Note Title 30-08-2011

EC 5142 Aug. 30 2011

Time Scaling:

$$y(t) = x(ct)$$

$y(t) \longleftrightarrow b_k$ period T_y

$x(t) \longleftrightarrow a_k$ period T_x

$$T_y = \frac{T_x}{c}$$

$$\Omega_y = c\,\Omega_x$$

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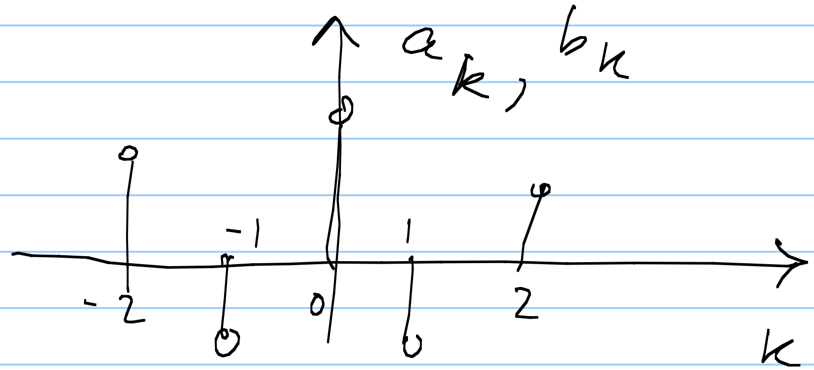
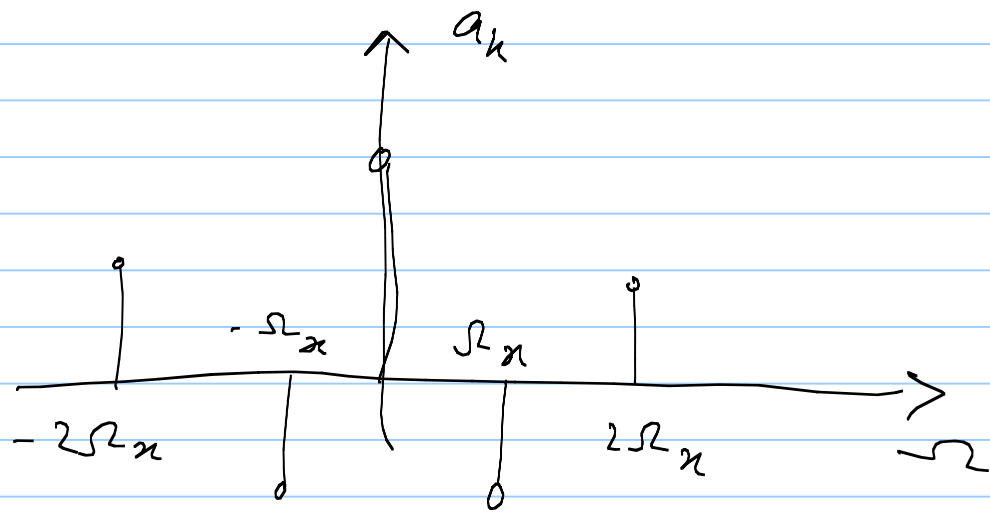
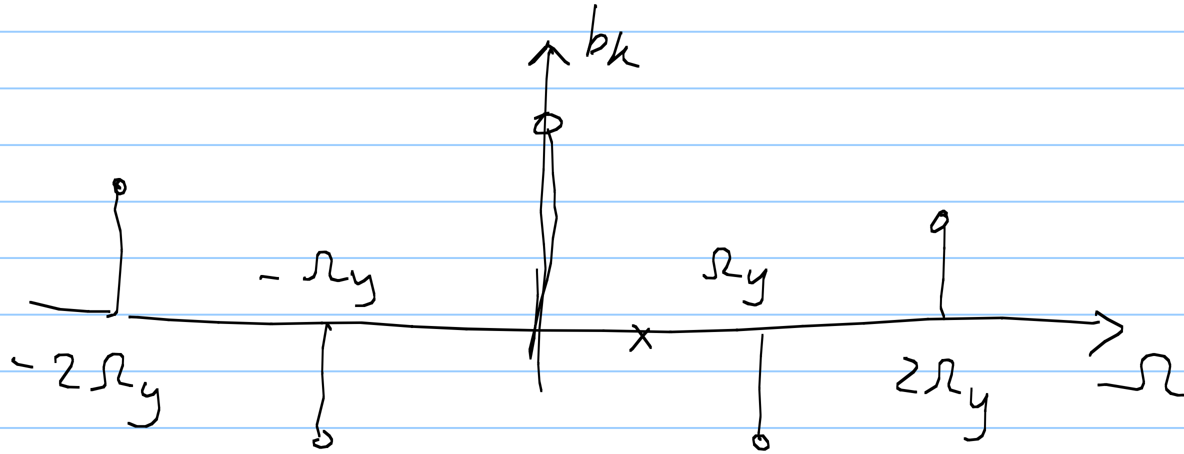
$$b_k = a_k$$







