EC 5142 Oct. 10 2011

Note Title 10-10-2011

The RoC for a general $x[n]$ is of the form

$$r_1 < |z| < r_2$$

r_1 can be as small as zero

r_2 can be as large as ∞

For finite duration sequences, $X(z)$ is

$$x[-M]z^{M} + x[-M+1]z^{-M+1} + \dots + x[-1]z + x[0]$$
$$x[1]z^{-1} + x[2]z^{-2} + \dots + x[N]z^{-N}$$

$z = 0 \notin RoC$ $\quad z = \infty \notin RoC$

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RoC = entire Z-plane except 0 and/or ∞

In general if $z = re^{j\omega} \in RoC$, then

$re^{j\omega} \in RoC \quad \forall \omega \in [-\pi, \pi)$

$$X(z) = \frac{P(z)}{Q(z)}$$

$P(z)$ & $Q(z)$ are polynomials in z

then $X(z)$ is said to be rational

roots of $P(z)$ are called ZEROS of $X(z)$

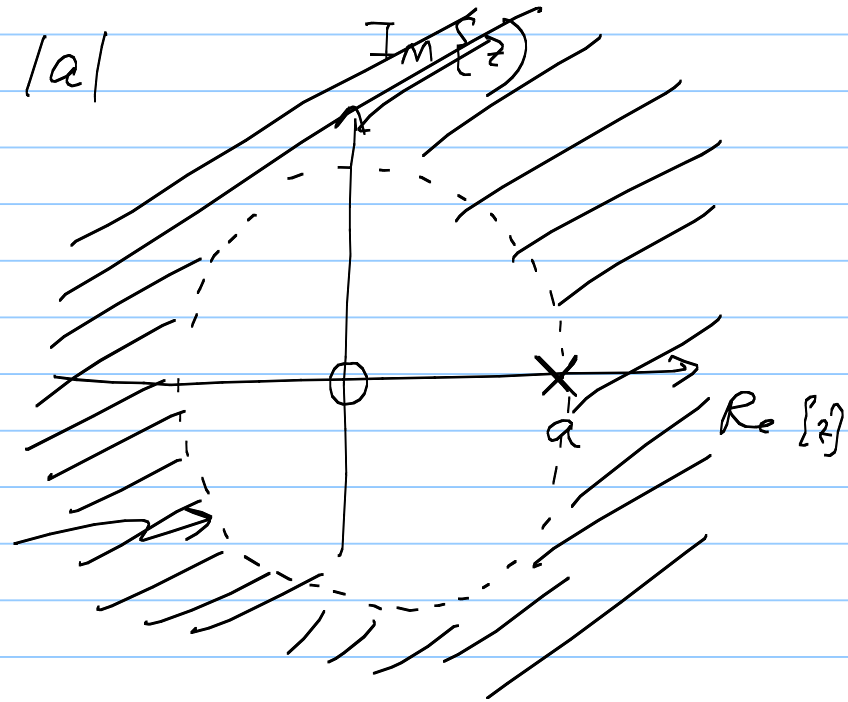
roots of $Q(z)$ are called POLES of $X(z)$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

