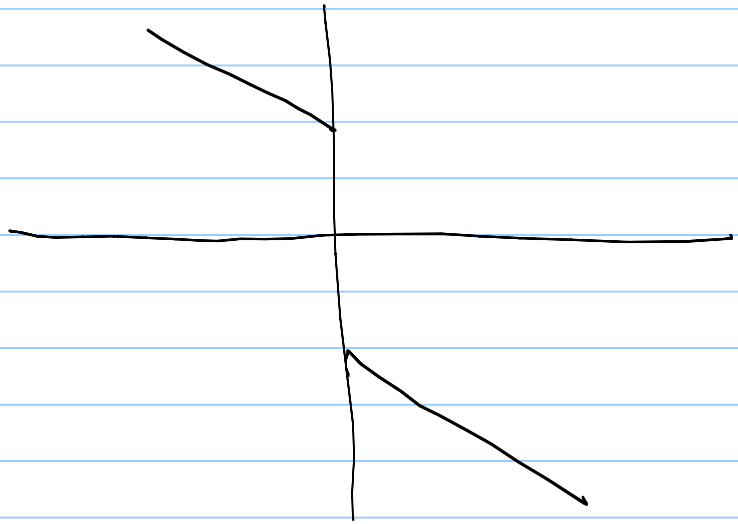
EC5142 Nov 10 2011

Note Title 10-11-2011



Generalized linear phase.

$$\phi(\omega) = -\alpha\omega + \beta \qquad \omega > 0$$

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j(-\alpha\omega+\beta)} = \sum_n h[n] e^{-j\omega n}$$

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$$\sum_n h[n] \sin\left(\overline{-\alpha + n}\,\omega + \beta\right) = 0$$

$h[n] = 0 \;\; \forall n$ is the trivial solution

First, assume $\beta = 0$

$$\sum_n h[n] \sin\left(\overline{-\alpha + n}\,\omega\right) = 0$$

Assume $h[n] = 0$ for n outside $[0, N-1]$

i.e. $h[n]$ is an FIR filter

$$\sum_{n=0}^{N-1} h[n] \sin\left(\overline{-\alpha+n}\,\omega\right) = 0$$

$$h[0] \sin(-\alpha\omega)$$

$$h[N-1] \sin\left(\overline{-\alpha+N-1}\,\omega\right)$$

$$\alpha = -\alpha + N-1 \Rightarrow \alpha = \frac{N-1}{2}$$

$$h[0] = h[N-1]$$

$$h[1] \sin\left(\overline{-\alpha+1}\,\omega\right)$$

$$h[N-2] \sin\left(\overline{-\alpha+N-2}\,\omega\right)$$

$$\alpha - 1 = -\alpha + N - 2 \Rightarrow \alpha = \frac{N-1}{2}$$

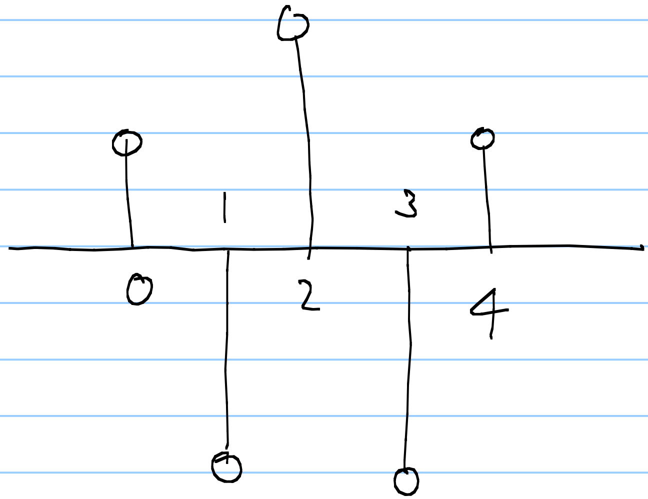
$$h[1] = h[N-2]$$

In general, if $h[N-1-n] = h[n]$

& $\alpha = \frac{N-1}{2}$ = centre of symmetry pairwise cancellation will occur.

