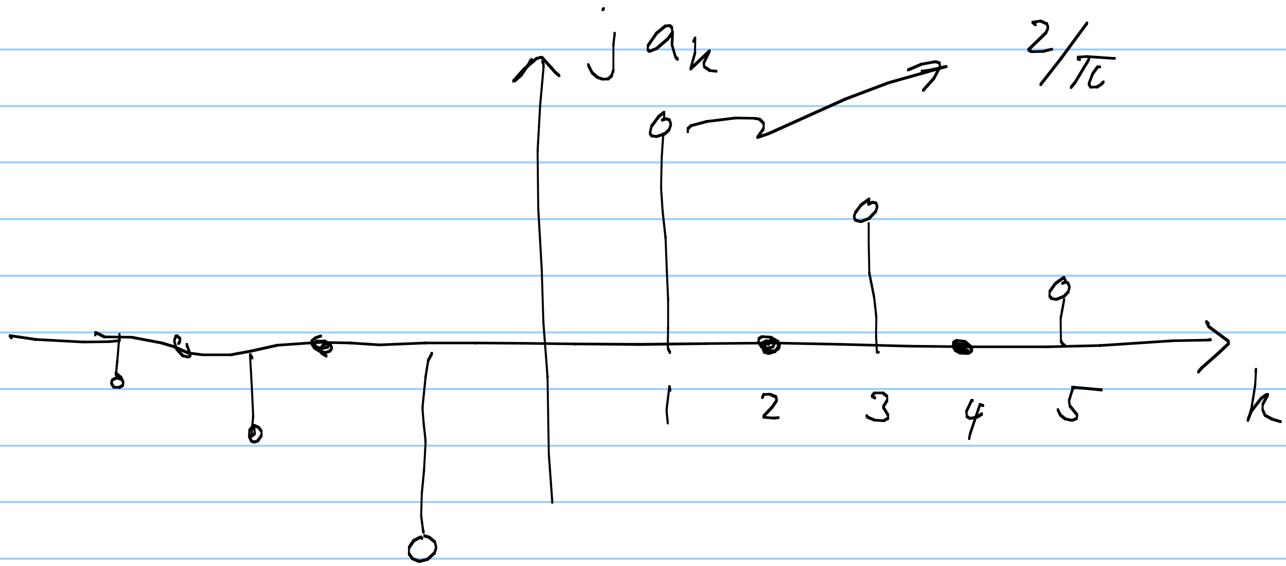
EC 5142 29th Aug. 2011

Note Title 29-08-2011

$$a_k = \begin{cases} \dfrac{2}{j\pi k} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$



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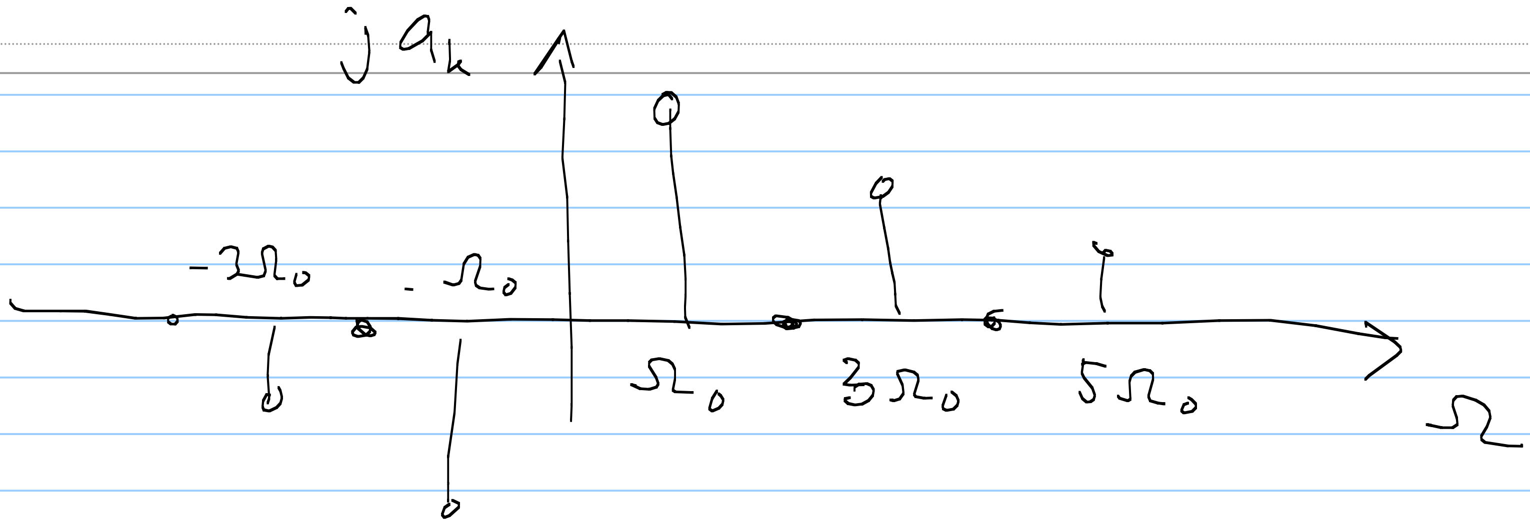