$$= \frac{z}{z-a}$$

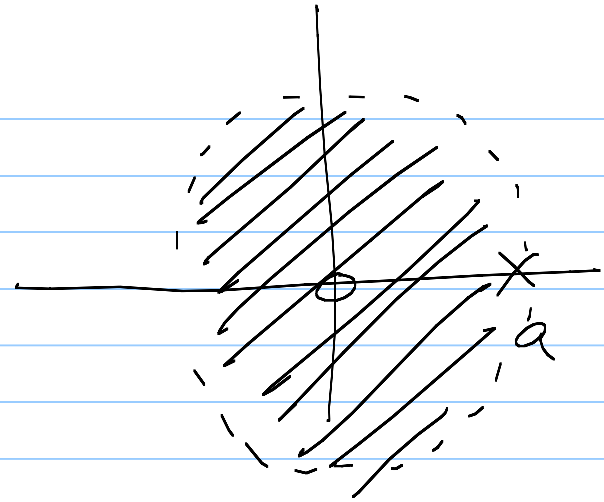
pole: $z = a$

zero: $z = 0$



$|z| = |a|$

$-a^n u[-n-1] \longleftrightarrow \dfrac{1}{1-az^{-1}} \quad |z|<|a|$



$\boxed{\text{RoC cannot contain poles}}$

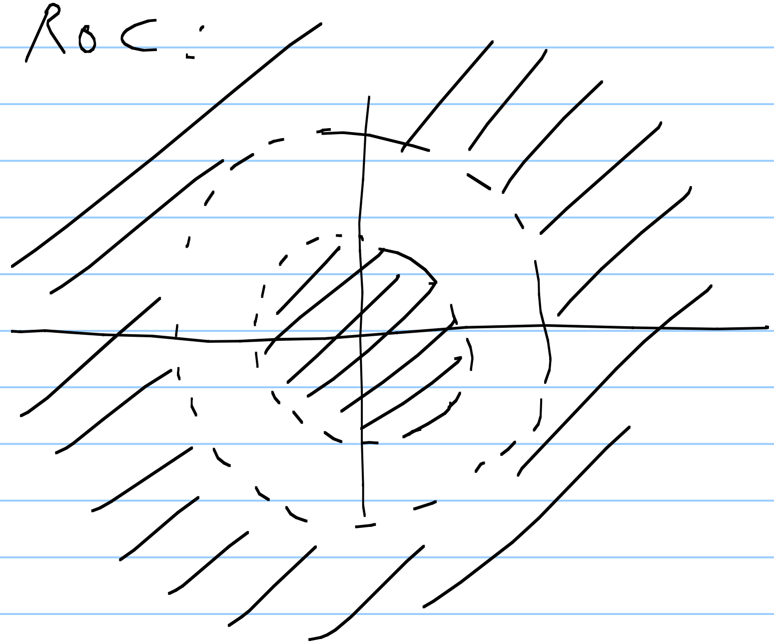
For rational $X(z)$:

(a) RoC is an annular region

(b) RoC cannot contain poles

(c) RoC must be a connected region

Invalid RoC:



(d) For finite duration sequences, RoC is the entire z-plane except possibly for 0 and/or ∞

(e) For right sided sequences, the RoC is a circle with radius R_1 s.t. RoC is of the from $R_1 < |z| \leq \infty$

(f) For left sided sequences, the RoC is bounded by a circle of radius R_2 s.t. the RoC is of the from $0 \leq |z| < R_2$

$X(z)$ is an analytic function in the RoC

$$z = x + jy$$

$$X(z) = U(x, y) + j\,V(x, y)$$

$$\left.\begin{aligned} U_x &= V_y \\ U_y &= -V_x \end{aligned}\right\} \text{Cauchy-Riemann Equations}$$

