Note Title 23-08-2011

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$$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\Omega_k t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} \overbrace{H(j\Omega_k)}^{H(\Omega_k)} a_k e^{j\Omega_k t}$$

No new frequencies are introduced by an LTI system

If $x(t+T) = x(t)$, then

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t} \qquad \Omega_0 = \frac{2\pi}{T}$$

Fourier Series representation

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}}$$ synthesis eqn

Inner product of $x(t)$ & $y(t)$: $\int_{R} x(t)\, y^{*}(t)\, dt$

$$\int_{0}^{T} x(t) \cdot e^{-jl\Omega_0 t}\, dt = \int_{0}^{T} \left[\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t} \right] e^{-jl\Omega_0 t}\, dt$$

$$\overset{?}{=} \sum_{k=-\infty}^{\infty} a_k \underbrace{\int_{0}^{T} e^{j(k-l)\Omega_0 t}\, dt}$$

$$\hookrightarrow \delta_{kl} = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases}$$

$$= a_k\, \delta_{kl}\, T$$

$= a_\ell T$

$$\boxed{a_\ell = \frac{1}{T}\int_0^T x(t)\, e^{-j\ell\Omega_0 t}\, dt}$$

analysis equation

$$\cos \Omega_0 t = \frac{1}{2} e^{j\Omega_0 t} + \frac{1}{2} e^{-j\Omega_0 t}$$

$$a_1 = \frac{1}{2} = a_{-1}$$

$\sin \Omega_0 t$: $\quad a_1 = \frac{1}{2j} \qquad a_{-1} = -\frac{1}{2j}$

$$\cos \Omega_0 t + \sin^2 \Omega_0 t = \cos \Omega_0 t + \frac{1}{2}\left(1 - \cos 2\Omega_0 t\right)$$

$$= \cos \Omega_0 t + \underbrace{\frac{1}{2}}_{a_0} - \frac{1}{2} \cos 2\Omega_0 t$$

$a_0 = \frac{1}{2} \qquad a_2 = a_{-2} = -\frac{1}{4}$

$a_1 = \frac{1}{2} = a_{-1}$