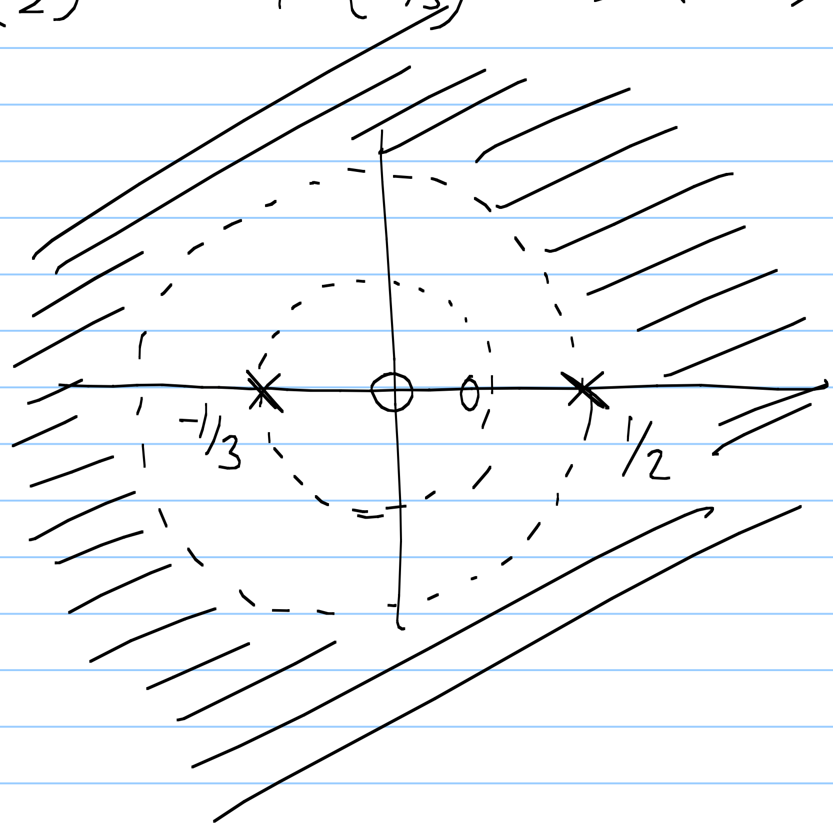
EC 5142 Oct. 11, 2011

Note Title 11-10-2011

$$\left(\tfrac{1}{2}\right)^n u[n] + \left(-\tfrac{1}{3}\right)^n u[n] \longleftrightarrow \frac{2\left(1-\frac{1}{12}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)} \qquad |z| > \tfrac{1}{2}$$



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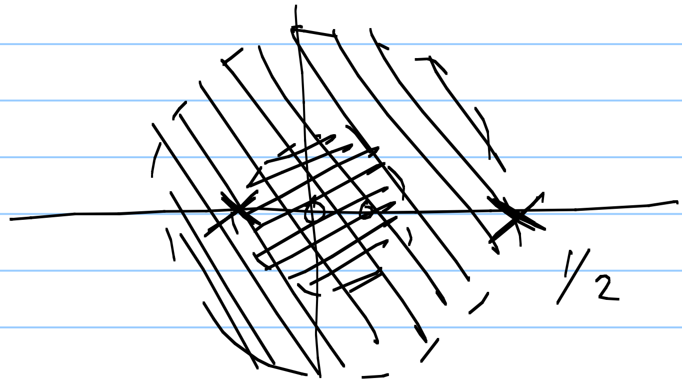
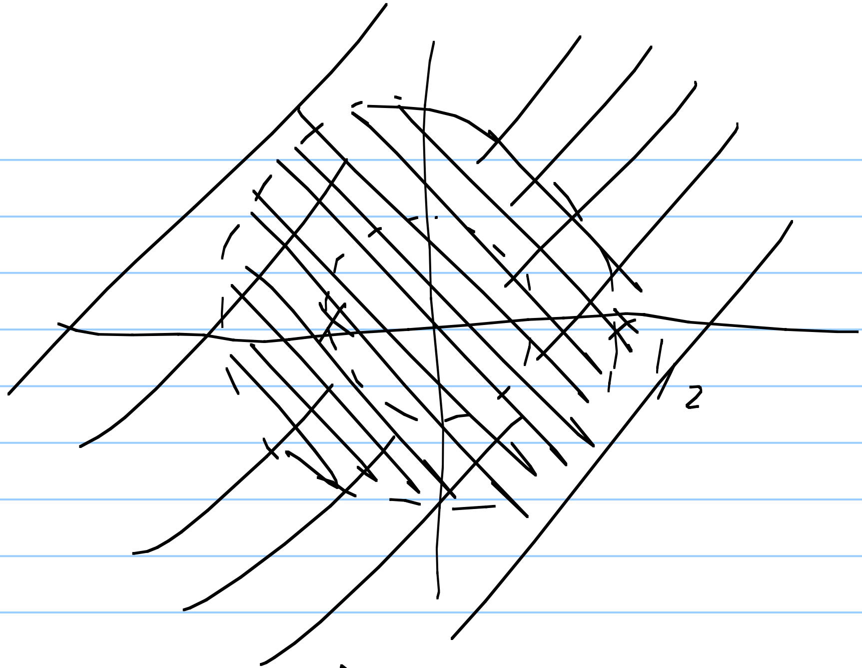