Eg:

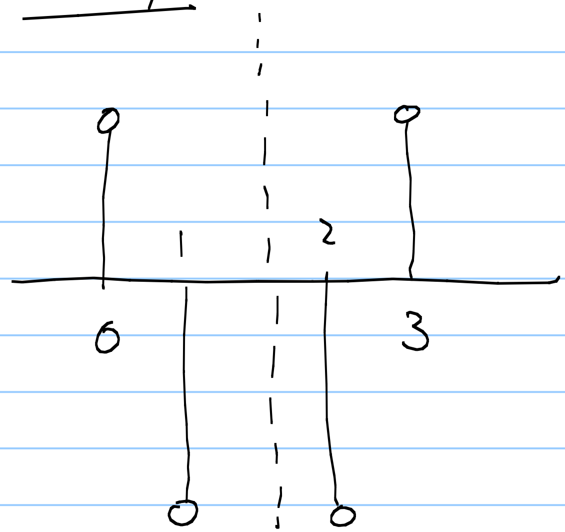
$N = 5$



$$h[2]\sin\left(\left(-\frac{5-1}{2}+\overbrace{n}^{2}\right)\omega\right) = 0$$

Since $n = \frac{N-1}{2}$ & $\alpha = \frac{N-1}{2}$

For $\underline{N=4}$



$$\phi(\omega) = -\alpha\omega$$

$$= -\frac{N-1}{2}\,\omega$$

$$\tau_g(\omega) = \frac{N-1}{2}$$

Now consider $\beta \neq 0$. Let $\beta = \frac{\pi}{2}$

$$\sum_{n=0}^{N-1} h[n] \sin\left(\overline{-\alpha+n}\,\omega + \pi/2\right) = 0$$

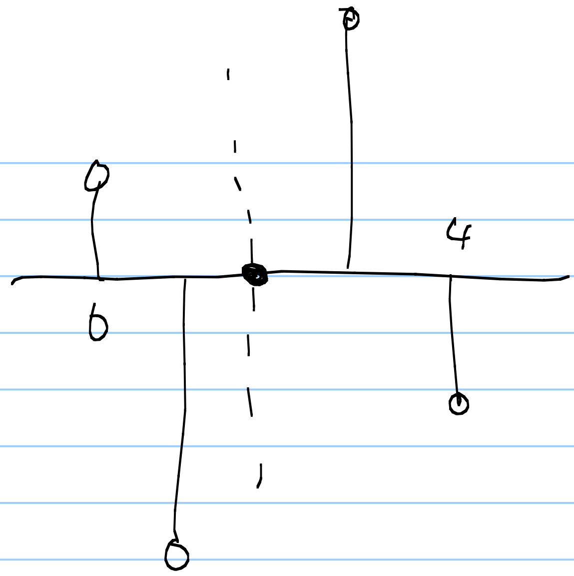
$$\sum_{n=0}^{N-1} h(n) \cos\left(\overline{-\alpha+n}\,\omega\right) = 0$$

$$h[0] \cos(-\alpha\omega)$$

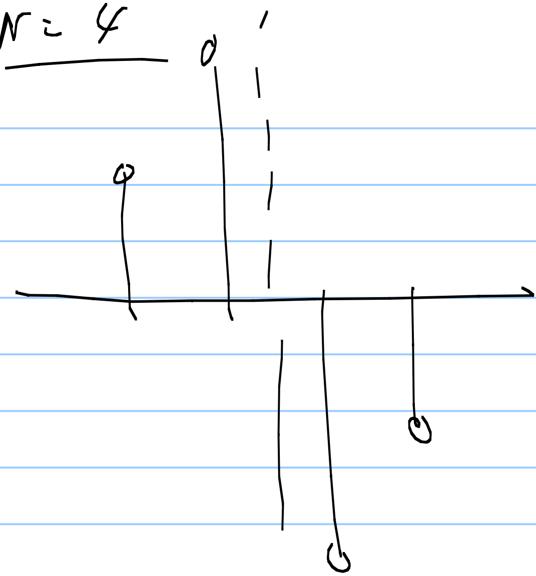
$$h[N-1] \cos\left(\overline{-\alpha+N-1}\,\omega\right) \qquad \alpha = \frac{N-1}{2}$$

