EC 5142 Aug. 24 2011

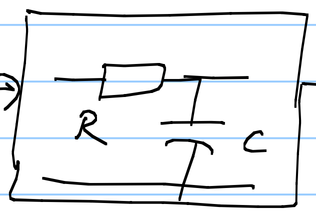
Note Title 24-08-2011

$$x(t) = \cos \Omega_0 t + \sin^2 \Omega_0 t$$

$$= \cos \Omega_0 t + \frac{1}{2}\left(1 - \cos 2\Omega_0 t\right)$$

$$a_0 = \frac{1}{2} \qquad a_1 = a_{-1} = \frac{1}{2} \qquad a_2 = a_{-2} = -\frac{1}{4}$$

$x(t) \longrightarrow$



$\longrightarrow y(t)$

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

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$$H(j\Omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\Omega t}\, dt = \frac{1}{1 + j\Omega RC}$$

$$y(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\Omega_0 t}$$

$$c_0 = a_0\, H(j\Omega)\Big|_{\Omega=0} = \frac{1}{2}$$

$$c_1 = \frac{1}{2}\, \frac{1}{1 + j\Omega_0 RC} \qquad c_{-1} = \frac{1}{2}\, \frac{1}{1 - j\Omega_0 RC}$$

$$C_2 = -\frac{1}{4}\,\frac{1}{1+j\,2\Omega_0 RC} \qquad C_{-2} = -\frac{1}{4}\,\frac{1}{1-2j\Omega_0 RC}$$

For real-valued $x(t)$, $x(t) = x^*(t)$

$$x^*(t) = \left[\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}\right]^*$$

$$= \sum_{k=-\infty}^{\infty} a_k^* e^{-jk\Omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_{-k}^{*} e^{jk\Omega_0 t}$$

$$= x(t)$$

$$\Rightarrow \quad a_k = a_{-k}^{*} \quad \Rightarrow \quad |a_k| = |a_{-k}|$$

Again, for real-valued $x(t)$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k e^{jk\Omega_0 t} + \sum_{k=1}^{\infty} a_{-k} e^{-jk\Omega_0 t}$$

$$= a_0 + \sum_{k=1}^{\infty} \left[a_k e^{jk\Omega_0 t} + a_k^* e^{-jk\Omega_0 t} \right]$$

$$a_k = A_k e^{j\theta_k}$$

$$= a_0 + \sum_{k=1}^{\infty} A_k e^{j(k\Omega_0 t + \theta_k)} + A_k e^{-j(k\Omega_0 t + \theta_k)} \Big]$$

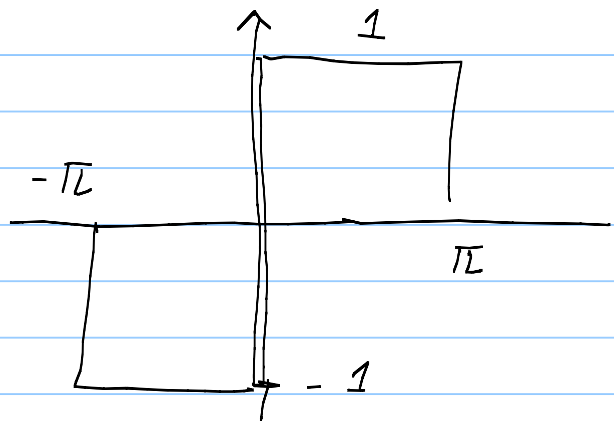
$$= a_0 + \sum_{k=1}^{\infty} 2 A_k \cos(k\Omega_0 t + \theta_k)$$

$$A_k \cos(k\Omega_0 t + \theta_k) = \underbrace{A_k \cos\theta_k}_{B_k} \cos k\Omega_0 t - \underbrace{A_k \sin\theta_k}_{C_k} \sin k\Omega_0 t$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} B_k \cos k\Omega_0 t - C_k \sin k\Omega_0 t$$

Eg:

