Note Title 04-10-2011

EC5142 Oct. 04 2011

$$z_0^n \longrightarrow \underset{LTI}{\boxed{h[n]}} \longrightarrow y[n]$$

$$y[n] = \sum_{m=-\infty}^{\infty} z_0^{n-m} h[m]$$

$$= z_0^n \underbrace{\sum_{m=-\infty}^{\infty} h[m] z_0^{-m}}_{H(z_0)}$$

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$$H(z) \equiv \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$ Z-transform of $h[n]$

$$h[n] \xleftrightarrow{Z} H(z)$$

Eg: $x(n) = a^n u[n]$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= \frac{1}{1 - a z^{-1}} \qquad |a z^{-1}| < 1$$

$$|az^{-1}| < 1 = |a| < |z| = \underbrace{|z| > |a|}$$

Region of convergence (RoC)

Eg:

$$x[n] = -a^{n} u[-n-1]$$

$$X(z) = -\sum_{n=-\infty}^{-1} a^{n} z^{-n} = \frac{1}{1-az^{-1}} \quad |z| < |a|$$

Recall: $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \qquad s = \sigma + j\Omega$

$$e^{+at} u(t) \longleftrightarrow \frac{1}{s-a} \qquad Re\{s\} > Re\{a\}$$

$$-e^{+at} u(-t) \longleftrightarrow \frac{1}{s-a} \qquad Re\{s\} < Re\{-a\}$$

$$a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - ae^{-j\omega}} \qquad |a| < 1$$

$2^n u[n]$ does <u>NOT</u> possess DTFT

$$2^n u[n] \overset{Z}{\longleftrightarrow} \frac{1}{1-2z^{-1}} \qquad |z| > |2|$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = X(\omega)$$

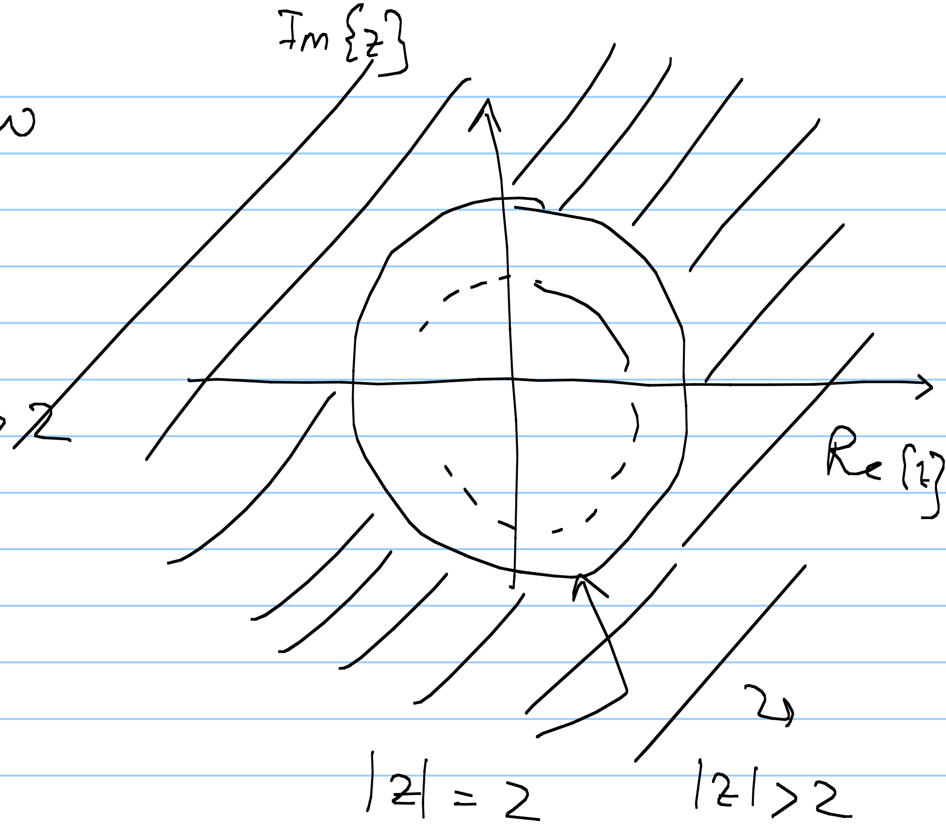
$$X(e^{j\omega}) = X(z)\Big|_{z=e^{j\omega}}$$