$$h[0] = -h[N-1]$$

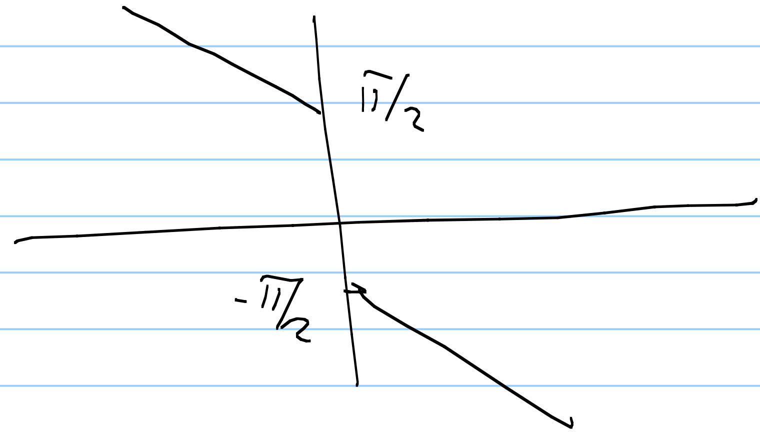
N = 5



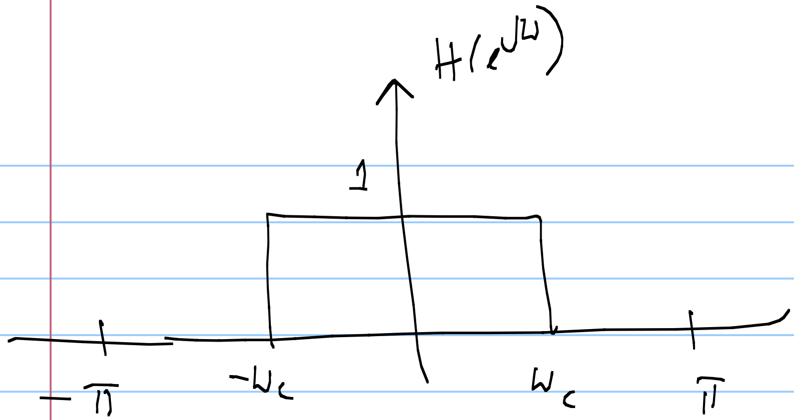
N = 4



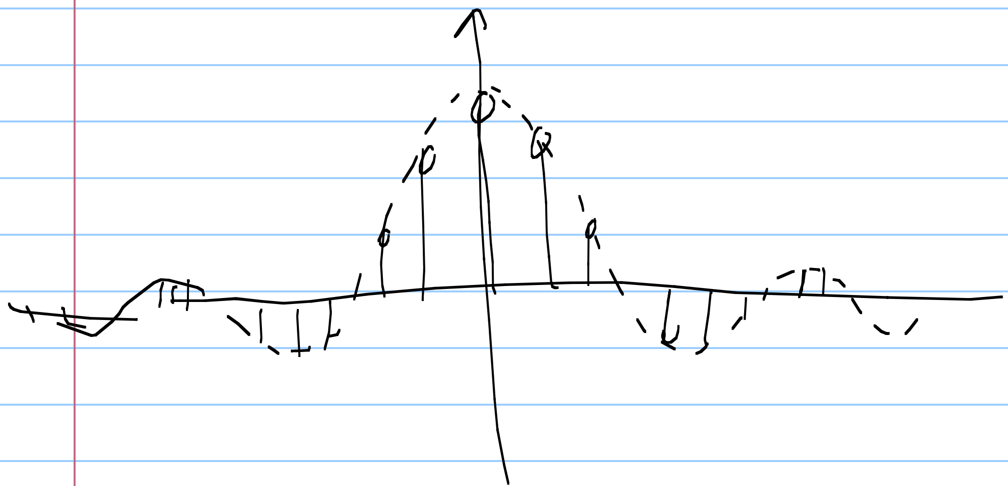
$$\phi(\omega) = -\frac{N-1}{2}\,\omega + \frac{\pi}{2}$$