$x(t)$



$$\Omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

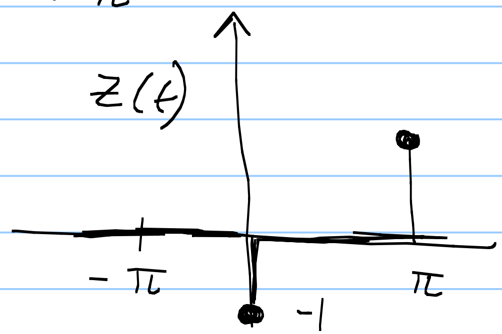
$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\Omega_0 t}\,dt = \begin{cases} \dfrac{2}{j\pi k} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

$$x(t) = \frac{4}{\pi}\left[\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \ldots\ldots\right]$$

$$x(t) = \begin{cases} -1 & -\pi < t < 0 \\ 1 & 0 < t < \pi \end{cases}$$

$$y(t) = \begin{cases} -1 & -\pi < t \leq 0 \\ 1 & 0 < t \leq \pi \end{cases}$$

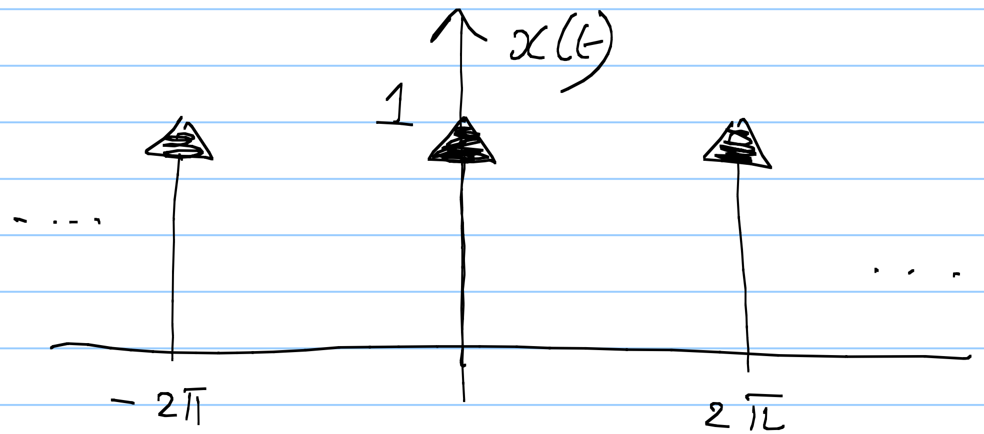
$$y(t) = x(t) +$$



$y(t)$ has the same set of FS coeffs. as $x(t)$

since $z(t)$'s FS coeffs. are ZERO.

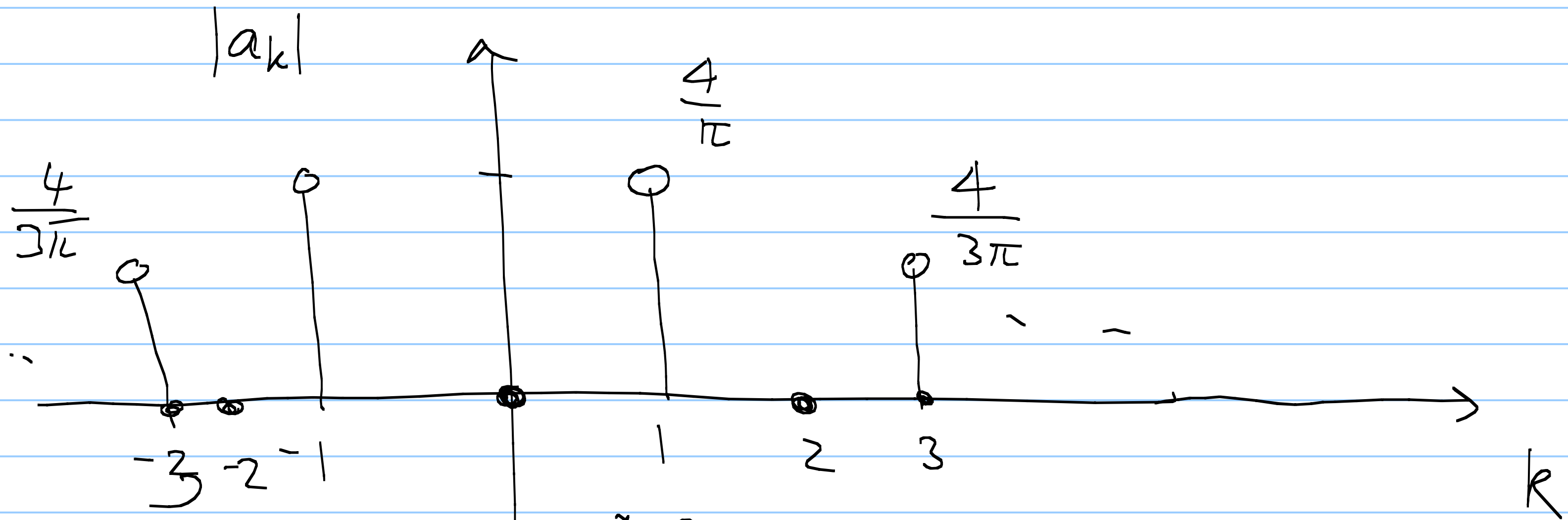
Eg:

