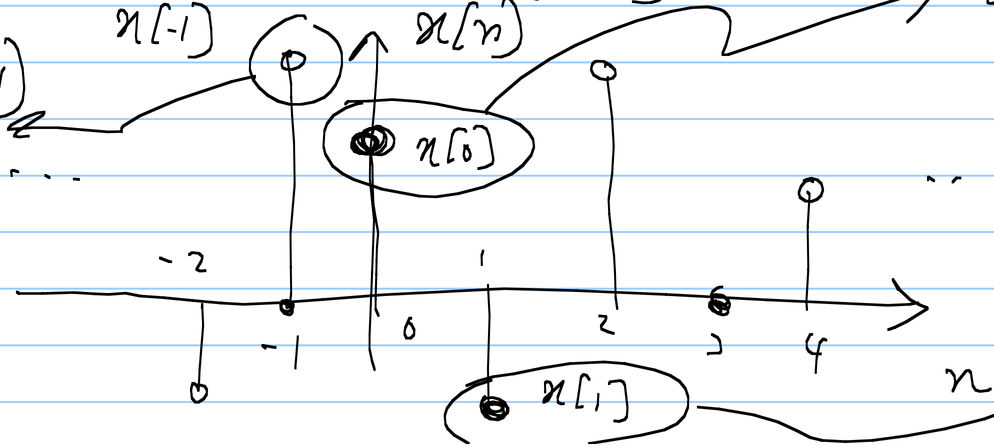
Systems that are both linear & time invariant form an important class LTI

$$\delta(t),\ \delta[n] \longrightarrow \boxed{\begin{matrix} h(t) \\ h[n] \end{matrix}} \longrightarrow y(t),\ y[n]$$



$x[-1]\,\delta[n+1]$

$x[0]\,\delta[n]$

$x[1]\,\delta[n-1]$

Note Title 17-08-2011

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$\delta[n] \longrightarrow h[n]$$

$$\delta[n-k] \longrightarrow h[n-k] \quad \text{time invariance}$$

$$x[k]\,\delta[n-k] \longrightarrow x(k)\,h[n-k] \quad \text{homogeneity}$$

$$\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \longrightarrow \sum_{k=-\infty}^{\infty} x(k)\,h[n-k] \quad \text{additivity}$$

$$\underbrace{\sum x[k]\,\delta[n-k]}_{}$$

$$x[n] \longrightarrow \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = y[n]$$

$$x(t) \longrightarrow \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = y(t)$$

For causal systems, $h(t) = 0 \quad t < 0$

necessary & sufficient

Necessity: $\delta(t)$ is applied at $t = 0$

& hence $h(t)$ cannot be non zero for $t < 0$

for a causal system

Sufficiency: $h(t) = 0 \quad t < 0$

$$y(t_0) = \int_{-\infty}^{\infty} x(\tau)\, h(t_0 - \tau)\, d\tau$$

$$= \int_{-\infty}^{t_0} x(\tau)\, h(t_0 - \tau)\, d\tau$$

$\Rightarrow$ $y(t_0)$ depends only on i/p values $t \leq t_0$

Stability $\quad |x(t)| \leq B_x$ bounded i/p

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau \right|$$

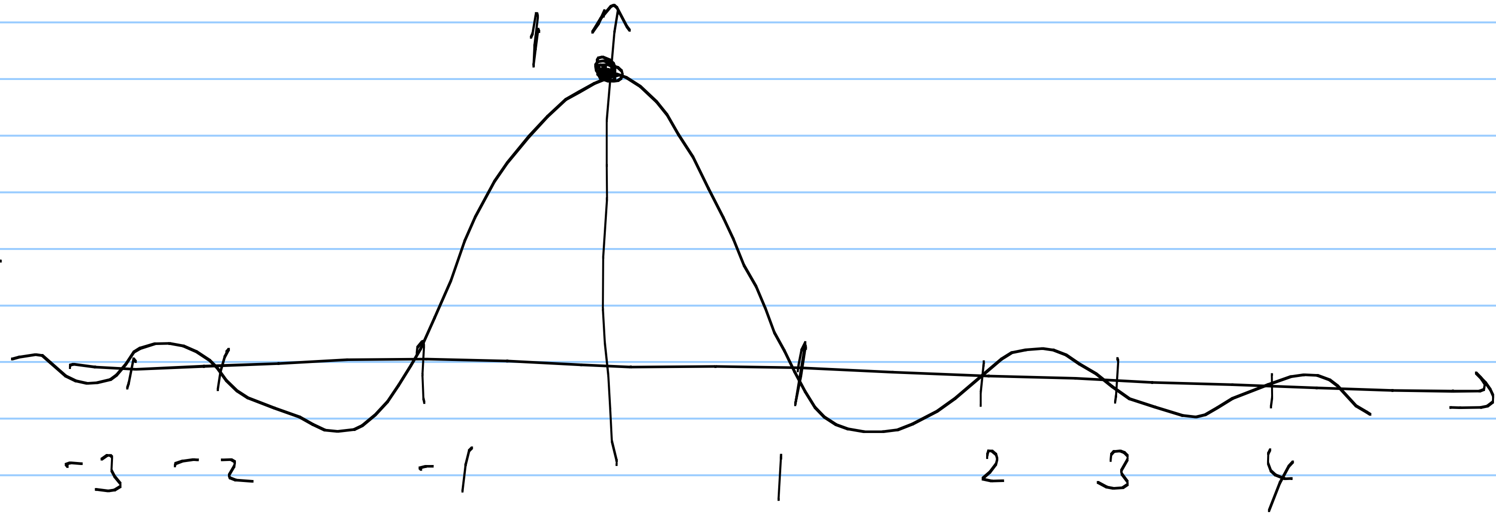
$$\leq \int_{-\infty}^{\infty} \underbrace{|x(t-\tau)|}_{\leq B_x} \, |h(\tau)|\, d\tau$$