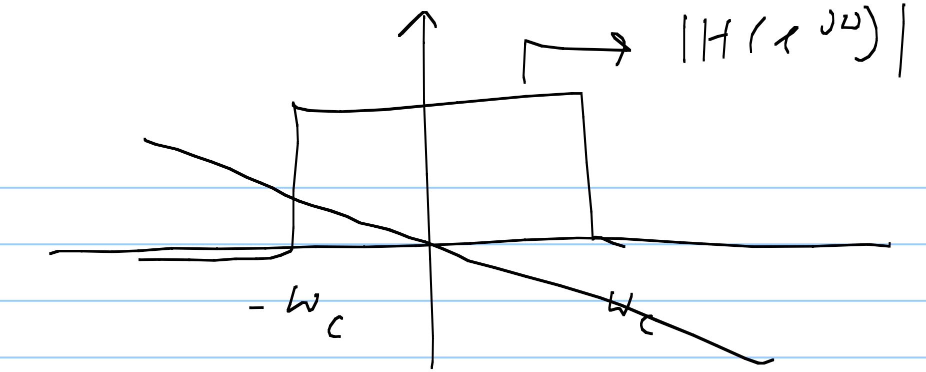
$$\tau_g(\omega) = \frac{N-1}{2}$$



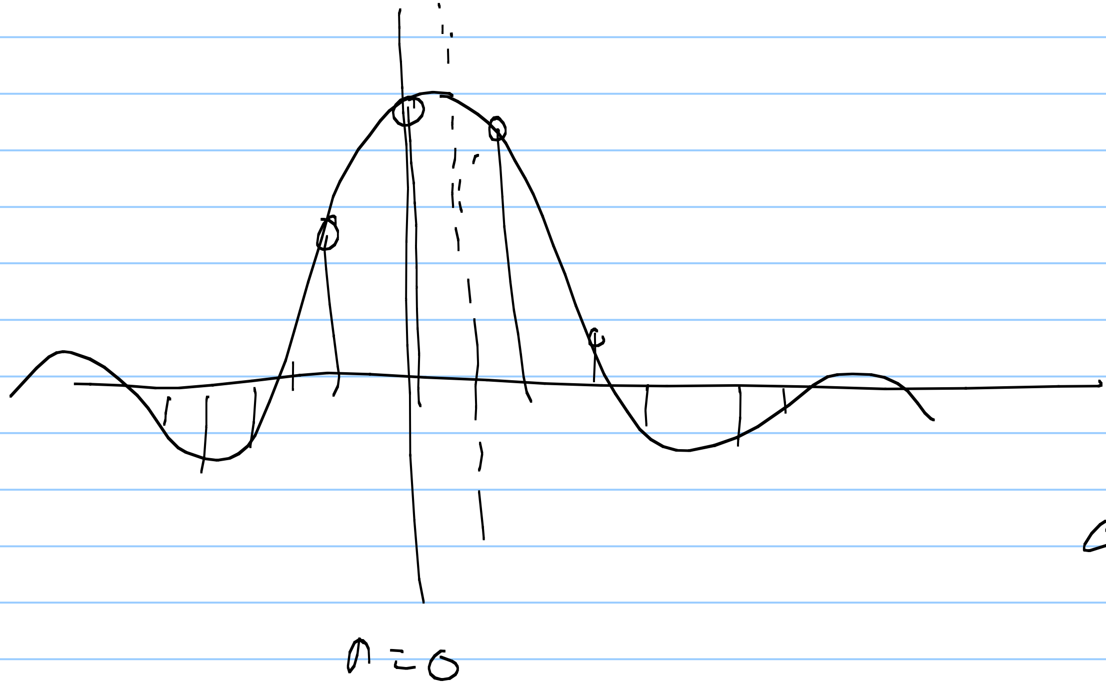
$$h(n) = \frac{\sin \omega_c n}{\pi n}$$



