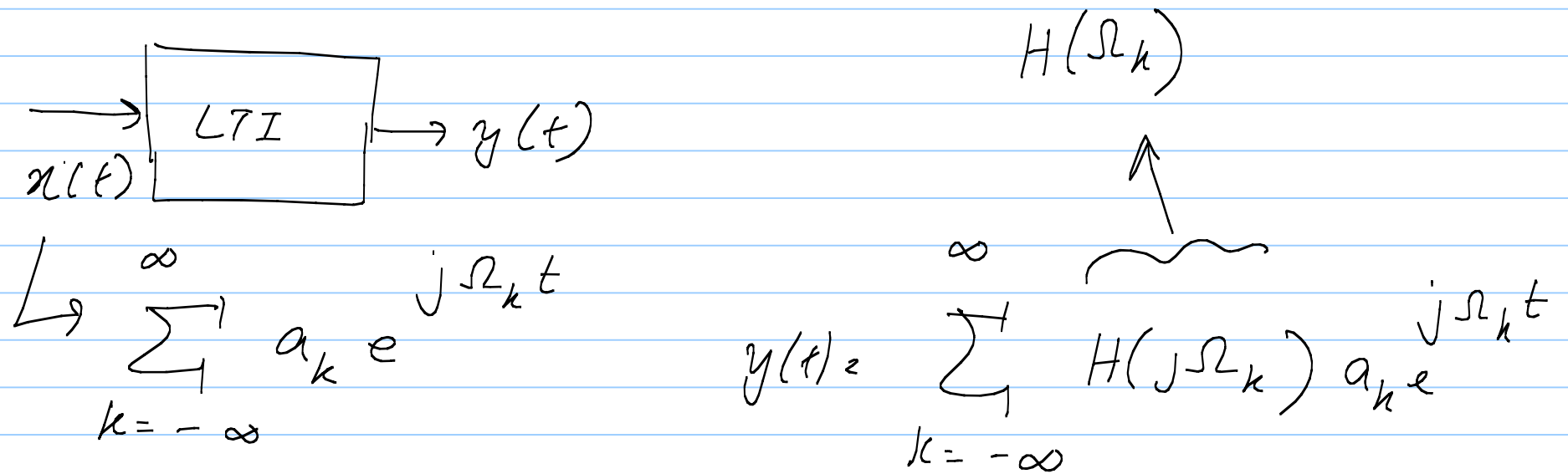


EC 5142 23rd Aug. 2011

Note Title

23-08-2011



No new frequencies are introduced by an LTI system

If $x(t+T) = x(t)$, then

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

$$\Omega_0 = \frac{2\pi}{T}$$

Fourier Series representation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

synthesis eqn

Inner product of $x(t)$ & $y(t)$: $\int_R x(t) y^*(t) dt$

$$\int_0^T x(t) \cdot e^{-j l \Omega_0 t} dt = \int_0^T \left[\sum_{k=-\infty}^{\infty} a_k e^{j k \Omega_0 t} \right] e^{-j l \Omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-l)\Omega_0 t} dt$$

$$= a_k \delta_{kl} T \quad \rightarrow \delta_{kl} = \begin{cases} 1 & k=l \\ 0 & k \neq l \end{cases}$$

$$= a_l T$$

$$a_l = \frac{1}{T} \int_0^T x(t) e^{-j l \Omega_0 t} dt$$

analysis
equation

$$\cos \Omega_0 t = \frac{1}{2} e^{j \Omega_0 t} + \frac{1}{2} e^{-j \Omega_0 t}$$

$$a_1 = \frac{1}{2} = a_{-1}$$

$$\sin \Omega_0 t : \quad a_1 = \frac{1}{2j} \quad a_{-1} = -\frac{1}{2j}$$

$$\cos \Omega_0 t + \sin^2 \Omega_0 t = \cos \Omega_0 t + \frac{1}{2}(1 - \cos 2\Omega_0 t)$$

$$= \cos \Omega_0 t + \underbrace{\frac{1}{2}}_{a_0} - \frac{1}{2} \cos 2\Omega_0 t$$

$$a_0 = \frac{1}{2}$$

$$a_2 = a_{-2} = -\frac{1}{4}$$

$$a_1 = \frac{1}{2} = a_{-1}$$