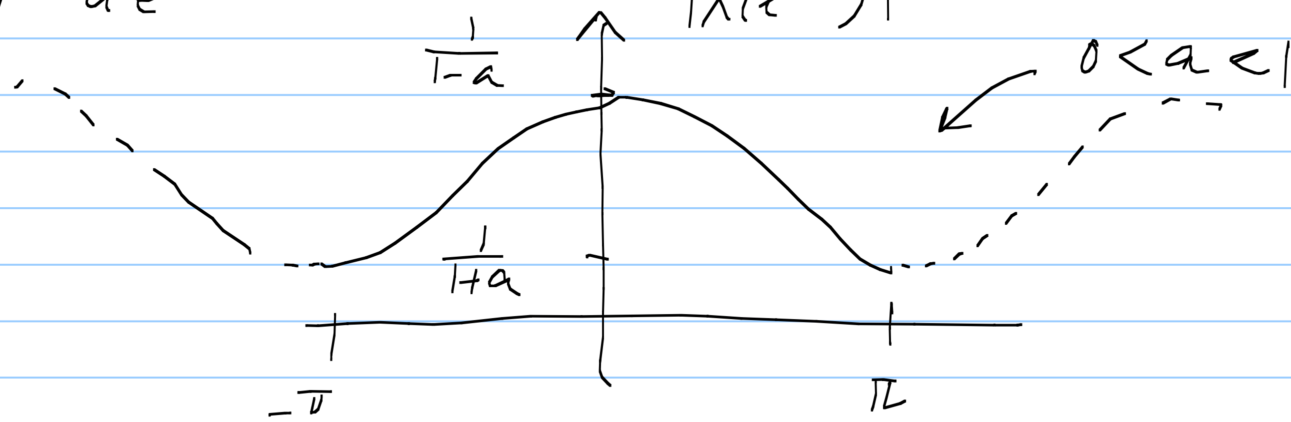
EC 5142 Oct. 03 2011

$$a^n u[n] \longleftrightarrow X(e^{j\omega})$$

$$\downarrow$$

$$\sum_{n=0}^{\infty} a^n e^{-j\omega n} = \frac{1}{1-ae^{-j\omega}}$$

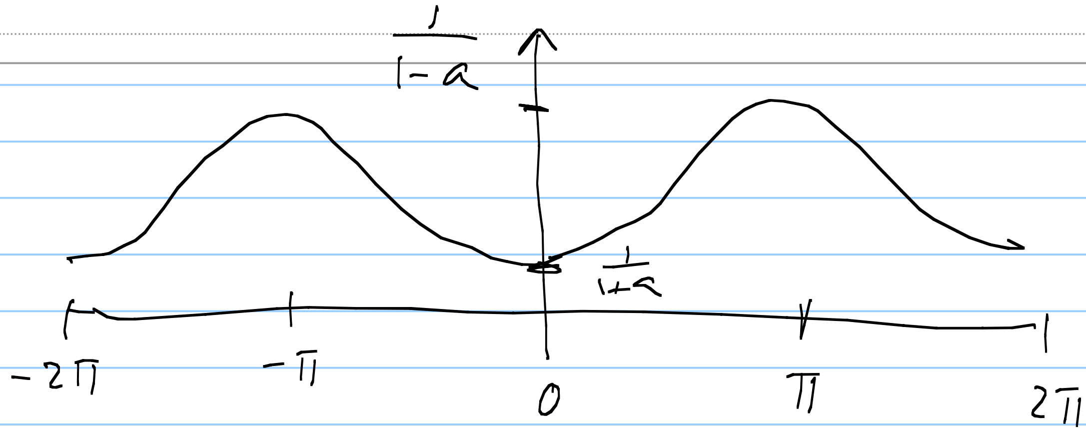
$|a| < 1$



$$(-1)^n a^n u[n]$$

$$\longleftrightarrow \frac{1}{1+ae^{-j\omega}}$$

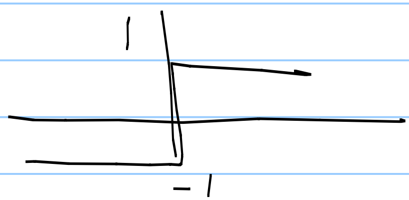
Note Title 03-10-2011



$u[n] \longleftrightarrow\ ?$