<u>IIR</u>



$$H(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

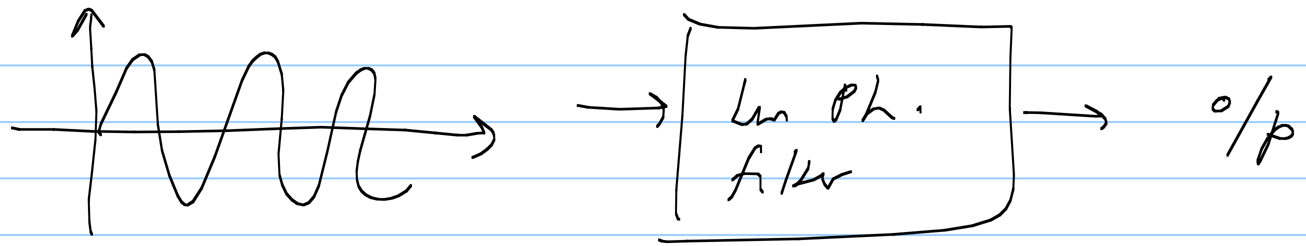


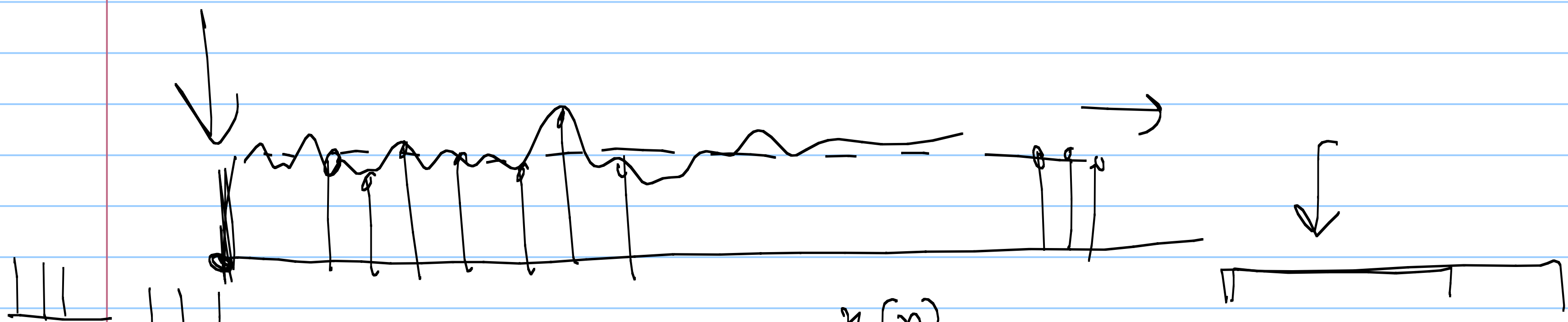
Symmetry is sufficient but not necessary for linear phase in the case for IIR filters

However, for FIR filters, symmetry is both necessary & sufficient for generalized linear phase

