EC 5142 Nov 03 2011

Note Title 03-11-2011

$$x[n] \longrightarrow \boxed{h[n]} \xrightarrow{w[n]} \boxed{h[-n]} \longrightarrow y[n]$$

$$W(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})\, H(e^{-j\omega})$$

For real-valued $h[n]$, $H(e^{-j\omega}) = H^*(e^{j\omega})$

$$Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})\, H^*(e^{j\omega})$$

$$= X(e^{j\omega})\, |H(e^{j\omega})|^2$$

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$|H(e^{j\omega})|^2$ = "zero phase filter"

Not practical because $h[-n]$ is non-causal if $h[n]$ is causal

$$x[n] \longleftrightarrow X(e^{j\omega})$$
$$x[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$$

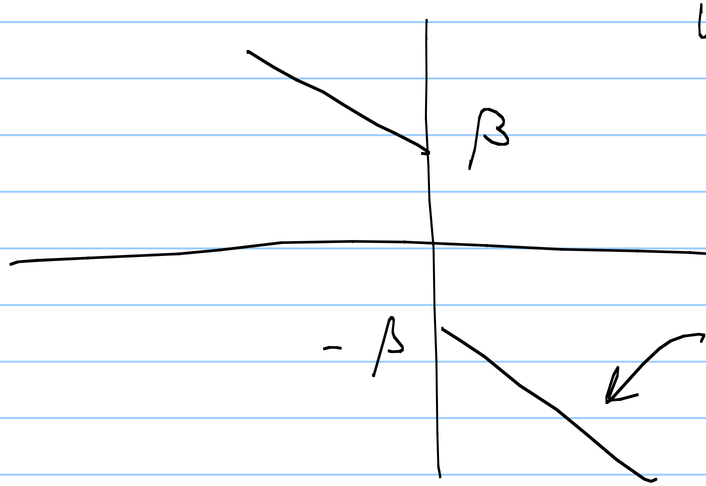
$$x[n] = A_1 \cos \omega_1 n + A_2 \cos \omega_2 n$$
$$x[n-n_0] = A_1 \cos \omega_1 (n-n_0) + A_2 \cos \omega_2 (n-n_0)$$

$$= A_1 \cos(\omega_1 n - \underbrace{\omega_1 n_0}_{\phi_1}) + A_2 \cos(\omega_2 n - \underbrace{\omega_2 n_0}_{\phi_2})$$

$$\phi(\omega) = -\alpha\omega + \beta$$

generalized linear phase

$\omega > 0$



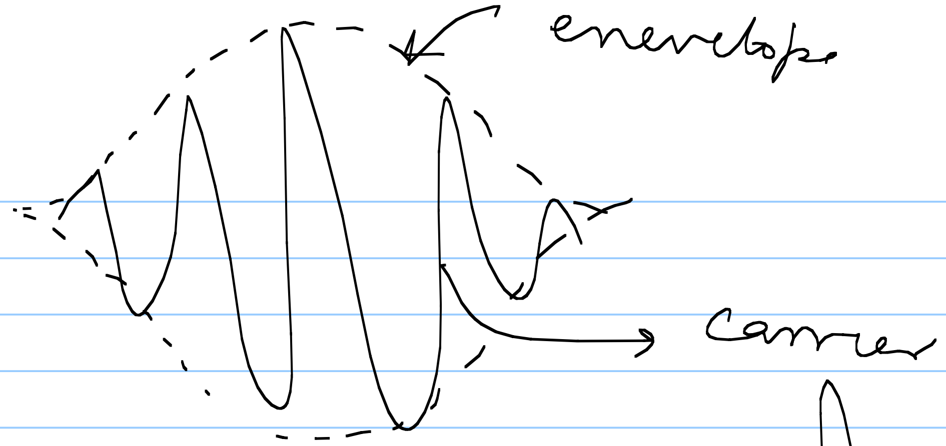
Slope $= -\alpha$

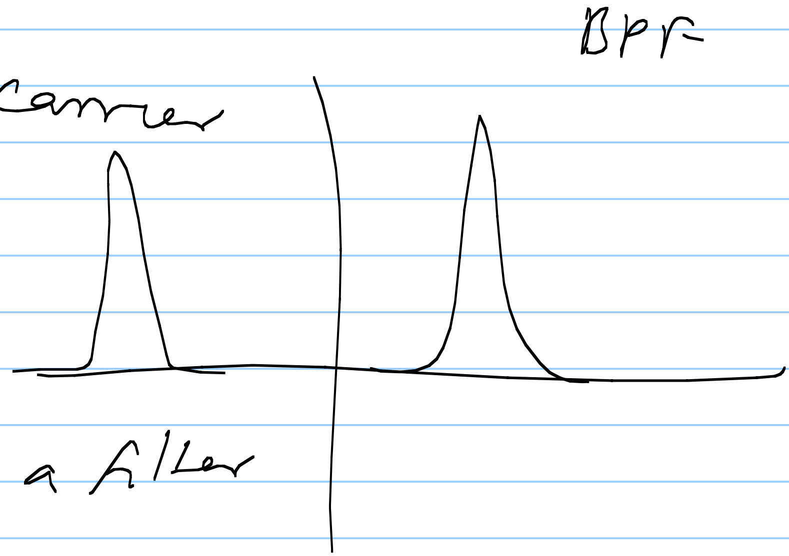
$$\tau_g(\omega) = -\frac{d\phi}{d\omega} = \alpha$$

Phase delay : $\tau_{ph}(\omega) = -\frac{\phi(\omega)}{\omega}$

For $\phi(\omega) = -\alpha\omega$ $\quad \tau_g(\omega) = \alpha$

$\tau_{ph}(\omega) = \alpha$





Under what conditions is a filter linear phase?

$$\phi(\omega) = -\alpha\omega + \beta$$

$$H(e^{j\omega}) = \sum_n h[n] e^{-j\omega n}$$

$$= |H(e^{j\omega})| e^{j(-\alpha\omega+\beta)}$$

$$\sum_n h[n] \cos\omega n - j \sum_n h[n] \sin\omega n = |H(e^{j\omega})| e^{j(-\alpha\omega+\beta)}$$

$$\frac{-\sum_n h[n] \sin\omega n}{\sum_n h[n] \cos\omega n} = \frac{\sin(-\alpha\omega+\beta)}{\cos(-\alpha\omega+\beta)}$$

$$\sum_n{}' h(n) \sin \omega n \cos(-\alpha\omega + \beta)$$

$$+ \sum{}' \sin(-\alpha\omega + \beta)\, h(n) \cos \omega n = 0$$

$$\sum_n{}' h(n) \sin\left((-\alpha + n)\omega + \beta\right) = 0$$