$$|z| < 1/3 \ \cap \ |z| < 1/2 \ = \ |z| < 1/3$$

$$-\left(-1/3\right)^n u[-n-1] \qquad -\left(1/2\right)^n u[-n-1]$$



$$|z| > 1/3 \cap |z| < 1/2$$

$$= 1/3 < |z| < 1/2$$

$$\left(-1/3\right)^n u[n] \qquad -\left(1/2\right)^n u[-n-1]$$

Time Delay

$$x[n-n_0] \longleftrightarrow z^{-n_0} X(z)$$

RoC is identical except possibly for addition or deletion of 0 and/or ∞

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z|>|a|$$

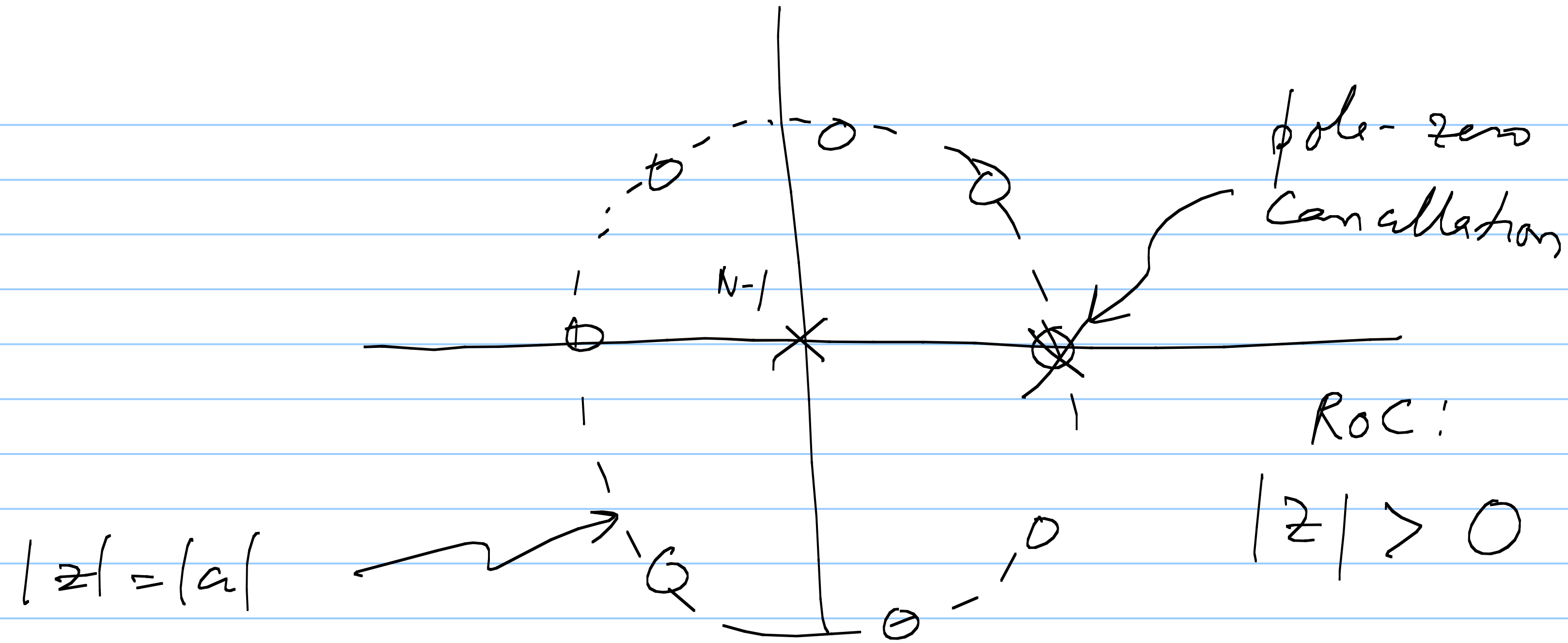
$$a^{n-N} u[n-N] \longleftrightarrow \frac{z^{-N}}{1-az^{-1}}$$

