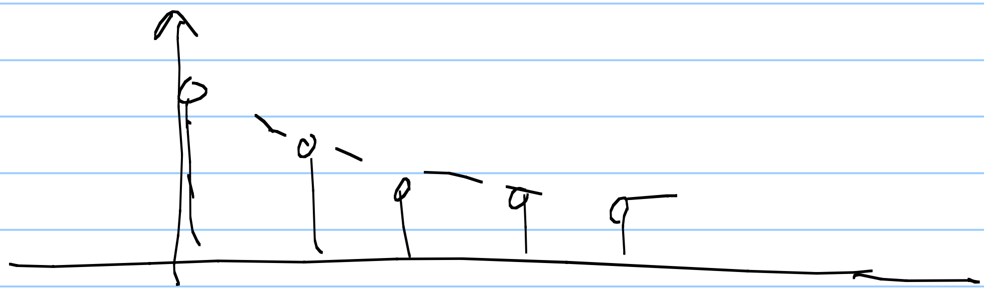
$$v[\infty] = \lim_{z \to 1} (1 - z^{-1}) V(z)$$

Eg: $x[n] = (1/3)^n u[n] + 2u[n] + (-1/2)^n u[n]$

$x[\infty] = 2$

Verify Final Value Theorem for above Example

$a^n u[n] \longleftrightarrow \dfrac{1}{1 - az^{-1}} \qquad |z| > |a|$



$a^n \xleftrightarrow{z}$ does not exist

$1 \xleftrightarrow{z}$ — do —