Symmetry

$x(t)$ is real-valued

$$x(t) = \sum_{k=0}^{\infty} A_k \cos k\Omega_0 t - \sum_{k=1}^{\infty} B_k \sin k\Omega_0 t$$

Suppose $x(t)$ is even, i.e. $x(t) = x(-t)$

$$x(-t) = \sum_{k=0}^{\infty} A_k \cos k\Omega_0 t + \sum_{k=1}^{\infty} B_k \sin k\Omega_0 t$$

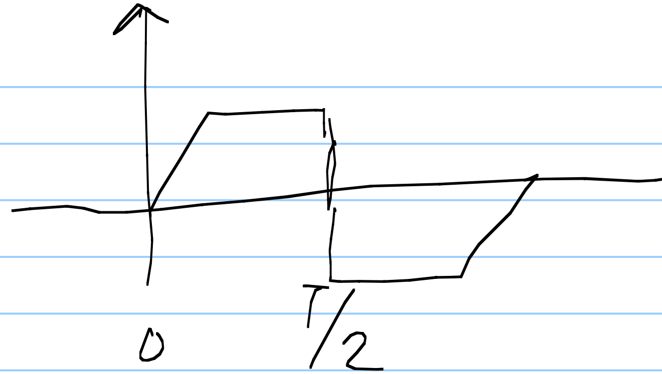
$$= x(t)$$

$$\Rightarrow \quad B_k = 0$$

Suppose $x(t) = -x(-t)$ i.e. odd

$$\Rightarrow \quad A_k = 0$$

Halfwave (odd) symmetry)



$$x(t + T/2) = -x(t)$$

$$\frac{1}{T}\int_0^{T/2} x(t)\, e^{-jk\Omega_0 t}\, dt + \underbrace{\frac{1}{T}\int_{T/2}^{T} x(t)\, e^{-jk\Omega_0 t}\, dt}_{t + T/2 = \lambda}$$

$a_k = 0 \quad k = \text{even}$

What about halfwave even symmetry?