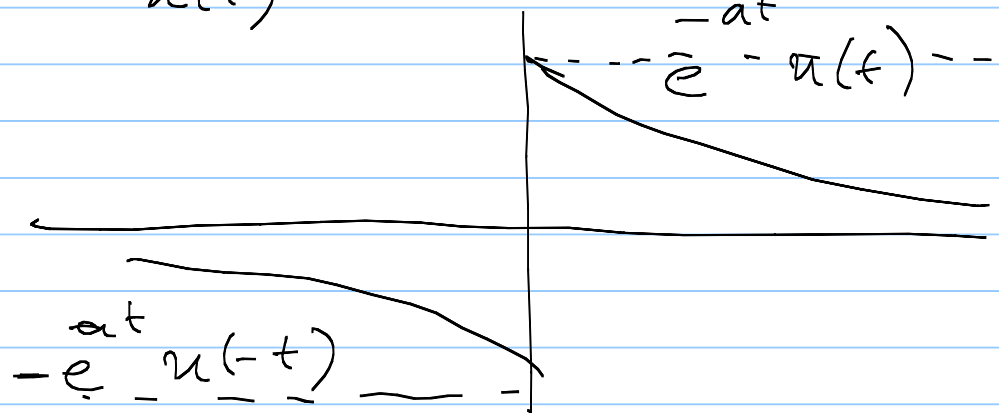
Recall:

$$\mathrm{sgn}(t) \longleftrightarrow \frac{2}{j\Omega}$$



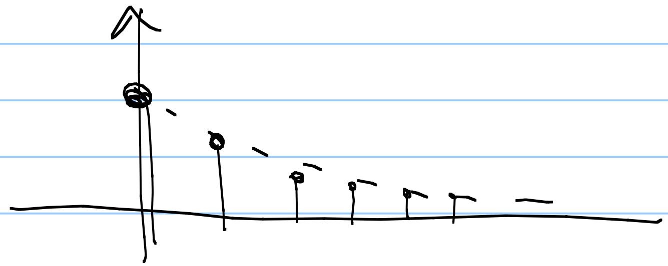
$$\frac{1}{2}\operatorname{sgn}(t) \longleftrightarrow \frac{1}{j\Omega}$$

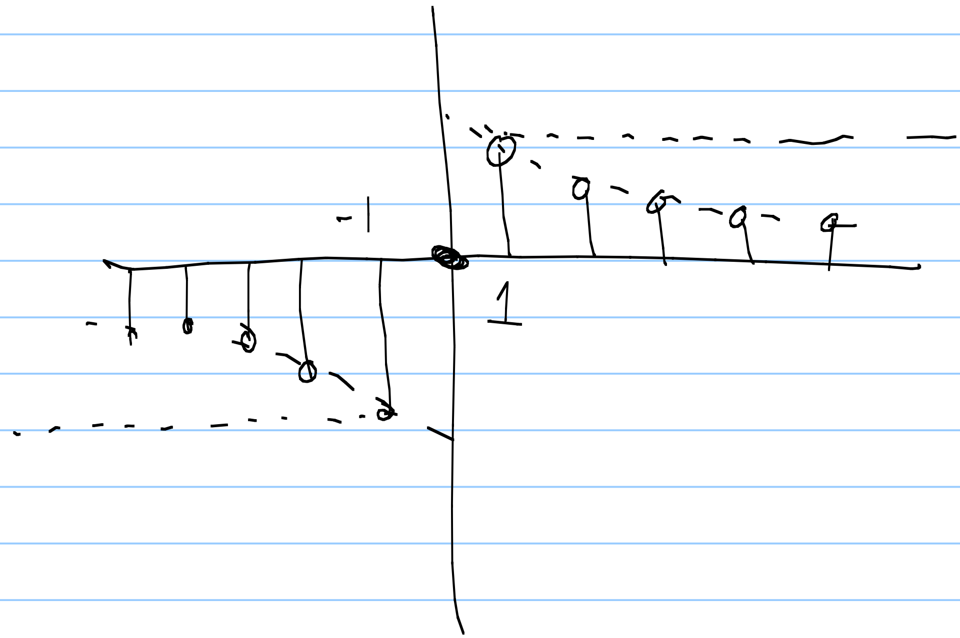
$$\underbrace{\frac{1}{2}\operatorname{sgn}(t) + \frac{1}{2}}_{u(t)} \longleftrightarrow \frac{1}{j\Omega} + \pi\,\delta(\Omega)$$