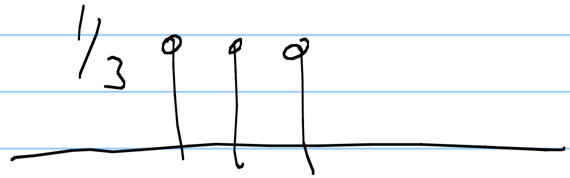
Length: N

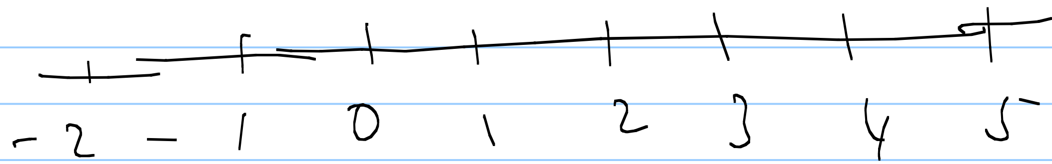
Symmetry	Length	Name	$\tau_g(\omega)$	
Even	Odd	Type I	$\frac{N-1}{2}$	integer
Even	Even	Type II	"	integer + 1/2
Odd	Odd	Type III	"	integer
Odd	Even	Type IV	"	integer + 1/2





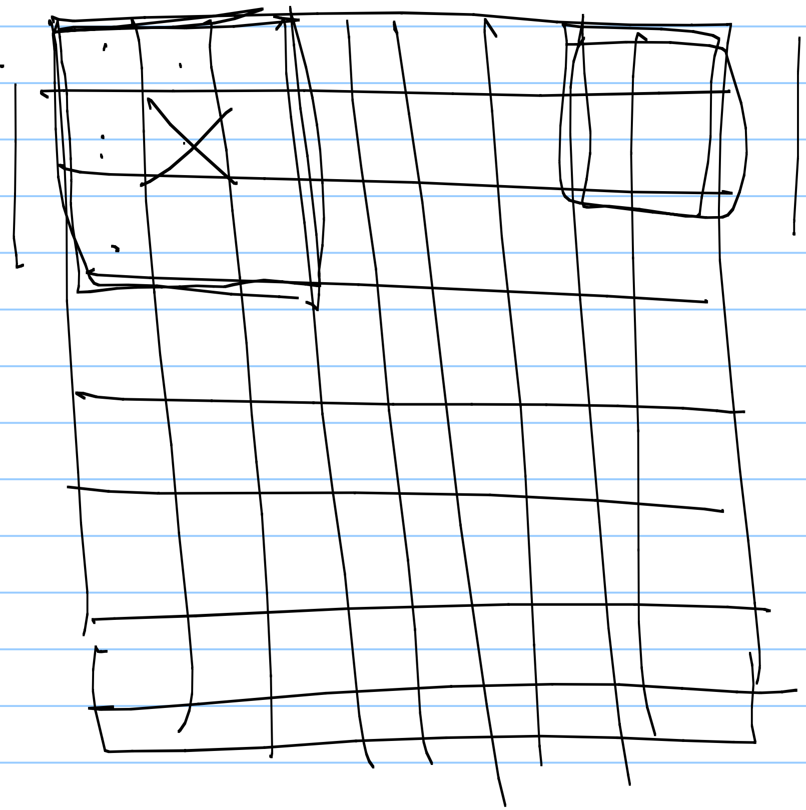
x(n)





y(n)

1/9



1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$$H(e^{j\omega}) = |H(e^{j\omega})| \, e^{j\theta(\omega)}$$ $\theta(\omega)$: linear phase

