EC 5142 22nd Aug. 2011

$$\frac{d^k}{dt^k} a_k e^{st} = a_k s^k e^{st}$$

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{l=0}^{M} b_l \frac{d^l}{dt^l} x(t)$$

Homogeneous soln is obtained by solving

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = 0$$

Note Title 22-08-2011

If $y(t) = e^{st}$

$$e^{st}\left[\sum_{k=0}^{N} a_k s^k\right] = 0$$

$$\Rightarrow \sum_{k=0}^{N} a_k s^k = 0$$

If the roots are distinct,

$$y_h(t) = \sum_{k=1}^{N} c_k e^{s_k t}$$

c_k : are determined by the auxiliary conditions

$e^{s_k t}$: natural modes of the system

Recall.

$$Y_I e^{-at} + \frac{1}{a-b}\left[-e^{-at} + e^{-bt}\right] u(t)$$

For $Y_I = 0$, we get $\underbrace{-\frac{1}{a-b} e^{-at}}_{\text{natural mode}} + \underbrace{\frac{1}{a-b} e^{-bt}}_{\text{forced response}} \quad t \geq 0$

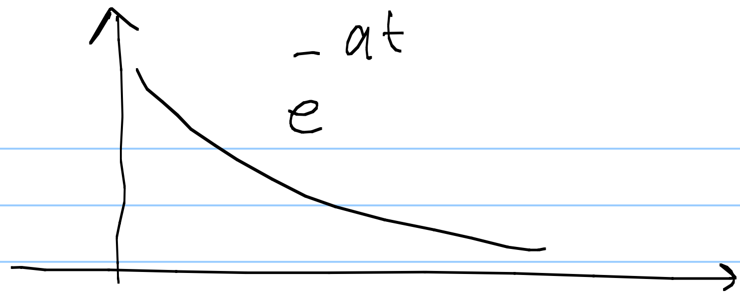
Repeated roots:

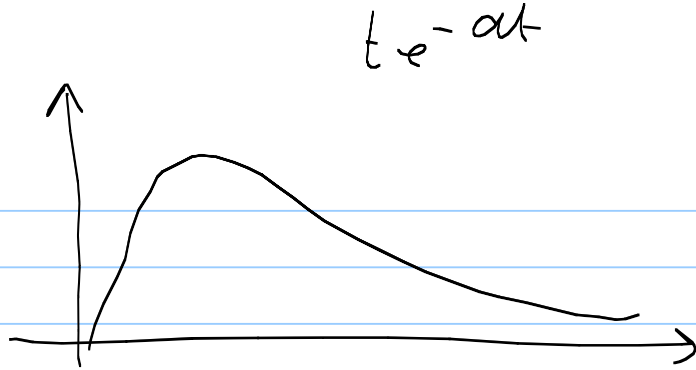
Eg 1 $\quad e^{st}(s+a)^2 = 0$

Modes are: $\quad c_1 e^{-at}, \quad c_2 \, t e^{-at}$

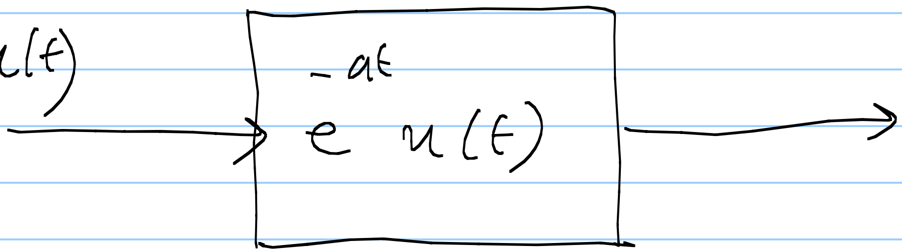
For multiplicity 'r'

$$c_1 e^{-at}, \; c_2 t e^{-at}, \; c_3 t^2 e^{-at}, \ldots, \; c_r t^{r-1} e^{-at}$$

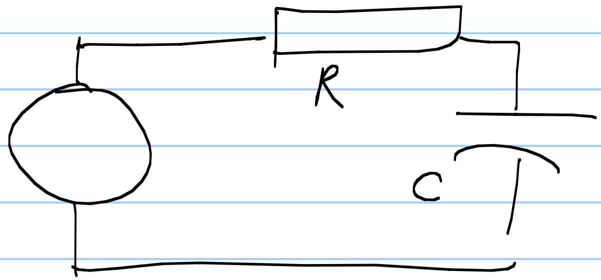




$e^{-bt}u(t)$



$e^{-bt}u(t)$



$$\frac{1}{a-b}\left[e^{-bt}-e^{-at}\right]u(t)$$

If $a = b$, sol^n is of the form $\underbrace{K e^{-at}}_{\text{natural mode}} + C \cdot t e^{-at}$

$$e^{-at} u(t) * e^{-at} u(t) = \int_0^t e^{-a\tau} e^{-a(t-\tau)} d\tau$$

$$= e^{-at} \int_0^t d\tau$$

$$= t e^{-at} u(t)$$

Look up Lathi for zero-input & zero state responses.

$$x(t) = e^{st} \longrightarrow \boxed{T} \longrightarrow T e^{st}$$

For LTI: $$y(t) = e^{st}\, \underbrace{H(s)}_{\int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau}$$

$$= \lambda \cdot x(t)$$

Reminiscent of $\underline{A}\,\underline{x} = \lambda\,\underline{x}$

$$x(t) = e^{j\Omega t} \longrightarrow \boxed{} \longrightarrow y(t) = e^{j\Omega t} \int_{-\infty}^{\infty} h(\tau) e^{-j\Omega \tau}\, d\tau$$

$H(j\Omega)$

$H(j\Omega)$ is also denoted by $H(\Omega)$

$\hookrightarrow$ just notation

$$\sum_{k=1}^{N} a_k e^{s_k t} \longrightarrow \boxed{LTI} \longrightarrow \sum_{k=1}^{N} H(s_k) a_k e^{s_k t}$$

$$\sum_{k=1}^{N} a_k e^{j\Omega_k t} \longrightarrow \boxed{LTI} \longrightarrow \sum_{k=1}^{N} a_k H(j\Omega_k) e^{j\Omega_k t}$$

No new frequencies !!

We will assume

$$\underbrace{\sum_{k=-\infty}^{\infty} a_k e^{j\Omega_k t}}_{\text{very powerful way of representing certain types of } x(t)} \longrightarrow \boxed{\text{LTI}} \longrightarrow \sum_{k=-\infty}^{\infty} a_k H(j\Omega_k) e^{j\Omega_k t}$$