Take limit as $a \to 0$

$$a^{n}\, u[n] \longleftrightarrow \frac{1}{1-ae^{-j\omega}}$$



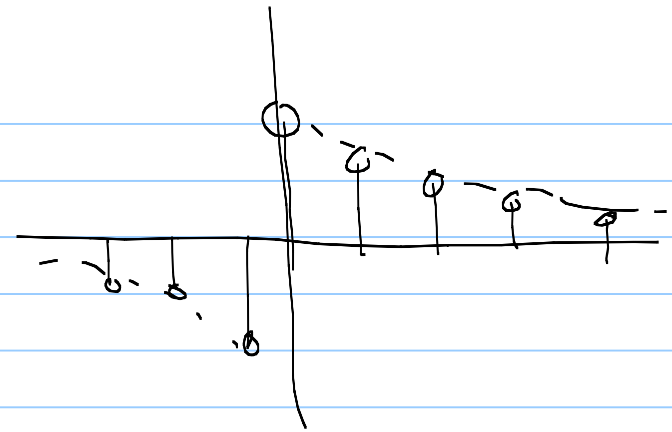


$$x[n] \longleftrightarrow X(e^{j\omega})$$

$$x[-n] \longleftrightarrow X(e^{-j\omega})$$

$$a^{-n}\, u[-n] \longleftrightarrow \frac{1}{1-ae^{j\omega}}$$

$$a^{-(n+1)} u[-n-1] \longleftrightarrow \frac{e^{jw}}{1-ae^{jw}}$$



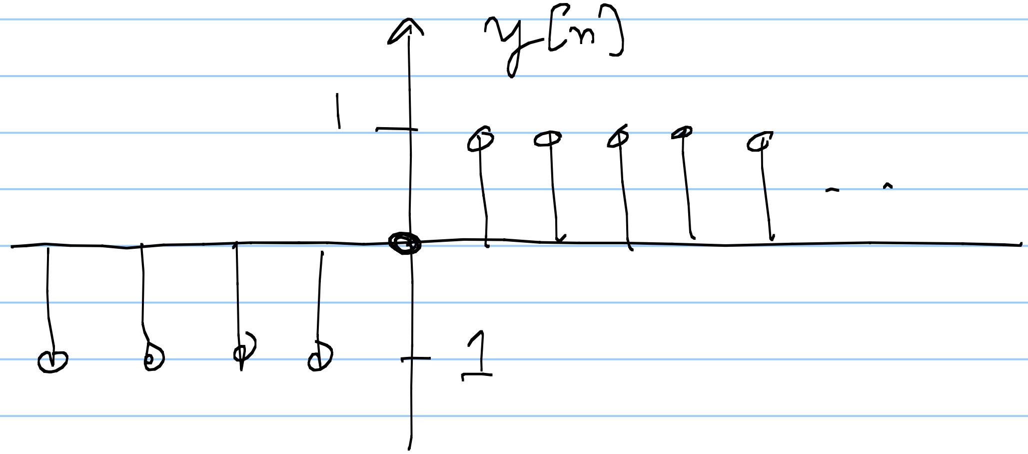
$$x[-n-n_0] \longleftrightarrow e^{-jwn_0} X(e^{jw})$$

$$a^{-n} u[-n-1] \longleftrightarrow \frac{ae^{jw}}{1-ae^{jw}}$$

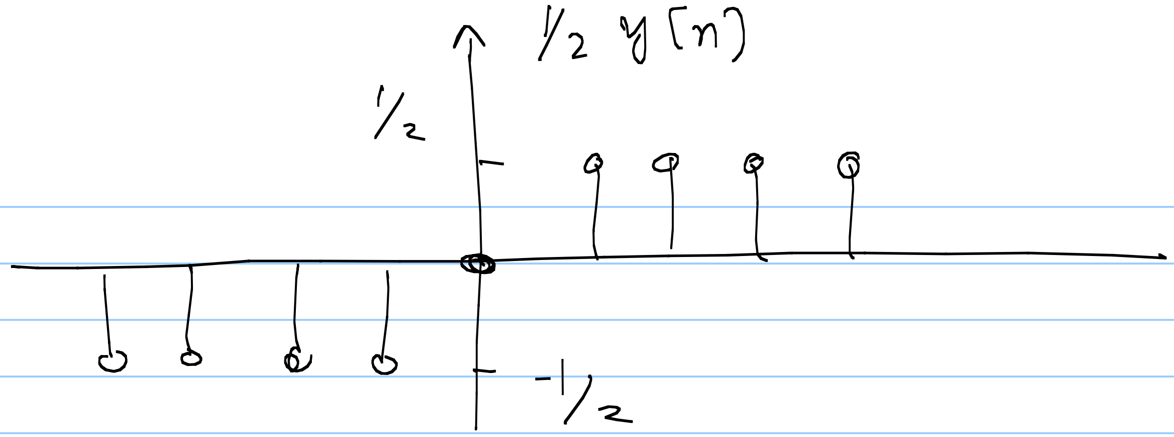
$$a^{n} u[n] - a^{-n} u[-n-1] \longleftrightarrow \frac{1}{1-ae^{-jw}} - \frac{ae^{jw}}{1-ae^{jw}}$$