$x(t+T/2) = x(t)$

Properties

(1) Linearity $\quad x_1(t+T) = x_1(t) \longleftrightarrow a_k$

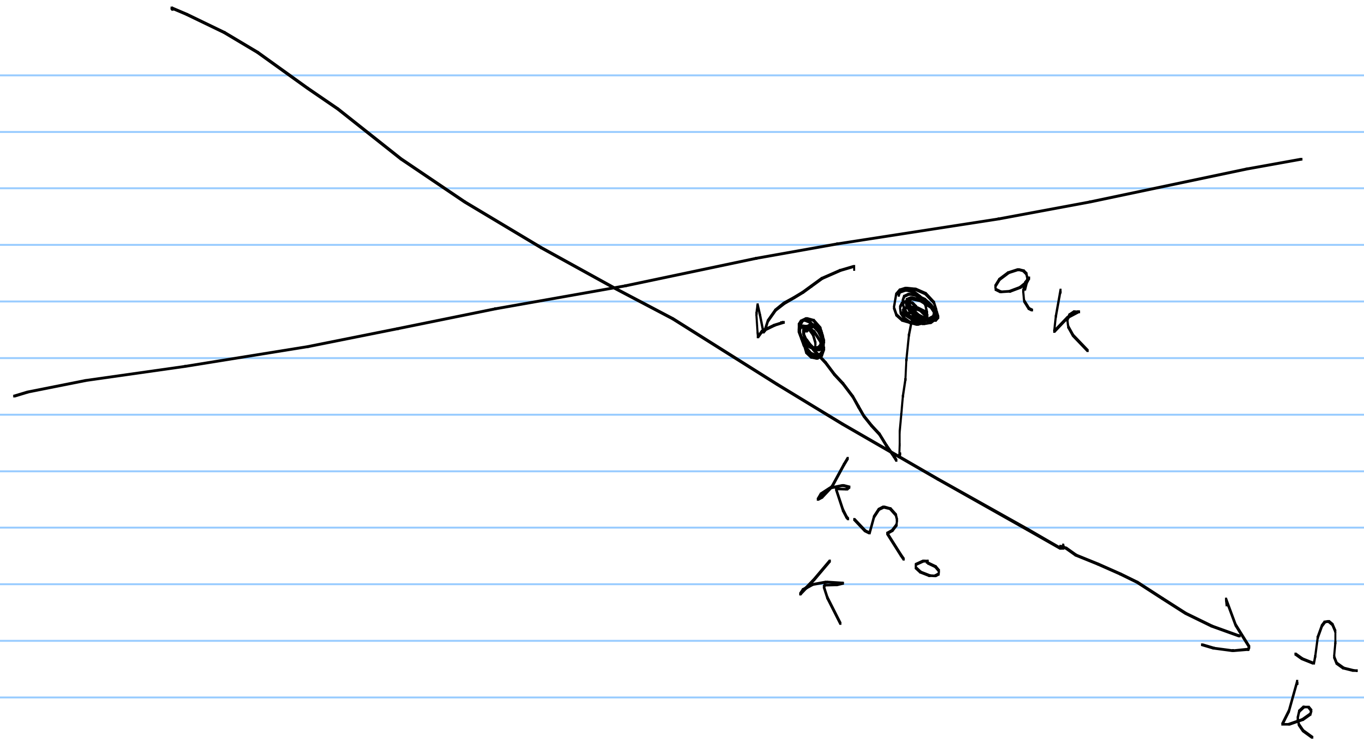
$x_2(t+T) = x_2(t) \longleftrightarrow b_k$

$$c_1 x_1(t) + c_2 x_2(t) \xleftrightarrow{FS} c_1 a_k + c_2 b_k$$

(2) Time Shift

$$y(t) = x(t - t_0) \longleftrightarrow e^{-jk\Omega_0 t_0} a_k = b_k$$

$$|b_k| = |a_k|$$



a k
$k\Omega_0$
k
Ω
k

$$y(t) \longleftrightarrow b_k$$

(3) $= x(-t) \longleftrightarrow b_k = a_{-k}$

$x(t)$ is real valued $\Rightarrow$ $a_{-k} = a_k^*$

$x(t)$ is even $\Rightarrow$ $a_{-k} = a_k$

$$a_k = a^*_k = a_{-k} \quad \text{real \& even}$$

(4) $y(t) = x(ct)$

$$x(t) = x(t + T_x)$$

$$y(t) = x(ct) = x(ct + T_x)$$

$$y\left(t - \frac{T_x}{c}\right) = x\left(c\left(t - \frac{T_x}{c}\right) + T_x\right)$$

$$= x(ct)$$

$$\Rightarrow \quad T_y = \frac{T_x}{c}$$

$$b_k = \frac{1}{T_y} \int_0^{T_y} y(t)\, e^{-jk\frac{2\pi}{T_y}t}\, dt$$

$$= \frac{1}{T_y} \int_0^{T_y} x(ct)\, e^{-jk\frac{2\pi}{T_y}t}\, dt$$

Letting $ct = \lambda$, we get

$$b_k = \frac{1}{T_y} \int_0^{T_y} y(t)\, e^{-jk\frac{2\pi}{T_y}t}\, dt = a_k$$