EC 5142 Oct 12 2011

Note Title 12-10-2011

$$\gamma^n x[n] \longleftrightarrow X(z/\gamma) \qquad |\gamma| r_1 < |z| < |\gamma| r_2$$

$$\sum_{n=-\infty}^{\infty} u[n] z^{-n} = \frac{1}{1-z^{-1}} = X(z) \quad |z| > 1$$

$$a^n u[n] \longleftrightarrow X(z/a) \qquad |z/a| > 1$$

$$\frac{1}{1-(z/a)^{-1}} = \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

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$$e^{j\omega_0 n} x[n] \longleftrightarrow X\left(z / e^{j\omega_0}\right) \quad \text{RoC is same}$$

$$\text{DTFT} : X(z)\Big|_{z = e^{j\omega}}$$

$$e^{j\omega_0 n} x[n] \xleftrightarrow{\text{DTFT}} X\left(e^{j\omega} / e^{j\omega_0}\right) = X\left(e^{j(\omega - \omega_0)}\right)$$

Differentiation in the Z-Domain

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$\frac{dX}{dz} = \sum_{n=-\infty}^{\infty} x[n] \left(-n z^{-n-1}\right)$$

$$-z \frac{dX}{dz} = \sum_{n=-\infty}^{\infty} n\, x[n]\, z^{-n}$$

$$n\,x[n] \longleftrightarrow -z\,\frac{dX}{dz}$$

RoC is same except possibly for the boundary circle

Eg

$$x[n] = \begin{cases} \frac{1}{n^2} & n \geq 1 \\ 0 & n \leq 0 \end{cases}$$

$X(z)$ — dilogarithm function $\quad |z| \geq 1$

$$n\,x[n] = \begin{cases} \frac{1}{n} & n \geq 1 \\ 0 & n \leq 0 \end{cases}$$

$X(z)$: RoC $|z| > 1$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

$$n\,a^n u[n] \longleftrightarrow -z \frac{d}{dz} \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

$$= (-z)\,\frac{-(-a)(-z^{-2})}{(1-az^{-1})^2}$$

$$= \frac{az^{-1}}{(1-az^{-1})^2} \qquad |z| > |a|$$

$$(n+1)\,a^{n+1}\,u[n+1] \longleftrightarrow \frac{a}{(1-az^{-1})^2} \qquad |z| > |a|$$

$$(n+1)\,a^{n}\,u[n+1] \longleftrightarrow \frac{1}{(1-az^{-1})^2} \qquad |z| > |a|$$

$$(n+1)a^n u[n] \longleftrightarrow \frac{1}{(1-az^{-1})^2} \qquad |z|>|a|$$

$$? \longleftrightarrow \frac{1}{(1-az^{-1})^m} \qquad |z|>|a|$$

Exponential mult.

$$\tfrac{1}{2} r^n e^{j\omega_0 n} u[n] \longleftrightarrow \frac{1/2}{1-\left(\frac{z}{re^{j\omega_0}}\right)^{-1}} \qquad |z|>r$$

$$= \frac{1/2}{1 - r e^{j\omega_0} z^{-1}}$$

$$\frac{1}{2} r^n e^{-j\omega_0 n} u[n] \longleftrightarrow \frac{1/2}{1 - r e^{-j\omega_0} z^{-1}} \qquad |z| > r$$

$$r^n \cos \omega_0 n \; u[n] \longleftrightarrow \frac{1 - r \cos\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$$

$$\left(1 - r e^{j\omega_0} z^{-1}\right)\left(1 - r e^{-j\omega_0} z^{-1}\right)$$

Suppose $x[n]$ is real-valued. $x[n] = x^*[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x^*[n]\, z^{-n}$$

$$= \left[\sum_{n=-\infty}^{\infty} x[n] \left(z^*\right)^{-n} \right]^*$$

$$= X^*(z^*)$$

$$X^*(z) = X(z^*)$$

Suppose z_0 is a root of $X(z)$.