$$a^n u[n-N] \longleftrightarrow \frac{a^N z^{-N}}{1-az^{-1}} \qquad |z| > |a|$$

$$a^n \left[u[n] - u[n-N]\right] \longleftrightarrow \frac{1-a^N z^{-N}}{1-az^{-1}}$$

$1, a, a^2, \dots, a^{N-1}$

$$= \frac{z^N - a^N}{z^{N-1}(z-a)}$$



pole or zero at the origin: "trivial" pole / zero

Uncancelled nontrivial pole: $\frac{1}{1-az^{-1}}$

gives rise to either $a^n u[n]$ or $-a^n u[-n-1]$

(depends on RoC) $\Rightarrow$ ∞-duration sequence.

If there are only trivial poles, then

the sequence is finite duration

$$1, a, a^2, \ldots, a^{N-1} \longleftrightarrow 1 + a z^{-1} + a^2 z^{-2} + \cdots + a^{-(N-1)} z^{-(N-1)}$$

"All zero filter"

$$\frac{z^{N-1} + a z^{N-2} + \cdots + a^{-(N-1)}}{z^{N-1}}$$

N-1 zeros

N-1 trivial poles

$$X(z) = \frac{1}{1 + a_1 z^{-1} + \cdots + a_{N-1} z^{-(N-1)}}$$

"All pole filter"

$$X(z) = \frac{P(z)}{Q(z)}$$ "pole zero filter"

$$\frac{1}{(1-az^{-1})(1-bz^{-1})} = \frac{z^2}{(z-a)(z-b)}$$

$$\frac{z}{(z-a)(z-b)} \Rightarrow$$

poles: $z = a, b$

zero: $z = 0, \infty$

$$\frac{(z-a)(z-b)}{z}$$

zeros: $z = a, b$

poles: $z = 0, \infty$

Exponential Multiplication

$$x[n] \longleftrightarrow X(z) \qquad r_1 < |z| < r_2$$

$$a^n x[n] \longleftrightarrow \sum_{n=-\infty}^{\infty} x[n] \left(z/a\right)^{-n} \longleftrightarrow X\left(z/a\right)$$

$$r_1 < \left|z/a\right| < r_2$$

$$|a| r_1 < |z| < |a| r_2$$

Suppose z_0 is a zero of $X(z)$, i.e. $X(z_0) = 0$

$$y[n] = a^n x[n] \longleftrightarrow Y(z) = X(z/a)$$

$$Y(a z_0) = X(a z_0 / a) = X(z_0) = 0$$

$\Rightarrow$ $a z_0$ is a root of $Y(z)$