EC5142 11th Aug. 2011

Note Title 11-08-2011

Classification of Systems.

1) Linearity

Additivity + homogeneity = linearity

$v(t) = i(t) \cdot R$ linear system

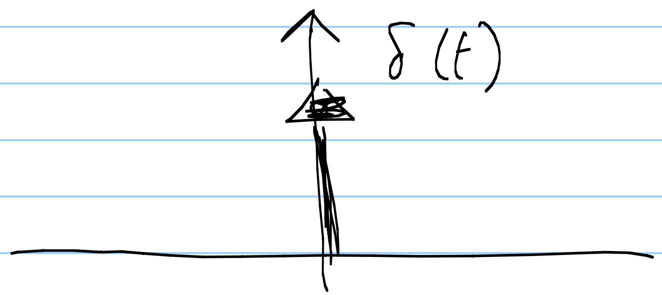
$\underbrace{a}_{} \cdot x \longrightarrow a \cdot y$

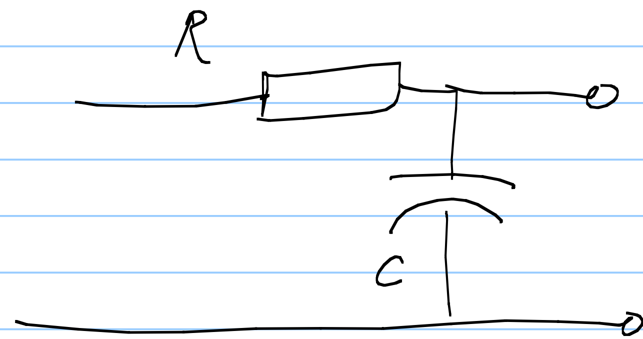
$a \in \mathbb{C}$

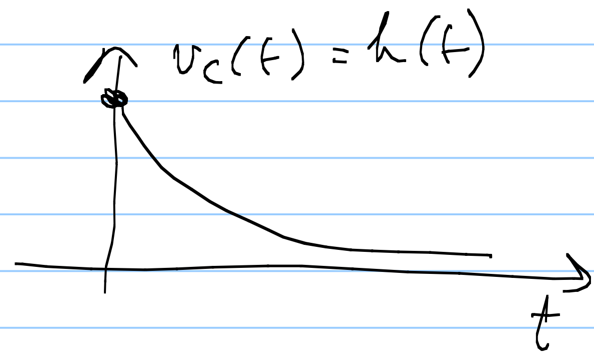
$0 \cdot x \longrightarrow 0 \cdot y$

If $x(t) = 0 \ \forall t$, then $y(t) = 0 \ \forall t$

If $x(t) = 0$ for $t < t_0$, then $y(t) = 0$ for $t < t_0$







$$h(t) = e^{-t/RC} u(t) = v_c(t)$$

$$v_c(0^-) = 0$$

2) Causality

If the Output $y(t)\Big|_{t=t_0}$ depends only on

i/p $x(t)$ for $t \le t_0$ & $y(t)$ for $t < t_0$

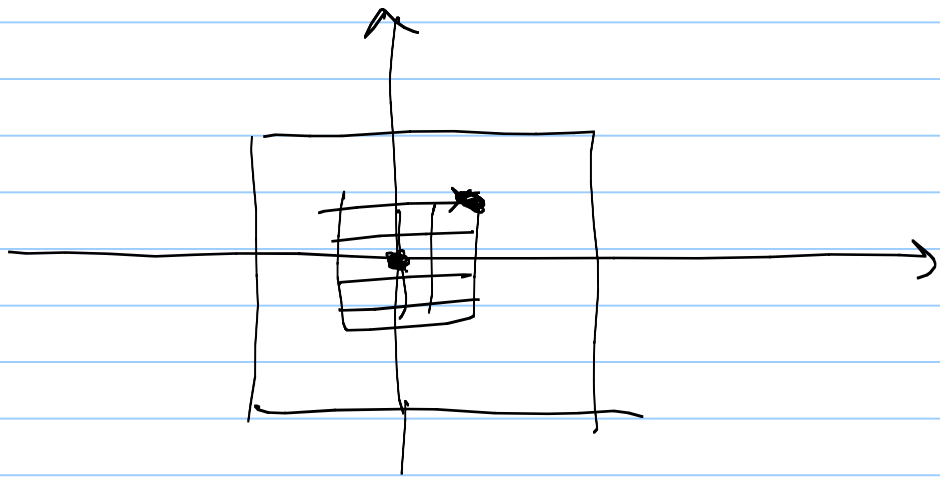
then the system is causal

Eg: $y(t) = c\,x(t)$: causal

Eg: $y(t) = x(-t)$ non causal

since for $t<0$, the o/p is a fn. of $x(t)$ for $t>0$.

Eg: $y(t) = x(t) \cos(t+1)$ causal



3) Memory or Memoryless

If o/p at $t=t_0$ depends only i/p only at $t=t_0$ then the system is memoryless.

$v(t) = i(t) \cdot R$ memoryless

$$v_c(t) = \int_{-\infty}^{t} i_c(\tau)\, d\tau$$ has memory

$y[n] = x[n] + a\, y[n-1]$ has memory
causal, linear

$y[n] = x[n] + b\, x[n+1] + a\, y[n-1]$ has memory
linear
non causal

4) <u>Time Invariance</u>

If $x(t) \longrightarrow y(t)$

& $x(t-t_0) \longrightarrow y(t-t_0)$

then the system is time invariant

$$y(t) = t\,x(t)$$

$$y(t-t_0) = (t-t_0)\,x(t-t_0) \qquad \text{——} \quad (1)$$

$$x(t) \longrightarrow x(t-t_0) = w(t) \longrightarrow t\,w(t)$$

$$t\,x(t-t_0) \quad -(2)$$

$\Rightarrow$ Time variant

$y(t) = x^2(t)$ nonlinear
time invariant

Linearity + time invariance : important
Sub class of systems
LTI systems

(5) <u>Stability</u> Bounded i/p Bounded o/p (BIBO) Stability

If $|x(t)| \le B_x$ produces $y(t)$ s.t.

$|y(t)| \le B_y$, then the system is BIBO stable

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

If $x(t) = u(t)$, $y(t) = t\, u(t)$

$e^{j\omega_0 n}$: $x[n+N] = x[n]$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{k}{N} \qquad k = 0, 1, \ldots, N-1$$

For some k & N, N need not be the fundamental period.

Even & Odd

If $x(t) = x(-t)$, then $x(t)$ is even

($x(t)$ is real valued)

$\cos(t)$: even

$\sin(t)$: odd

$x(t) = -x(-t)$ then $x(t)$ is odd

For complex signals

If $x(t) = x^*(-t)$, then it is conjugate even
conjugate symmetric

$x(t) = -x^*(-t)$, conjugate antisymmetric

$$e^{j\Omega_0 t} \longrightarrow \left(e^{-j\Omega_0 t}\right)^* = e^{j\Omega_0 t}$$: conjugate symmetric

Real Valued :

Any signal can be expressed as a sum of even & odd parts.

$$x(t) = \underbrace{x_o(t)}_{\text{odd part}} + \underbrace{x_e(t)}_{\text{even part}}$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$