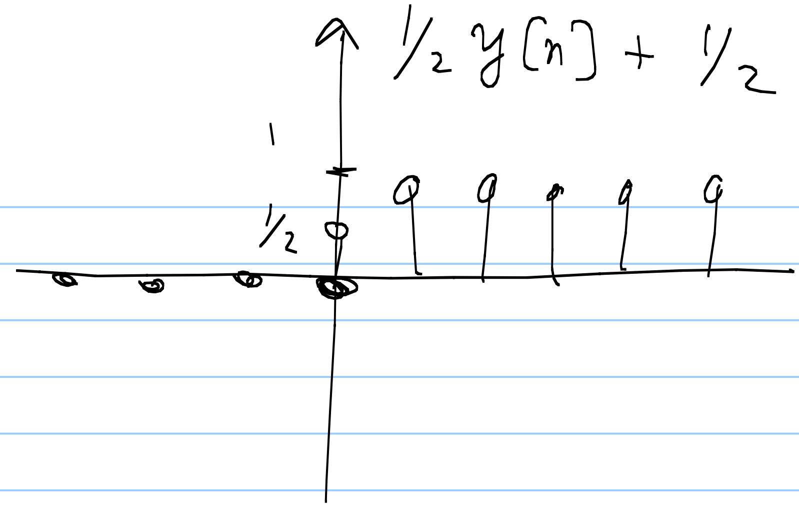
$$a^{n} u[n] - a^{-n} u[-n-1] - \delta[n] \longleftrightarrow \frac{1}{1-ae^{-j\omega}} - \frac{ae^{j\omega}}{1-ae^{j\omega}} - 1$$

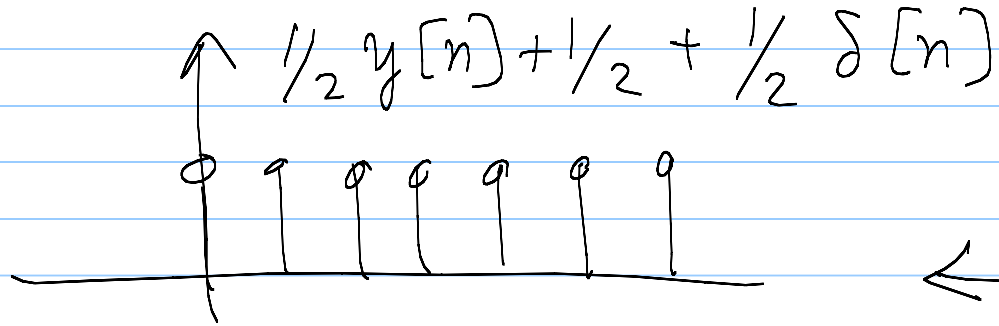
Take Limit as $a \to 1$ & simplify



$$\longleftrightarrow \quad \frac{-j\sin\omega}{1-\cos\omega}$$







$$\longleftrightarrow \quad \frac{1}{1-e^{-j\omega}} + \pi\delta(\omega)$$

$$-\pi \leq \omega < \pi$$

$$\longleftrightarrow \frac{1}{1-e^{-jw}} + \sum_{k=-\infty}^{\infty} \pi \, \delta(w - 2\pi k)$$

$$X(e^{jw}) \equiv X(w)$$

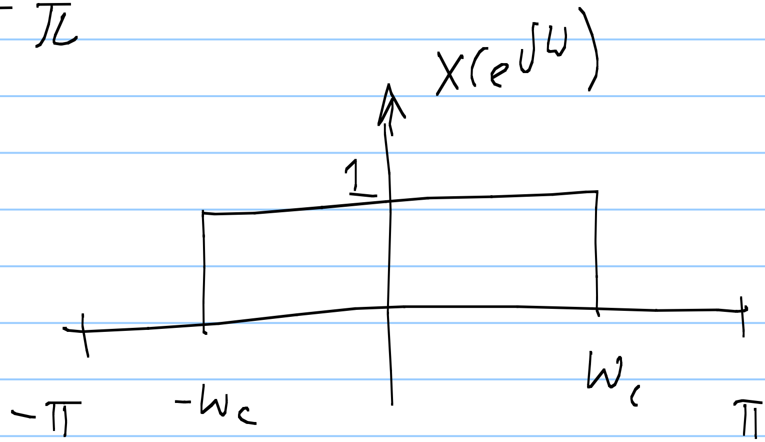
<u>Properties</u> (cont'd)

$$x[n] * y[n] \longleftrightarrow X(e^{jw}) \, Y(e^{jw})$$

$$x[n] \, y[n] \longleftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta) \, Y(w-\theta) \, d\theta$$

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

$$\frac{\sin \omega_c n}{\pi n} \longleftrightarrow$$



$$\sum_{n=-\infty}^{\infty} x[n] = X(e^{j\omega})\Big|_{\omega=0}$$

$$\sum_{n=-\infty}^{\infty} \frac{\sin \omega_c n}{\pi n} = 1$$

$$\sum_{n=-\infty}^{\infty} \frac{\sin^2 \omega_c n}{\pi^2 n^2} = \frac{\omega_c}{\pi}$$