Multiplication in the time domain

$$x(t)\,y(t) \longleftrightarrow ?$$

$$\frac{1}{T}\int_T \left[\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}\right]\left[\sum_{l=-\infty}^{\infty} b_l e^{jl\Omega_0 t}\right] e^{-jm\Omega_0 t}\,dt$$

$$= \frac{1}{T}\int_T \left[\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} a_k b_l e^{j(k+l)\Omega_0 t}\right] e^{-jm\Omega_0 t}\,dt$$

Let $k + l = r$ & simplify

$$x(t)\, y(t) \longleftrightarrow a_k * b_k$$

Convolution in the time domain

$$z(t) = x(t) \circledast y(t) \longleftrightarrow a_k \cdot b_k$$

$$z(t) = \int_T x(\tau)\, y(t-\tau)\, d\tau$$

Differentiation:

$$x(t) \longleftrightarrow a_k$$

$$x'(t) \longleftrightarrow$$

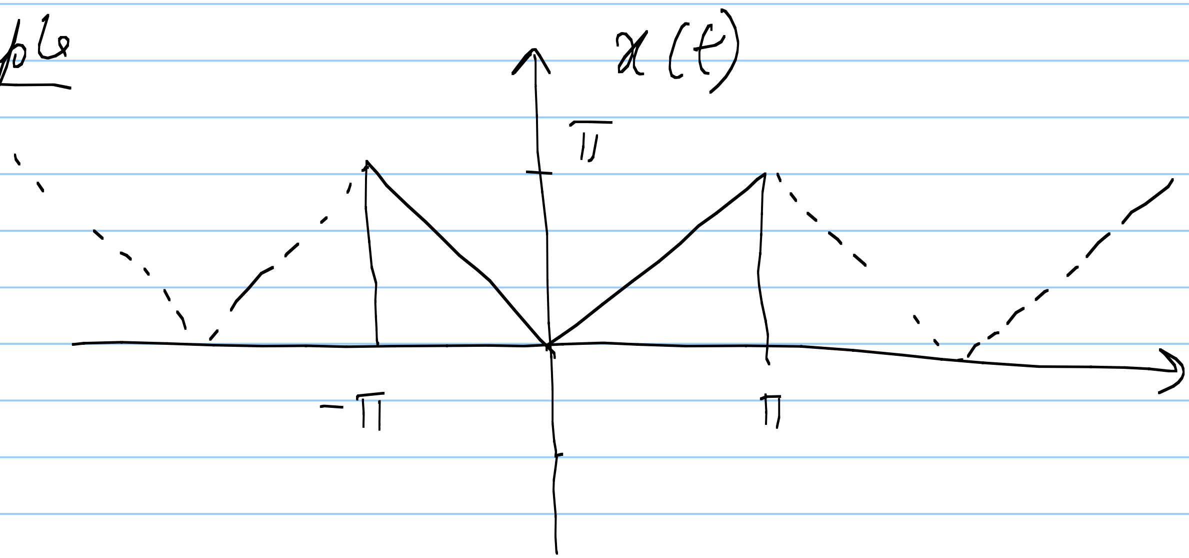
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