$$|y(t)| \leq B_x \int_{-\infty}^{\infty} |h(\tau)|\, d\tau$$

For bounded o/p we require $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \quad \text{for DT systems}$$

$$\frac{\sin \pi t}{\pi t}$$



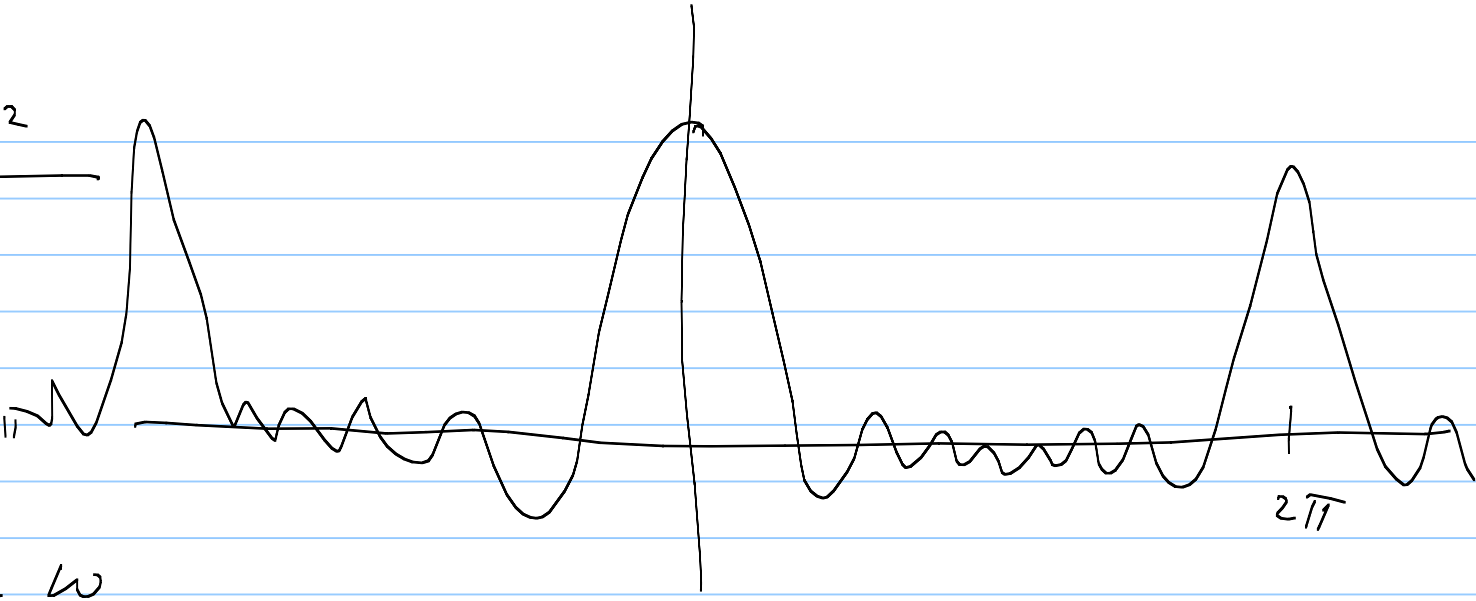
"Sinc" function

sometimes also called "analog sinc"

non periodic

$$\frac{\sin (2N+1)\,\omega/2}{\sin \omega/2}$$

"digital sinc"



periodic in ω

w/ period 2π

Dirichlet kernel

Matlab: diric

Systems represented by Linear Const. Coeff.
Differential Equations (LCCDE) are a very
Differnce
important class