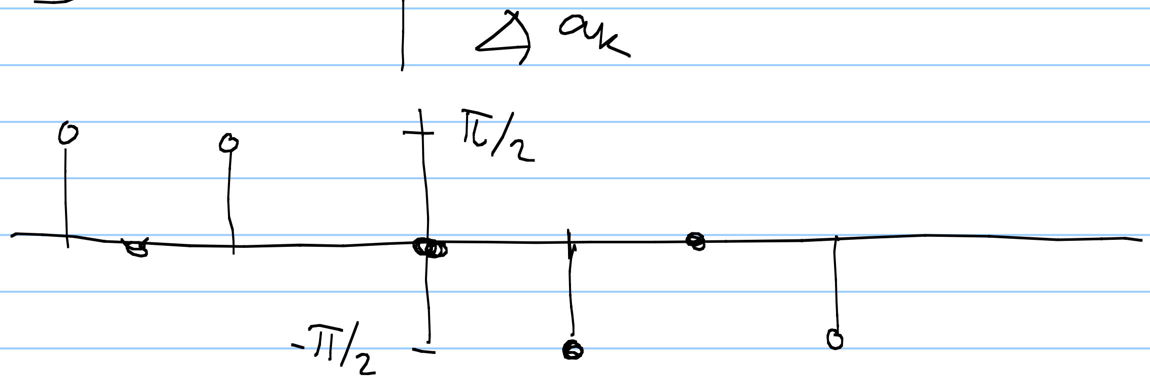


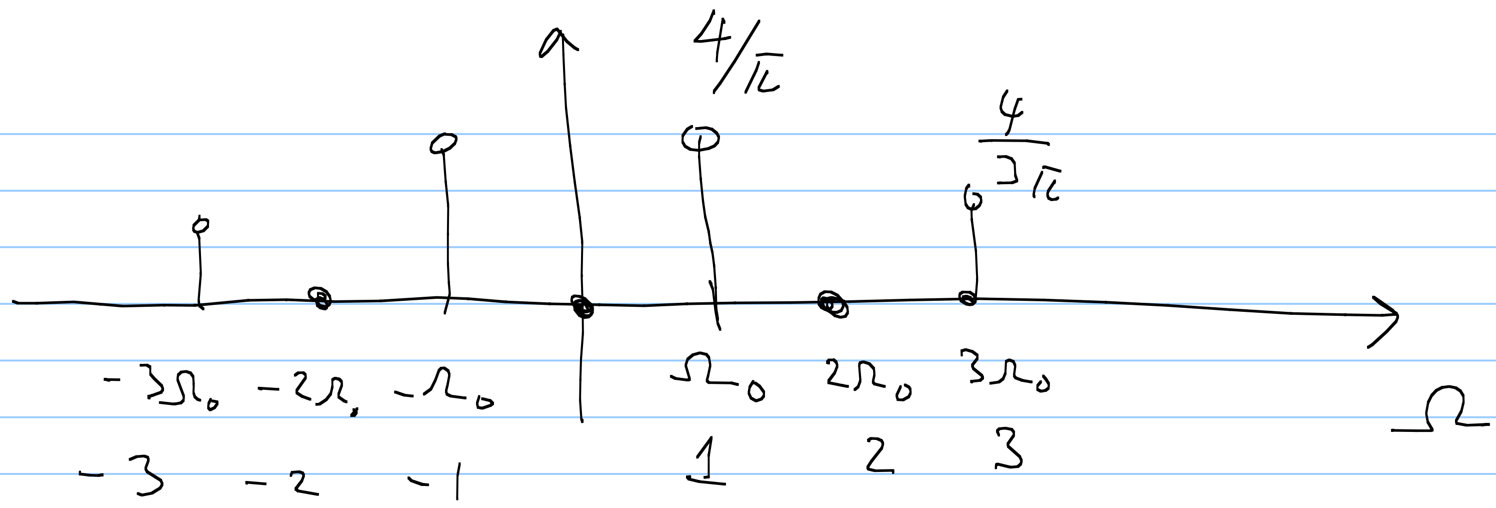
$$a_k = \frac{1}{2\pi} \quad \forall k$$

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t + 2k\pi)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$






4/π
4/3π
−3Ω₀
−2Ω₀
−Ω₀
Ω₀
2Ω₀
3Ω₀
Ω
−3
−2
−1
1
2
3