EC 5142 Nov. 16 2011

Note Title 16-11-2011

$$x_c(t) \xleftrightarrow{CTFT} X_c(F)$$

$$x_s(t) \xleftrightarrow{CTFT} X_s(F) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(F - kF_s)$$

$$X_s(F) = \sum_{n=-\infty}^{\infty} x_c(nT_s) e^{-j 2\pi \frac{F}{F_s} n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \qquad x[n] \equiv x_c(nT_s)$$

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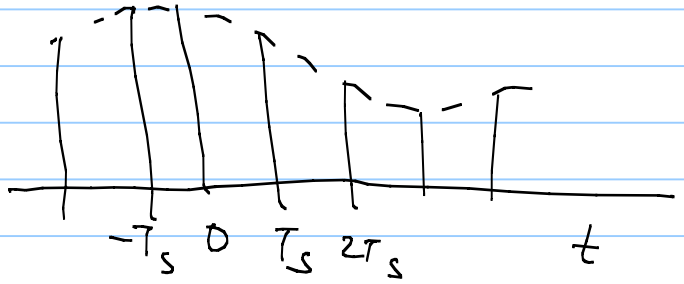
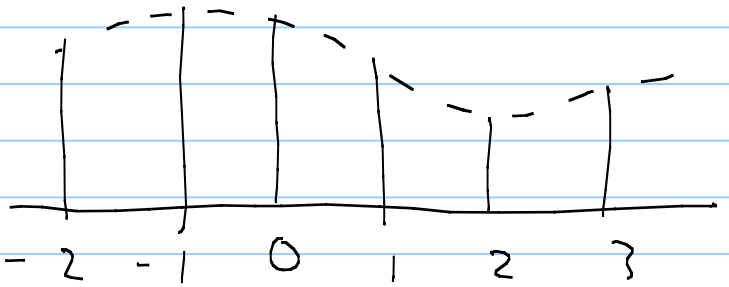
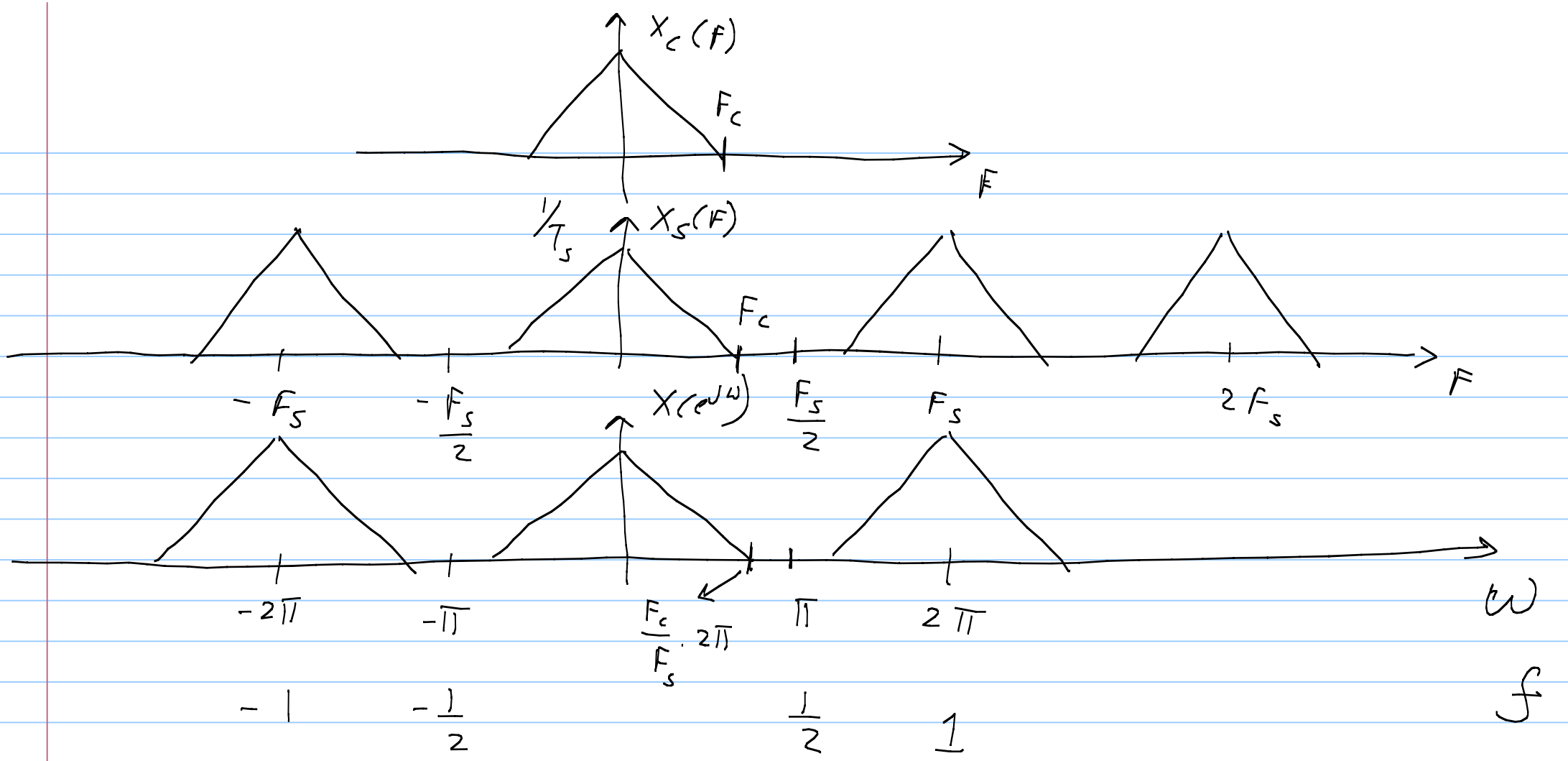
$$X_s(F)\Big|_{F \to \frac{\omega}{2\pi T_s}} = X(e^{j\omega})$$