$$X(z_0) = 0$$

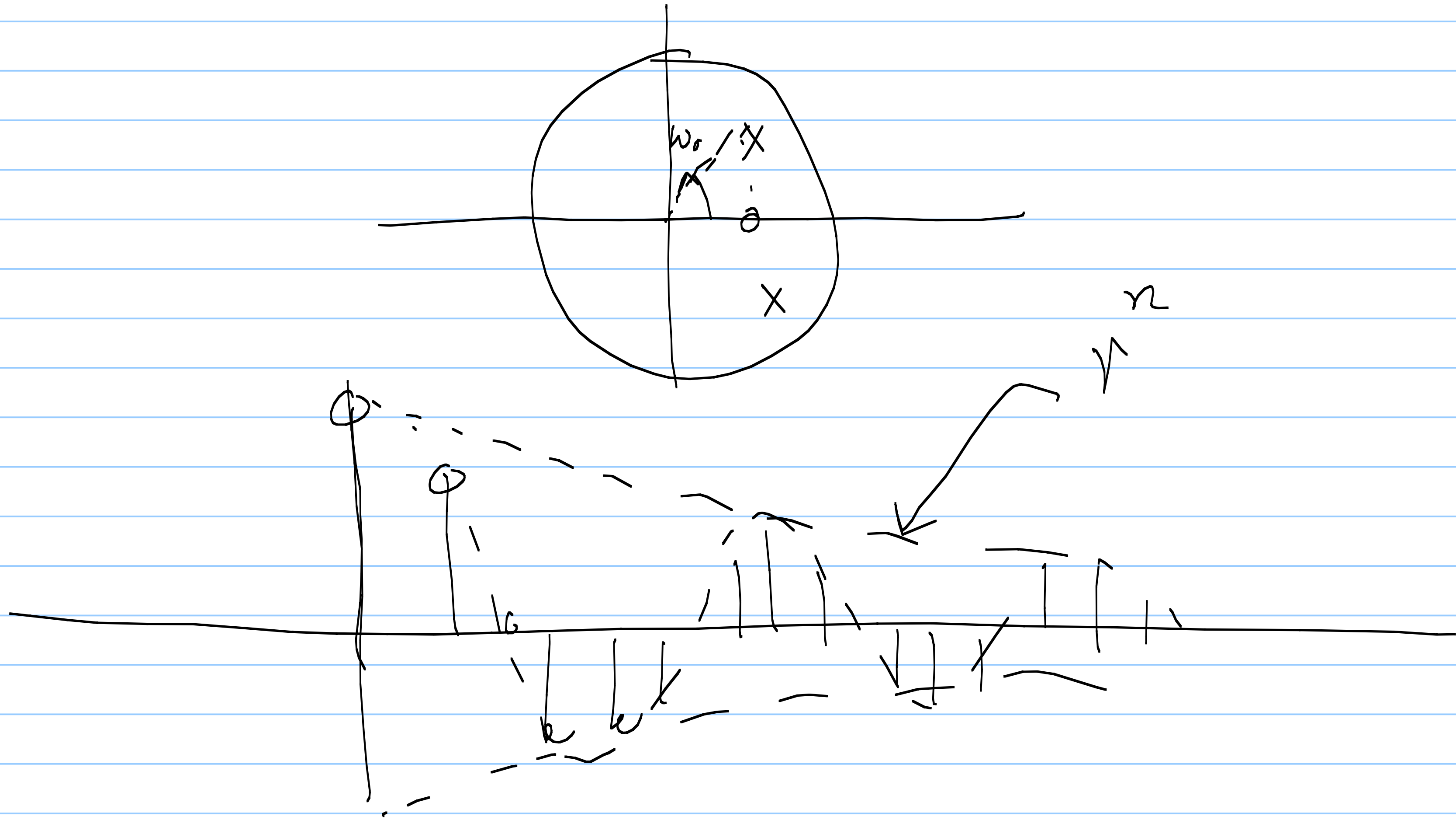
$$X^*(z_0) = 0$$

$$X(z_0^*) = 0$$

$\Rightarrow$ z_0^* is also a root

$$X(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

$b_k, a_k \in \mathbb{R}$



Time Reversal

$$x[n] \longleftrightarrow X(z) \qquad r_1 < |z| < r_2$$

$$x[-n] \longleftrightarrow X(z^{-1}) \qquad 1/r_2 < |z| < 1/r_1$$

$$a^{n} u[n] \longleftrightarrow \frac{1}{1 - a z^{-1}} \qquad |z| > |a|$$

$$a^{-n} u[-n] \longleftrightarrow \frac{1}{1 - a z} \qquad |z| < 1/|a|$$

$$= \frac{-a^{-1} z^{-1}}{1 - a^{-1} z^{-1}}$$

$$a^{-n-1} u[-n-1] \longleftrightarrow \frac{-a^{-1}}{1 - a^{-1} z^{-1}} \qquad |z| < |a^{-1}|$$

$$-(a^{-1})^{n} u[-n-1] \longleftrightarrow \frac{1}{1 - a^{-1} z^{-1}} \qquad |z| < |a^{-1}|$$

$$x(-n) \longleftrightarrow X(z^{-1}) \qquad 1/r_2 < |z| < 1/r_1$$

$$(-1)^n x(n) \longleftrightarrow X(-z) \qquad \text{RoC is same}$$

Convolution in the Time Domain

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

$$= x[n] * h[n]$$

$$Y(z) = X(z)\,H(z) \qquad ROC \supseteq ROC_x \cap ROC_h$$