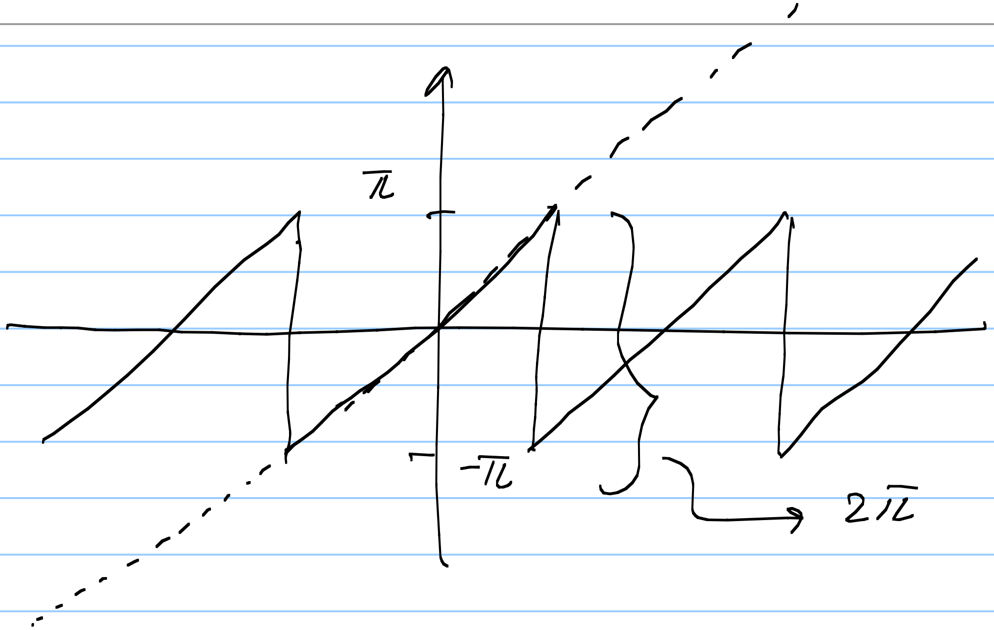
EC 5142 Nov 02 2011

Note Title 02-11-2011



Principal Phase: $-\pi \leq \Phi(\omega) < \pi$ "wrapped phase"

$\hookrightarrow$ principal $\measuredangle H(e^{j\omega})$

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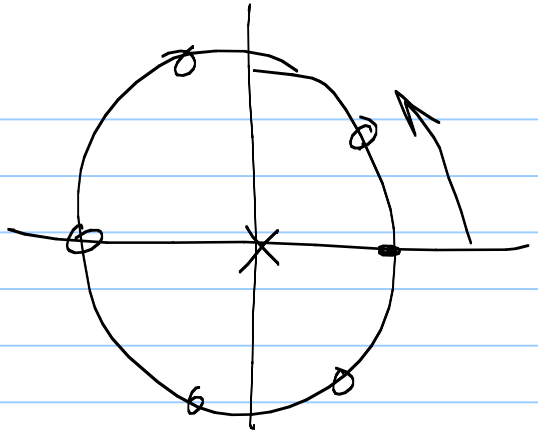
From $\Phi(\omega)$ we can get back $\phi(\omega)$ by adding or subtracting required multiples of 2π

For rational transfer function, the $\phi(\omega)$ is continuous except for possible jumps of π

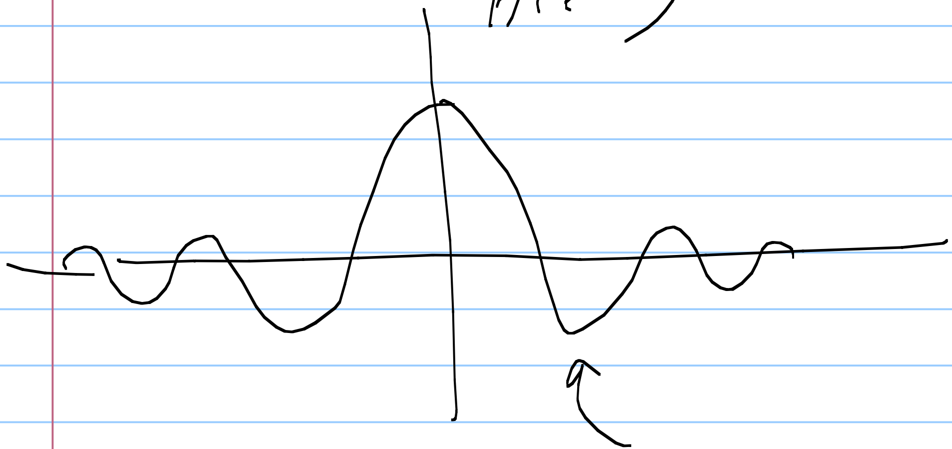
$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j \measuredangle H(e^{j\omega})}$$