$$2^n u[n] \longleftrightarrow \frac{1}{1-2z^{-1}} \quad |z|>2$$

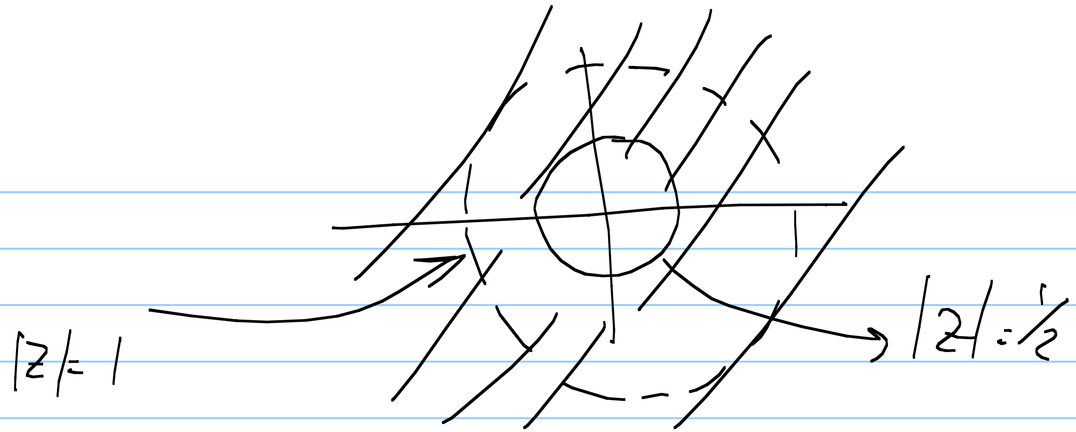
DTFT does not exist since

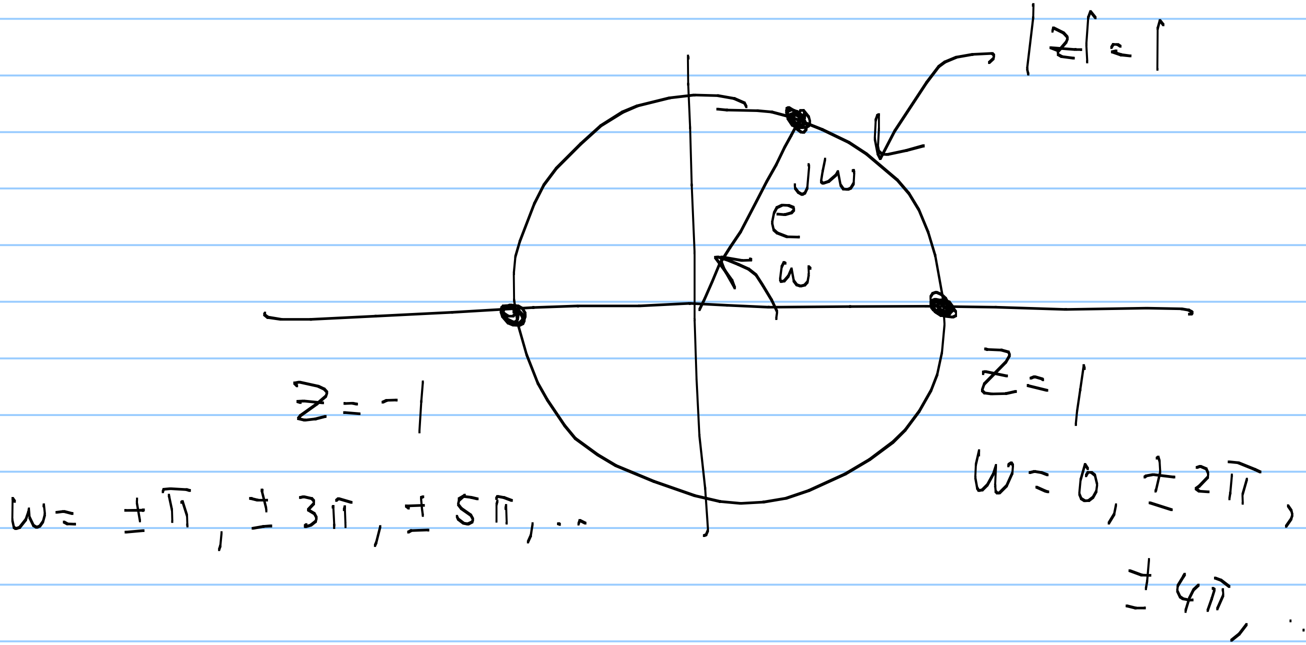
$$z = e^{j\omega} \notin \text{RoC}$$

$$\left(\tfrac{1}{2}\right)^n u[n] \longleftrightarrow \frac{1}{1-\frac{1}{2}z^{-1}} \quad |z|>\tfrac{1}{2}$$



$$\frac{1}{2}^{n} u[n] \longleftrightarrow \frac{1}{1-\frac{1}{2}e^{-jw}}$$





$$\sum_{n=-\infty}^{\infty} \underbrace{x[n]\, r^{-n}}\; e^{-j\omega n} \qquad z = r e^{j\omega}$$

$$2^{n} u[n] \longrightarrow \sum_{n=0}^{\infty} \left(\frac{2}{r}\right)^{n} e^{-j\omega n} \quad \text{exists for } r > 2$$

In general,

$$\sum_{n=-\infty}^{\infty} x[n]\, z^{-n} = \sum_{n=-\infty}^{\infty} x[n]\, r^{-n} e^{-j\omega n} = X(z)$$

$$|X(z)| = \left| \sum_{n=-\infty}^{\infty} x[n]\, r^{-n} e^{-j\omega n} \right|$$

$$\leq \sum_{n=-\infty}^{\infty} |x[n]|\, r^{-n} < \infty \Rightarrow \text{transform exists}$$

$$\sum_{n=0}^{\infty} \frac{|x[n]|}{r^{n}} + \sum_{n=-\infty}^{-1} \frac{|x[n]|}{r^{n}}$$

$$= \underbrace{\sum_{n=0}^{\infty} \frac{|x[n]|}{r^{n}}}_{\text{causal part}} + \underbrace{\sum_{n=1}^{\infty} |x[-n]|\, r^{n}}_{\text{anticausal part}}$$

(A) (B)

Suppose (A) exist for $r = r_1$

then (A) surely exist for all $r > r_1$

Suppose (B) exist for $r = r_2$

then (B) Surely exists for all $r < r_2$

$\therefore$ The RoC is, in general of the form

$$r_1 < |z| < r_2$$