$$\frac{\omega}{2\pi T_s} = \frac{\omega}{2\pi} F_s = f F_s$$



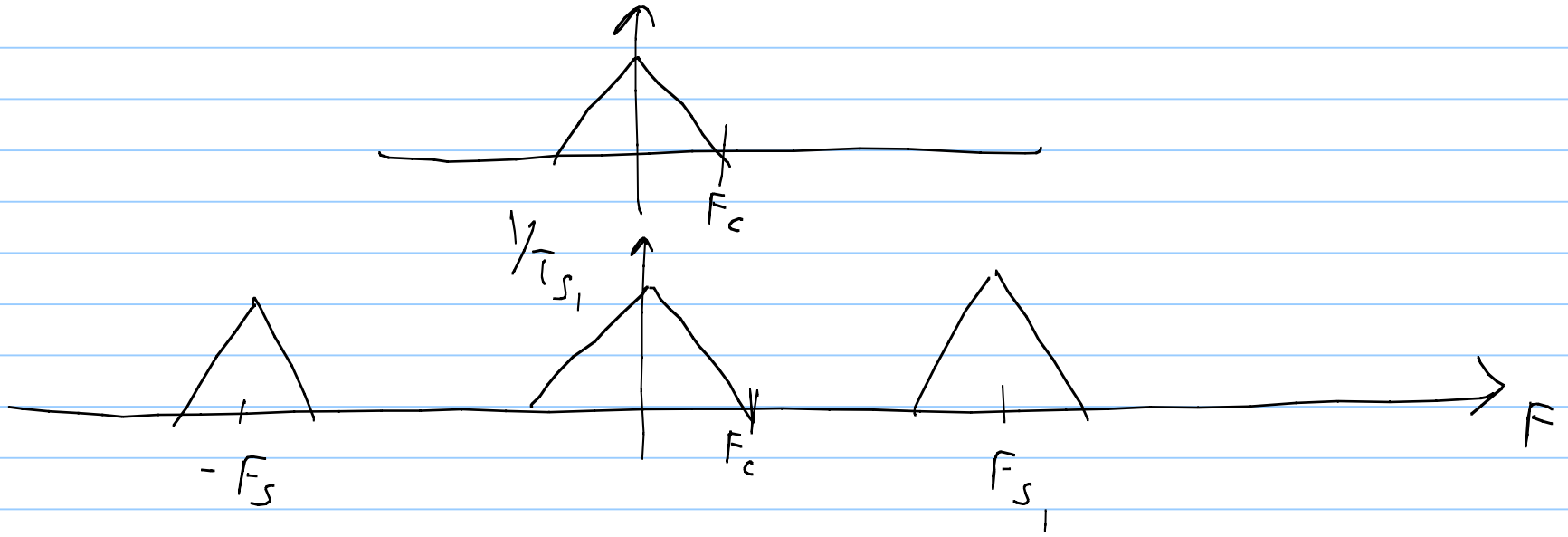


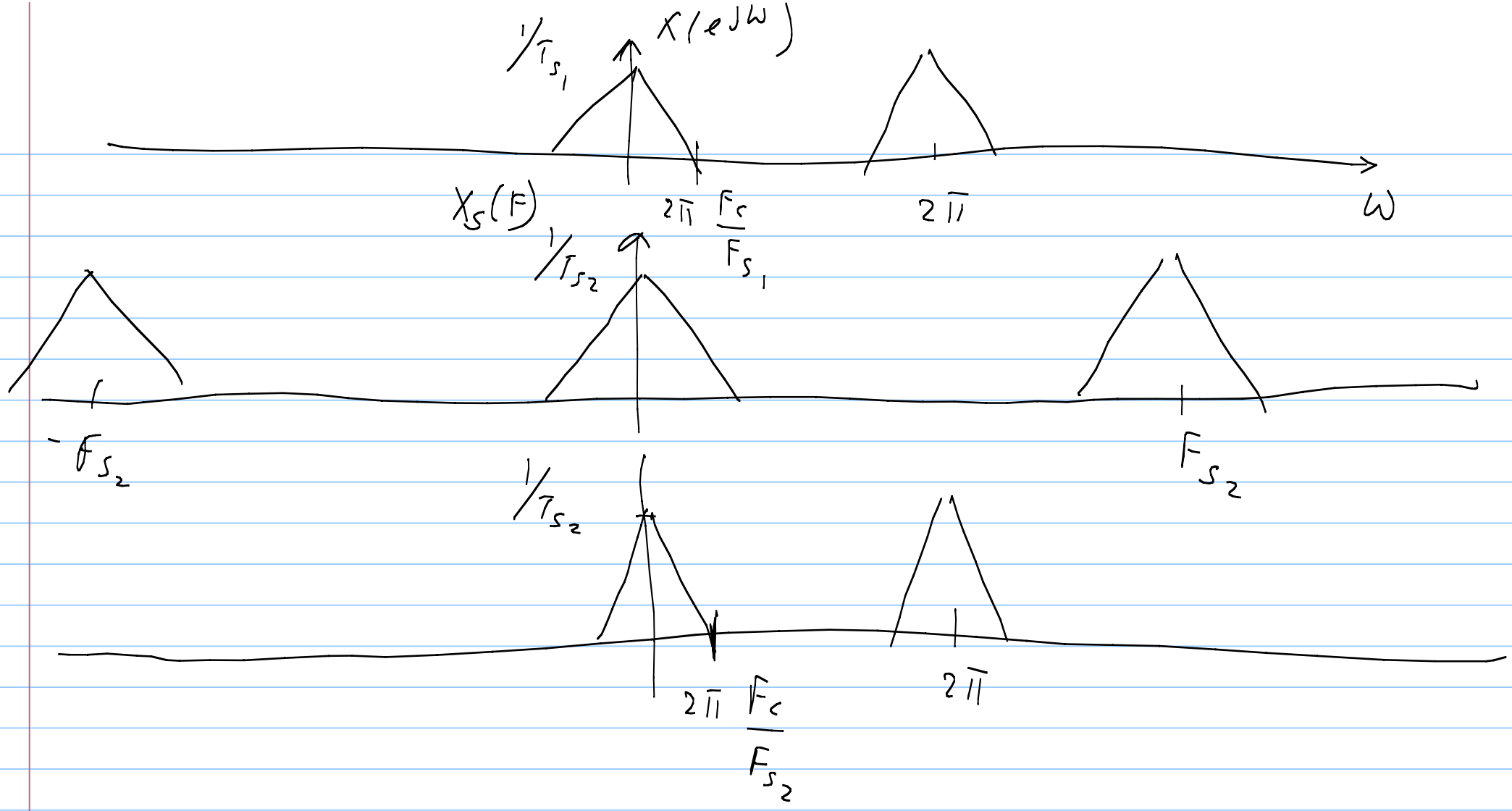
$$y(t) = x_c(t \cdot T_s) \longleftrightarrow \frac{1}{T_s} X_c\left(F/F_s\right) = \frac{1}{T_s} X_c\left(F \cdot F_s\right)$$