$$h[n] = \{1, 1, 1, 1, 1\}$$



$$\longleftrightarrow \quad \frac{\sin 5\omega/2}{\sin \omega/2}$$

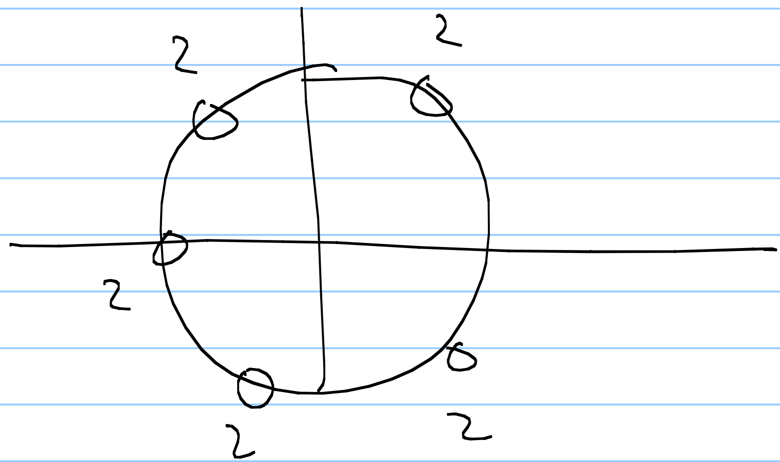
$H(e^{j\omega})$

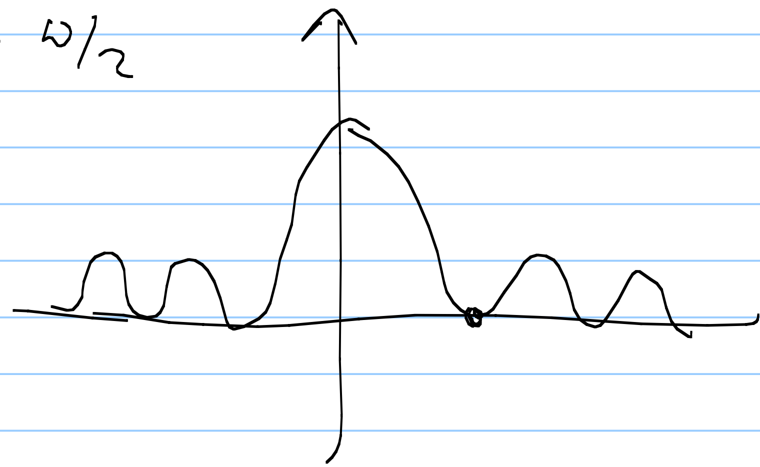


$|H(e^{j\omega})|$

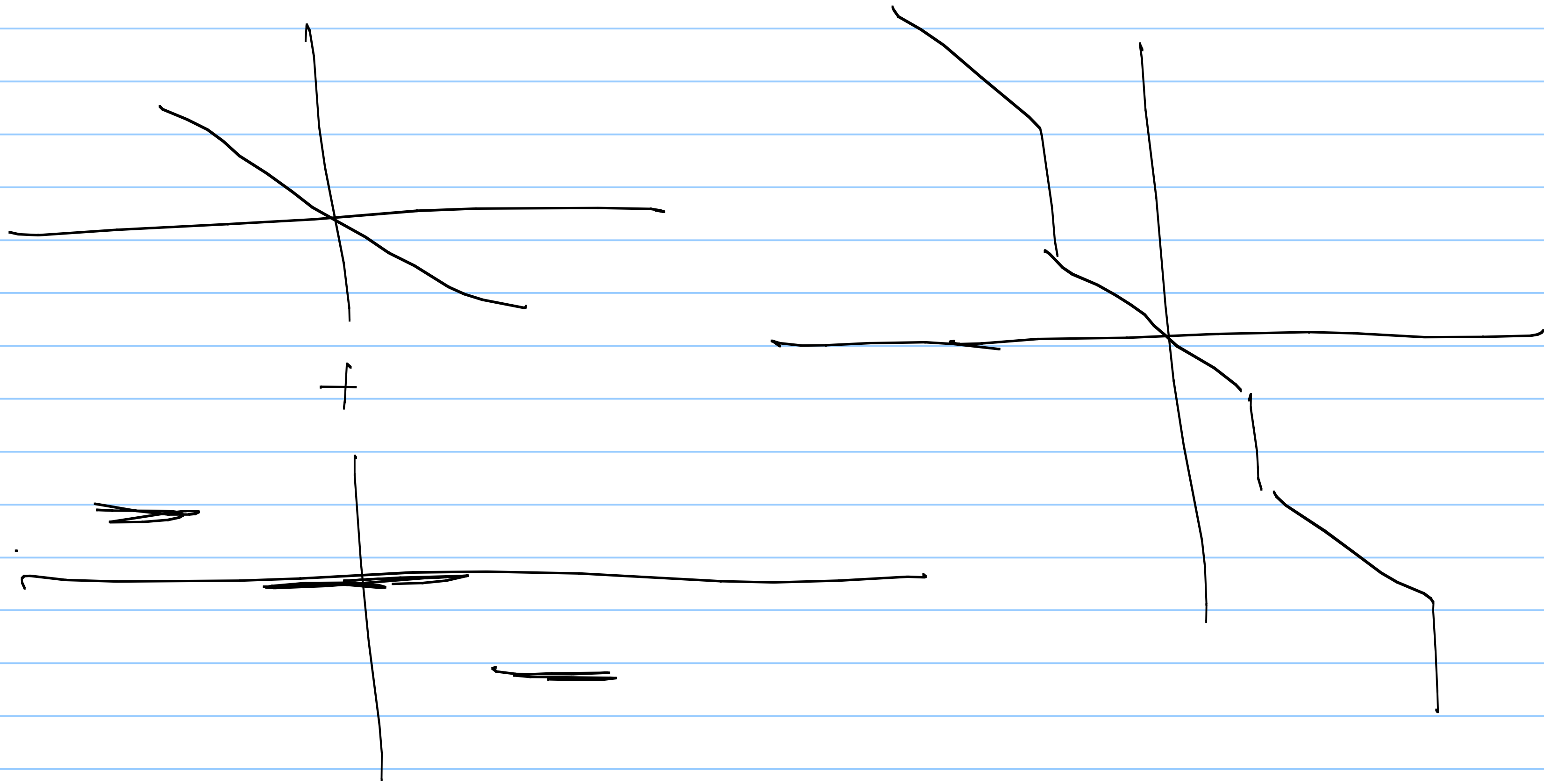
$$h[n] * h[n] \Longleftarrow \{1 \quad 2 \quad 3 \quad 4 \quad \underset{\uparrow}{5} \quad 4 \quad 3 \quad 2 \quad 1\}$$

$$H(e^{j\omega}) \cdot H(e^{j\omega}) = \frac{\sin^2 5\omega/2}{\sin^2 \omega/2}$$





$$h(n) = \{1, 1, 1, 1, 1\} \longleftrightarrow e^{-j2w} \overbrace{\frac{\sin 5w/2}{\sin w/2}}^{A(w)}$$



