EC 5142 Nov. 14 2011

Note Title 14-11-2011

For FIR filters, $h[N-1-n] = \pm h[n]$

gives rise to linear phase

For $\beta = \pi/2$, $h[N-1-n] = -h[n]$

i.e. sequence is real-valued & odd

$h[n] \longleftrightarrow H(e^{j\omega})$

$\hookrightarrow$ imaginary & odd

$H(e^{j\omega}) = j\, G(e^{j\omega})$.

Where $G(e^{j\omega})$ is real-valued

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$$H(e^{j\omega}) = e^{j\pi/2} G(e^{j\omega})$$

Type I length odd, symmetry is even $M = N-1$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1 \to M} h[n] e^{-j\omega n}$$

$$= h[0] + h[M] e^{-j\omega M}$$

$$+ h[1] e^{-j\omega} + h[M-1] e^{-j\omega(M-1)}$$

$$\vdots$$

$$+ \; h\left[\frac{M}{2}-1\right] e^{-j\omega\left(\frac{M}{2}-1\right)} + h\left[\frac{M}{2}+1\right] e^{-j\omega\left(\frac{M}{2}+1\right)}$$

$$+ \; h\left[\frac{M}{2}\right] e^{-j\omega M/2} \qquad e^{-j\omega\frac{M}{2}} \cdot e^{j\omega}$$

$$h[0] = h[M] \qquad h[1] = h[M-1] \quad , \text{etc.}$$

$$H(e^{j\omega}) = e^{-j\omega\frac{M}{2}} h[0]\left[e^{+j\omega M/2} + e^{-j\omega M/2}\right]$$

$$\vdots$$

$$+ \; e^{-j\omega\frac{M}{2}} \; h\left[\frac{M}{2}-1\right]\left[e^{j\omega} + e^{-j\omega}\right]$$

$$+$$

$$+ \; e^{-j\omega \frac{M}{2}} \, h\left[\frac{M}{2}\right]$$

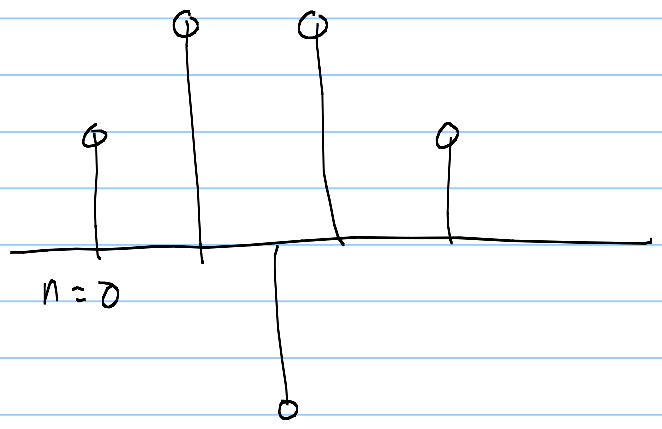
$$= e^{-j\omega \frac{M}{2}} \left[\sum_{n=0}^{\frac{M}{2}-1} h[n] \, 2\cos\left(\omega\left(\frac{M}{2}-n\right)\right) + h\left[\frac{M}{2}\right] \right]$$

$$= e^{-j\omega \frac{M}{2}} \underbrace{A(\omega)}$$

real-valued continuous function

Constrained Zeros

Type I $N = 5$



$+ \quad - \quad + \quad - \quad +$

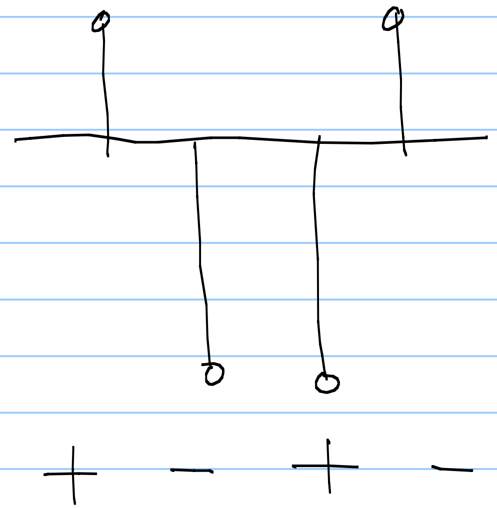
$$H(z)\Big|_{z=1} = \sum_{n=0}^{N-1} h(n)\, 1^{-n}$$

does not always add up to zero

$$H(z)\Big|_{z=-1} = \sum_{n=0}^{N-1} h(n)\, (-1)^{-n}$$

does not always add up to zero

Type II



$$\sum_{n=0}^{N-1} h(n) = 0 \ ?$$

$H(1) = 0$ not always

$H(-1) = 0$ always

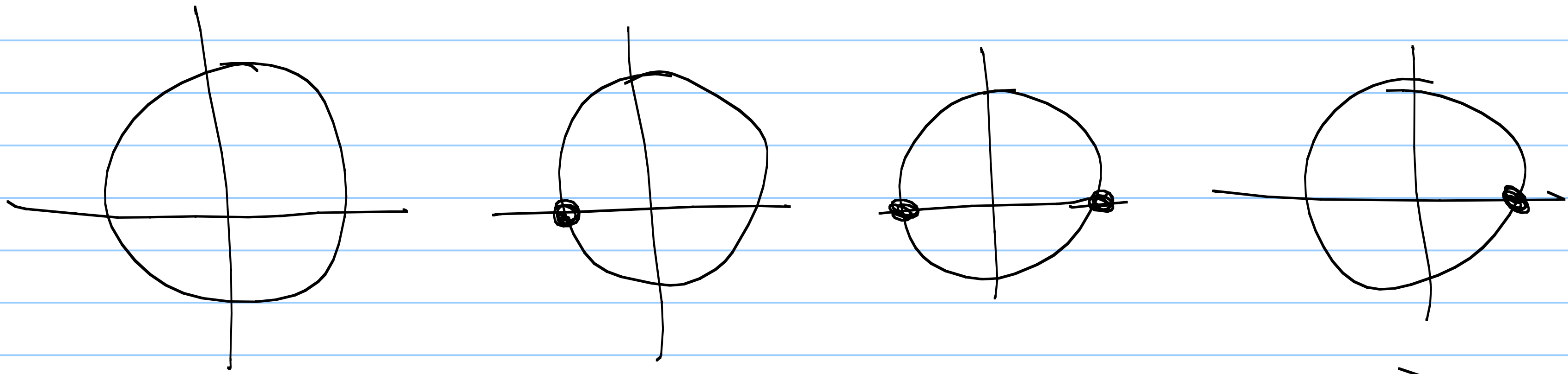
Type III $\quad \sum_{n=0}^{N-1} h(n) = 0$ since it is odd symmetric

$$\sum_{n=0}^{N-1} h(n)(-1)^n = 0$$

Type IV

$$\sum_{n=0}^{N-1} h(n) = 0$$ since it is odd symm

$$\sum_{n=0}^{N-1} h(n)\,(-1)^n$$ not always zero



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