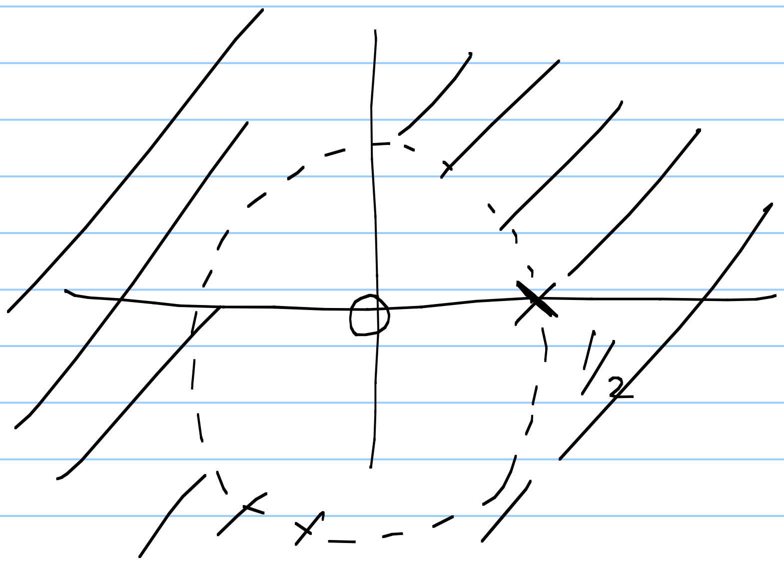
<u>Properties</u>

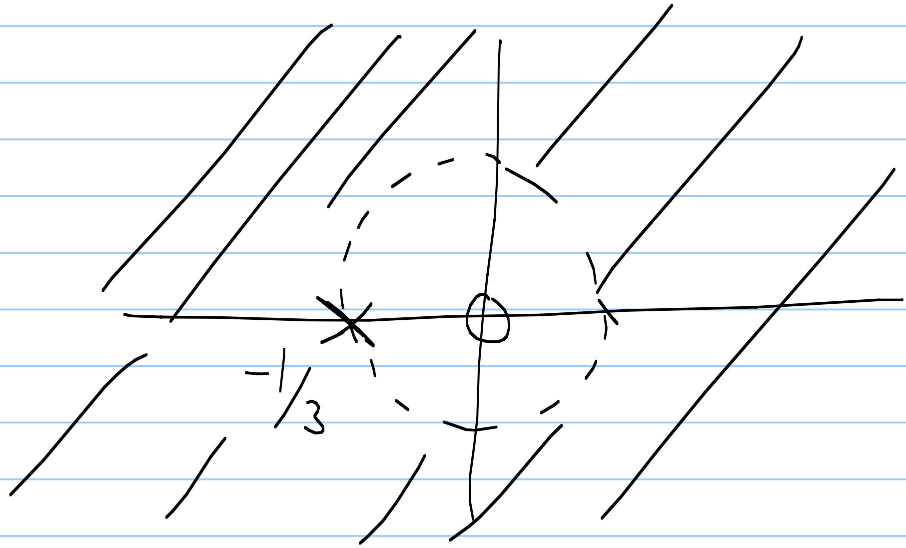
$$a_1 x_1[n] + a_2 x_2[n] \longleftrightarrow a_1 X_1(z) + a_2 X_2(z)$$

$$RoC \supseteq RoC_{x_1} \cap RoC_{x_2}$$

$$\left(\tfrac{1}{2}\right)^n u[n] \longleftrightarrow \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |z| > \tfrac{1}{2}$$

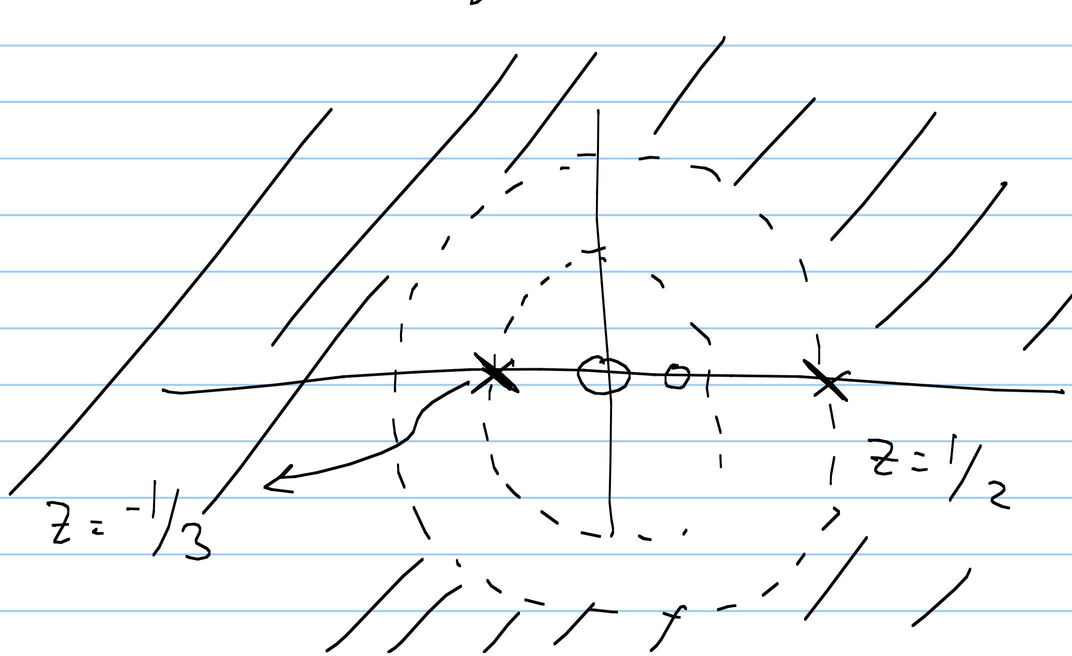
$$\left(-\tfrac{1}{3}\right)^n u[n] \longleftrightarrow \frac{1}{1+\frac{1}{3}z^{-1}} \qquad |z| > \tfrac{1}{3}$$





$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{3}z^{-1}} = \frac{2\left(1-\frac{1}{12}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}$$

$$|z| > \tfrac{1}{2} \quad \cap \quad |z| > \tfrac{1}{3} \qquad |z| > \tfrac{1}{2}$$



$$\frac{2z\left(z-\frac{1}{12}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$