$$H = \text{freqz}(b, a) \qquad \angle H(e^{j\omega})$$

$$H(e^{j\omega}) = |H(e^{j\omega})|\, e^{j\phi(\omega)}$$

$$= \underbrace{A(\omega)}_{\text{real-valued}}\, e^{j\phi_1(\omega)}$$

$\phi_1(\omega) \rightarrow$ continuous

true for rational TF

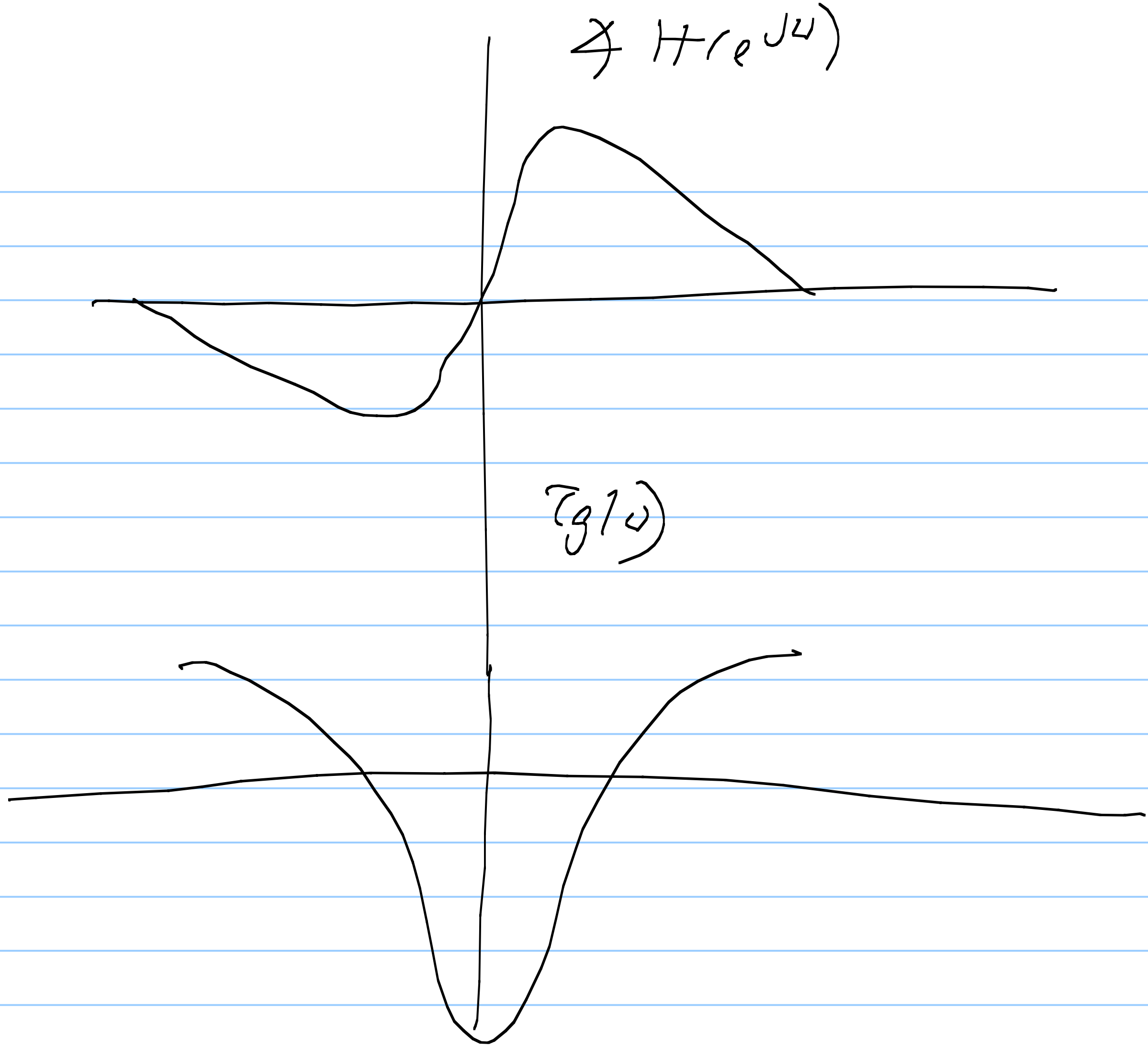
Group Delay: $$-\frac{d}{d\omega}\phi(\omega) = \tau_g(\omega)$$

Simple complex zero

$$\phi(\omega) = \tan^{-1} \frac{r \sin(\omega - \theta)}{1 - r \cos(\omega - \theta)}$$