$X_c(F)$
F_c
F
$\frac{1}{T_s}$
$X_s(F)$
F_c
F
$-F_s$
$-\frac{F_s}{2}$
$X(e^{j\omega})$
$\frac{F_s}{2}$
F_s
$2F_s$
-2π
$-\pi$
$\frac{F_c}{F_s} \cdot 2\pi$
π
2π
ω
-1
$-\frac{1}{2}$
$\frac{1}{2}$
1
f

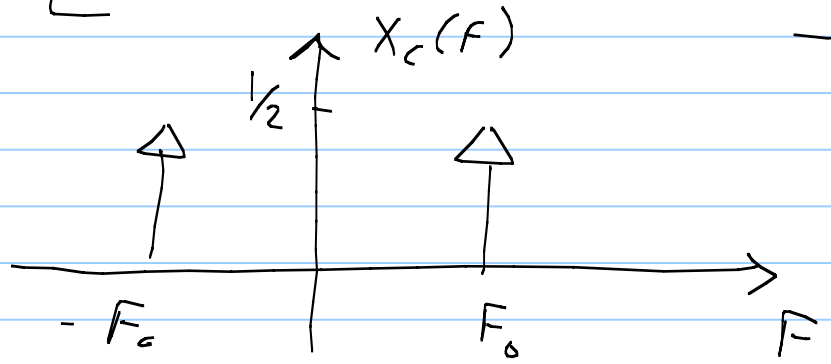
$$X_s(F) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c(F - kF_s)$$

$$X(e^{j\omega}) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega}{2\pi T_s} - kF_s\right)$$




1/Ts1
X(ejw)
Xs(F)
2π Fc/Fs1
2π
w
1/Ts2
-Fs2
Fs2
1/Ts2
2π Fc/Fs2
2π

$$\cos 2\pi F_0 t \longleftrightarrow \frac{1}{2}\left[\delta(F - F_0) + \delta(F + F_0)\right]$$



$$x[n] = x_c(nT_s) = \cos 2\pi F_0 n T_s$$

$$= \cos 2\pi \frac{F_0}{F_s} n$$

$$= \cos 2\pi f_0 n$$

$$= \cos \omega_0 n$$

$$\cos \omega_0 n \longleftrightarrow \pi \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right] \qquad -\pi \leq \omega < \pi$$

$$x_s(F) \longleftrightarrow \frac{1}{2T_s} \left[\delta(F - F_0) + \delta(F + F_0) \right] \qquad -\frac{F_s}{2} \leq F < \frac{F_s}{2}$$

$$x_s(F)\Big|_{F \to \frac{\omega}{2\pi T_s}} = X(e^{j\omega})$$

$$\frac{1}{2T_s}\,\delta(F - F_0) \longrightarrow \frac{1}{2T_s}\,\delta\left(\frac{\omega}{2\pi T_s} - F_0\right)$$

