EC 5142 Nov. 17 2011

Note Title 17-11-2011

FFT Algorithm

$$\underline{X} = \underline{W}\,\underline{x}$$

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ & & & & \\ 1 & & & & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} X_0 \\ X_1 \\ \\ \\ X_{N-1} \end{bmatrix}$$

$N \times N$ $\qquad$ $N \times 1$ $\qquad$ $N \times 1$

$$W_N = e^{-j\frac{2\pi}{N}}$$

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$$e^{-j\frac{2\pi kn}{N}} \longrightarrow n = 0, 1, \ldots, N-1$$

$$\Big| \; k = 0, 1, \ldots N-1$$

Requires roughly N^2 mult. & N^2 add

Decimation in Time Algorithm

$$X[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}} \qquad k = 0, 1, \ldots, N-1$$

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

Split the i/p seq into odd & even indices

$$= \sum_{l=0}^{\frac{N}{2}-1} x[2l]\, e^{-j\frac{2\pi}{N}(2l)k} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\, e^{-j\frac{2\pi k}{N}(2r+1)}$$

$$= \sum_{l=0}^{\frac{N}{2}-1} g[l]\, e^{-j\frac{2\pi lk}{N/2}} + e^{-j\frac{2\pi k}{N}} \sum_{r=0}^{\frac{N}{2}-1} h[r]\, e^{-j\frac{2\pi kr}{N/2}}$$

$$= \sum_{l=0}^{\frac{N}{2}-1} g[l]\, W_{\frac{N}{2}}^{lk} + e^{-j\frac{2\pi k}{N}} \sum_{r=0}^{\frac{N}{2}-1} h[r]\, W_{\frac{N}{2}}^{kr}$$

$k = 0, 1, \ldots, N-1$

$G[k]$ $\quad W_N^k \quad$ $H[k]$

$$N^2 \longrightarrow N + 2\left(\frac{N}{2}\right)^2$$

$N = 8 \qquad N^2 = 64$

$8 + 2 \cdot 16 = 40$

$$\left(\frac{N}{2}\right)^2 \longrightarrow \frac{N}{2} + 2\left(\frac{N}{4}\right)^2$$

$$N^2 \longrightarrow N + 2\left[\frac{N}{2} + 2\left(\frac{N}{4}\right)^2\right]$$

$$= N + N + 4\left(\frac{N}{4}\right)^2$$

$$N = 2^L \Rightarrow L = \log_2 N$$

We have L stages

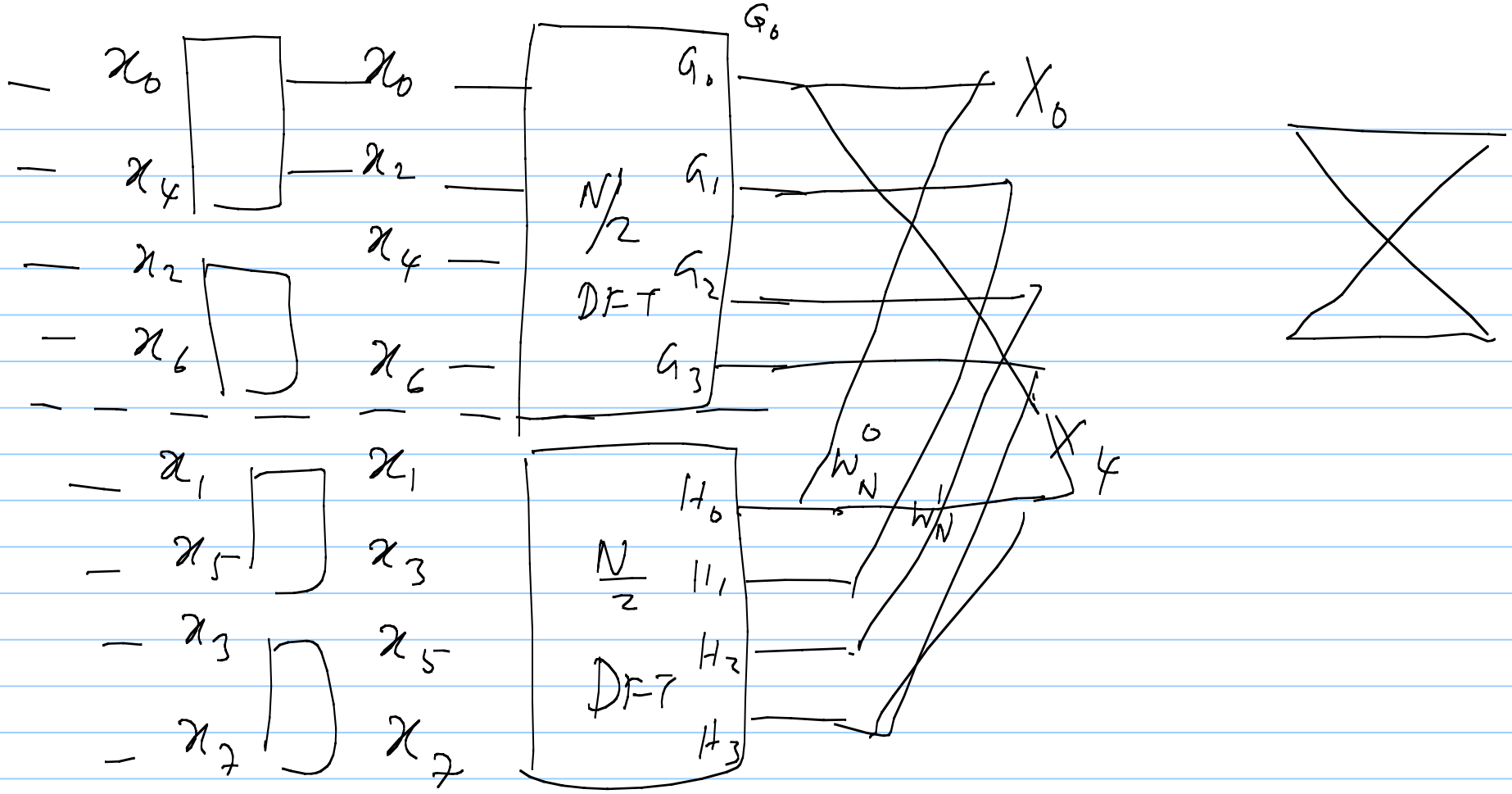
$$N^2 \longrightarrow NL = N \log_2 N$$

$N = 8$

2 pt. DFT

$x_0 + x_1$

$x_0 - x_1$


x_0
x_4
x_2
x_6
x_1
x_5
x_3
x_7
x_0
x_2
x_4
x_6
x_1
x_3
x_5
x_7
G_0
G_0
G_1
G_2
G_3
$N/2$
DFT
H_0
H_1
H_2
H_3
$N/2$
DFT
X_0
X_4
W_N^0
W_N^1

x_0	000	000	0
x_4	001	100	4
x_2	010	010	2
x_6	011	110	6
x_1	100	001	1
x_5	101	101	5
x_3	110	011	3
x_7	111	111	7