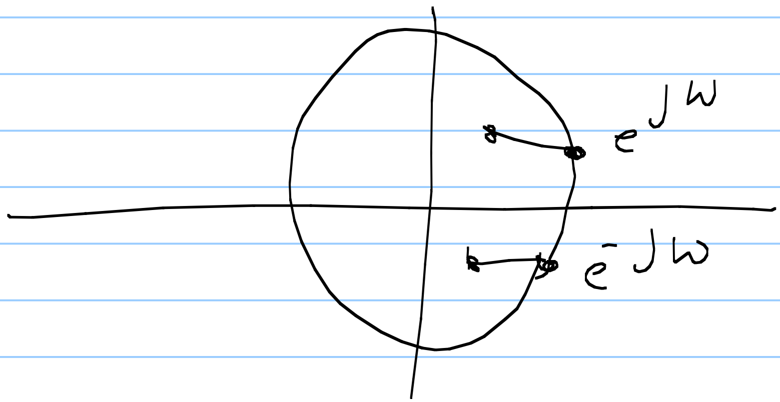
$$-\frac{d}{d\omega}\phi(\omega) = \frac{r\,(r - \cos(\omega - \theta))}{|1 - r e^{j\theta} e^{j\omega}|^2}$$

Single complex zero



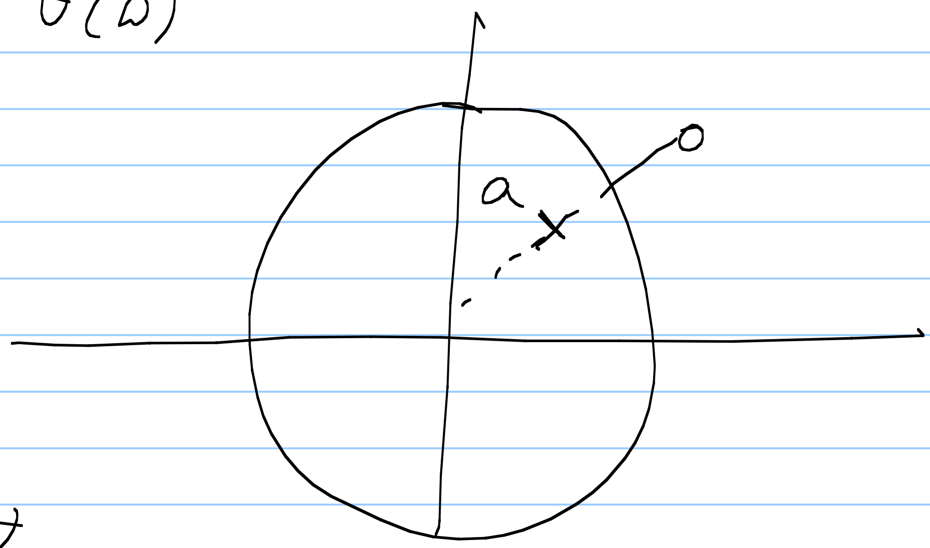
$$H(z) = \frac{-a^* + z^{-1}}{1 - a z^{-1}}$$

$$H(e^{jw}) = \frac{-a^* + e^{-jw}}{1 - a e^{-jw}} = \frac{e^{-jw} - a^*}{e^{-jw}(e^{jw} - a)}$$



$$|H(e^{j\omega})| = \left| \frac{e^{-j\omega} - a^*}{e^{j\omega} - a} \right| = \underline{1}$$

$$H(e^{j\omega}) = e^{j\theta(\omega)}$$



pole: $a = re^{j\theta}$

zero: $1/a^* = \frac{1}{r} e^{j\theta}$ reflected across the unit circle

$$H(z) = \frac{-a^{*}\left(1 - \frac{1}{a^{*}} z^{-1}\right)}{1 - a z^{-1}}$$

$|H(e^{j\Omega})| = 1$ "All pass" Section

$$H_{Ap}(z) = \frac{a_N^{*} + a_{N-1}^{*} z^{-1} + \cdots + a_1^{*} z^{-(N-1)} + z^{-N}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

$$= \frac{z^{-N} A^*(z^*)}{A(z)}$$

$$\begin{cases} 1 & |w| < w_c \\ 0 & \text{otherwise} \end{cases}$$