Consider $G(z) = \dfrac{1}{H(z)}$

$$G(e^{j\omega}) = \frac{1}{|H(e^{j\omega})|} \, e^{-j\theta(\omega)}$$

$$h[N-1-n] = h[n]$$

$$h[n] \longleftrightarrow H(z)$$

$$h[n+N] \longleftrightarrow z^{N} H(z)$$

$$h[N-n] \longleftrightarrow z^{-N} H(z^{-1})$$

$$h[N-1-n] \longleftrightarrow z^{-(N-1)} H(z^{-1})$$

$h[n] = \pm h[N-1-n]$ (real-valued)

$$H(z) = \pm z^{-(N-1)} H(z^{-1})$$

For the general case

$$h[n] = \pm h^{*}[N-1-n]$$

$$H(z) = \pm z^{-(N-1)} H^{*}(z^{*-1})$$

If z_0 is a zero of $H(z)$,

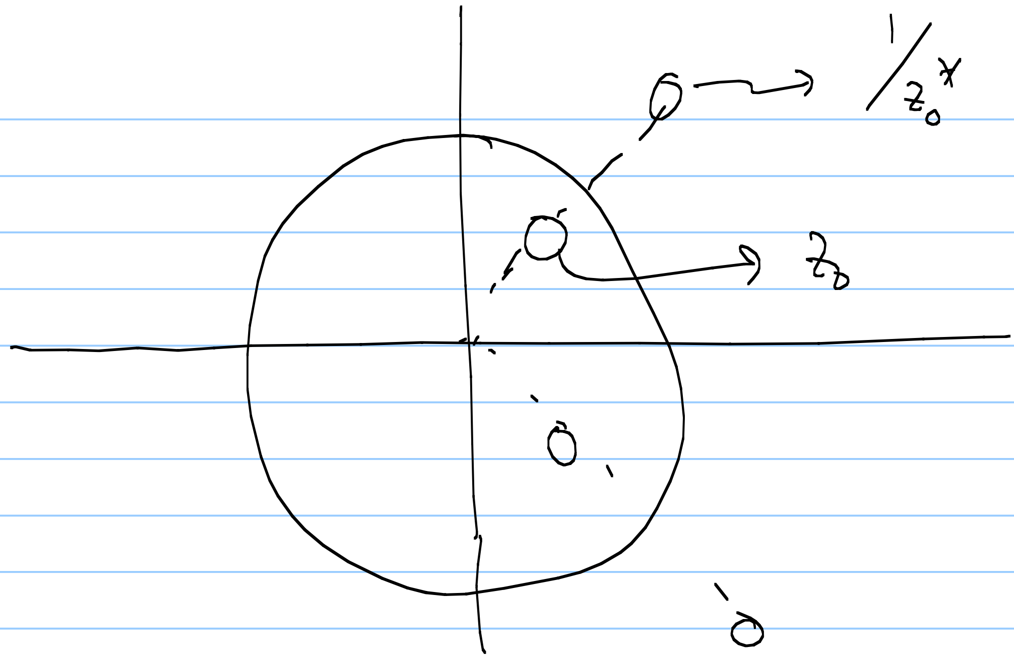
$$H(z_0) = 0$$

$$H(z_0) = \pm z_0^{-(N-1)} H^*(z_0^{*-1}) = 0$$

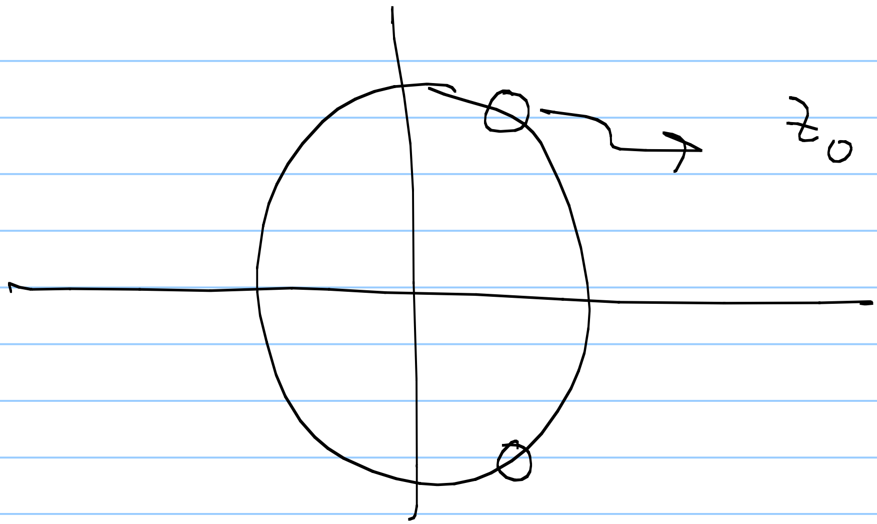
$$\Rightarrow \quad H^*(z_0^{*-1}) = 0$$