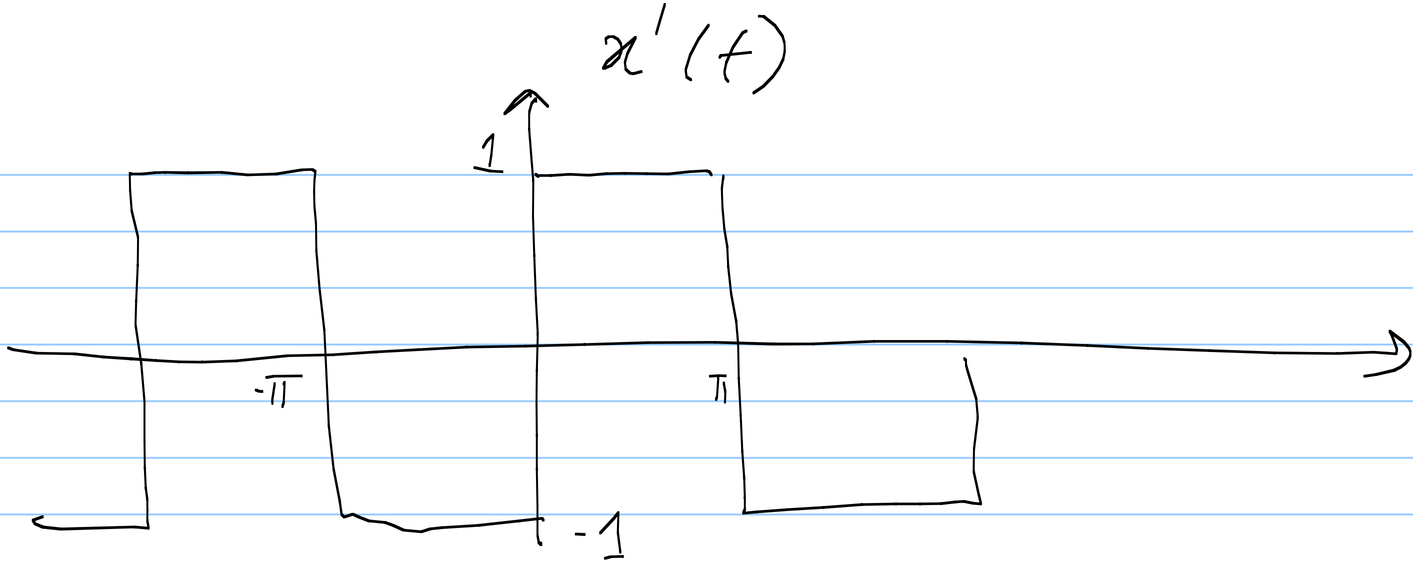
$$\frac{dx}{dt} = \frac{d}{dt}\left[\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}\right]$$

$$\stackrel{?}{=} \sum_{k=-\infty}^{\infty} jk\Omega_0 a_k \, e^{jk\Omega_0 t}$$

$$x'(t) \longleftrightarrow jk\Omega_0 a_k$$

<u>Example</u>





$$x'(t) \longleftrightarrow \begin{cases} \dfrac{2}{jk\pi} & k \text{ odd} \\[2ex] 0 & k \text{ even} \end{cases}$$

$$x(t) \longleftrightarrow a_k$$

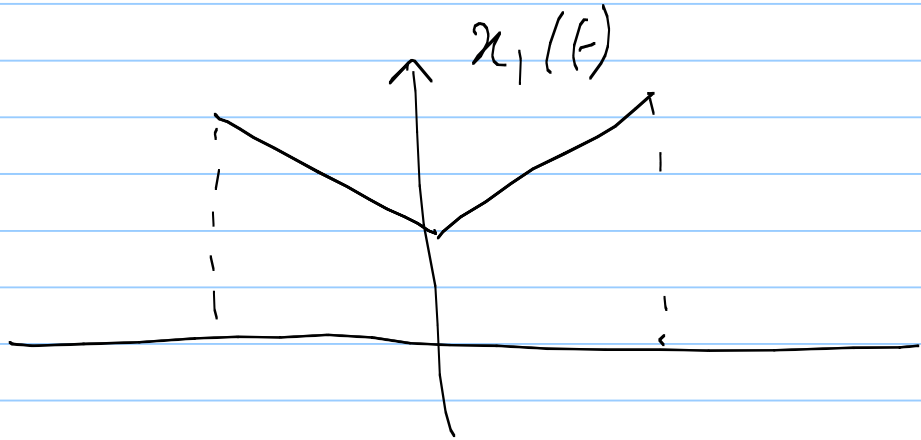
$$x'(t) \longleftrightarrow jk\Omega_0 a_k = b_k$$

$$b_k = \begin{cases} \dfrac{2}{jk\pi} & k = \text{odd} \\ 0 & k \text{ even} \end{cases}$$

$$a_k = \frac{2}{jk\pi \cdot jk\Omega_0} \qquad k \text{ odd}$$

$$= \frac{-2}{\pi \Omega_0 k^2} \quad k \neq 0 \quad \Omega_0 = 1$$

Now consider



$$x_1'(t) = x'(t)$$

Hence a_0 has to be computed separately

If the N^{th} derivative of $x(t)$ becomes impulsive, then the FS coefficients decay as $\frac{1}{K^N}$

Integration

$$\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{a_k}{jk\Omega_0} \qquad k \neq 0$$

$$a_0 = 0$$

Parsevals' Relation

$$\frac{1}{T}\int_T |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

$$\frac{1}{T}\int_T x(t)\,x^*(t)\,dt$$

$$= \frac{1}{T}\int_T \left[\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}\right]\left[\sum_{l=-\infty}^{\infty} a_l^* e^{-jl\Omega_0 t}\right] dt$$

$$\overset{?}{=} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} a_k a_l^* \frac{1}{T} \int_T e^{j(k-l)\Omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} |a_k|^2$$