$$\delta(at+b) = \frac{1}{|a|}\,\delta(t + b/a)$$

$$\frac{1}{2T_s}\,\delta\left(\frac{\omega}{2\pi T_s} - F_0\right) = \frac{1}{2T_s}\,2\pi T_s\,\delta(\omega - 2\pi T_s F_0)$$

$$= \pi\,\delta\left(\omega - 2\pi\frac{F_0}{F_s}\right)$$

$$= \pi\,\delta(\omega - 2\pi f_0)$$

$$= \pi \delta(\omega - \omega_0)$$

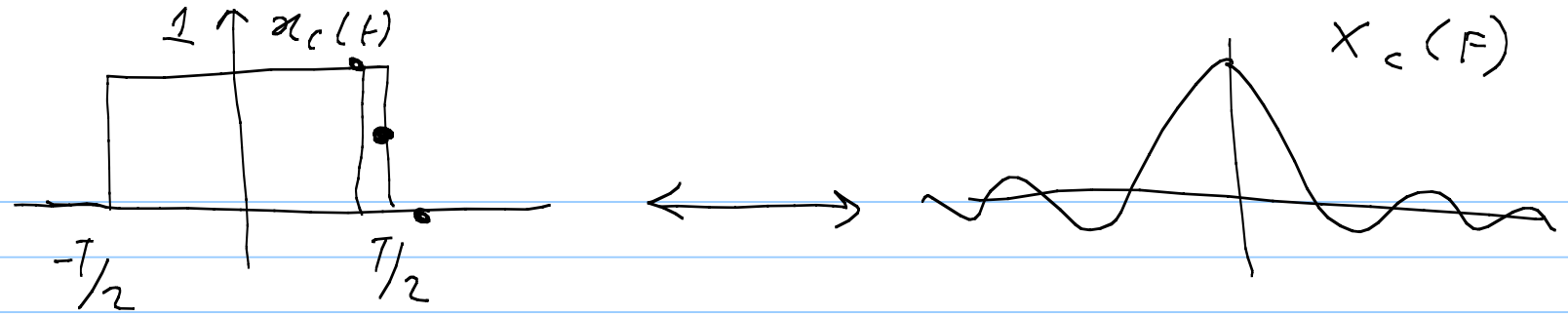
$\cos 2\pi F_0 t$

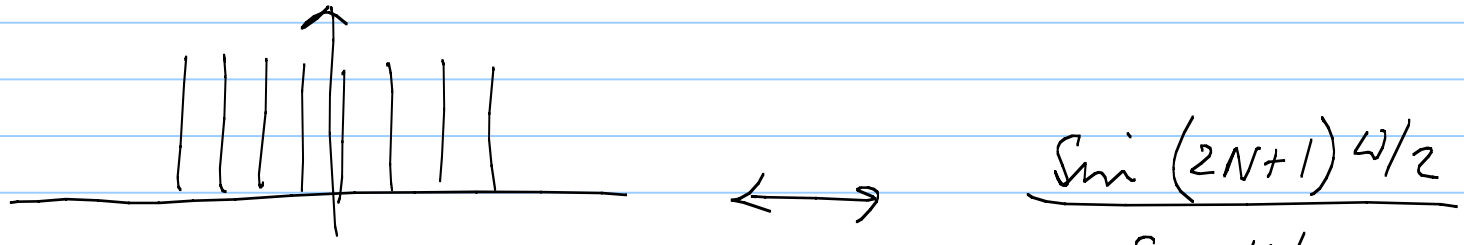
(i) $F_0 = 8\ kHz$ $\qquad F_s = 24\ kHz$

$$x[n] = x_c(nT_s) = \cos 2\pi \frac{8 \times 10^3}{24 \times 10^3} n = \cos \frac{2\pi n}{3}$$

(ii) $F_0 = 16\ kHz$ $\qquad F_s = 48\ kHz$

$$x[n] = x_c(nT_s) = \cos 2\pi \frac{16 \times 10^3}{48 \times 10^3} n = \cos \frac{2\pi n}{3}$$





$$\frac{\sin(2N+1)\omega/2}{\sin \omega/2}$$