$\Rightarrow \quad \dfrac{1}{z_0^*}$ is also a zero

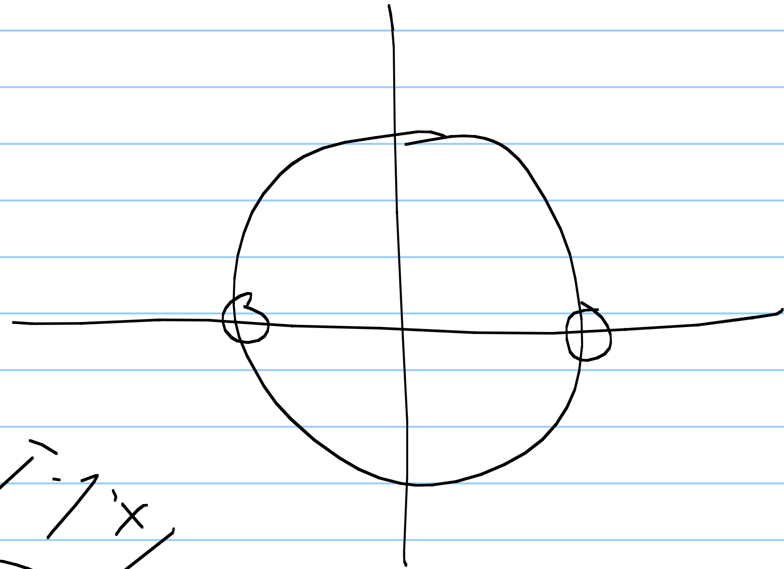
If $z_0 = re^{j\theta}$, then $\dfrac{1}{z_0^*} = \dfrac{1}{r} e^{j\theta}$



$r e^{j\theta}, \quad r e^{-j\theta}, \quad \frac{1}{r} e^{j\theta}, \quad \frac{1}{r} e^{-j\theta}$

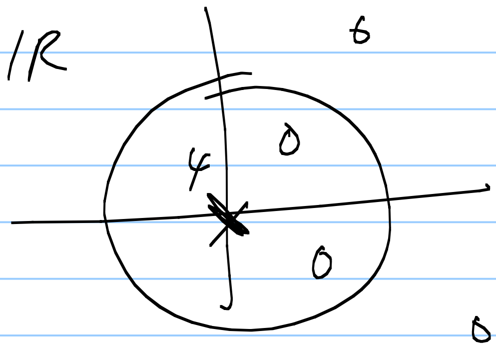


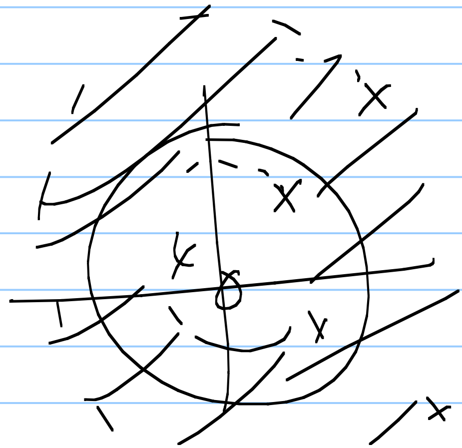
z_0 , $1/z_0^*$



Min Phase

FIR





Min. Phase IIR