Note Title 10-08-2011

EC 5142 10^{th} Aug. 2011

$x[n] = e^{j\omega_0 n}$: We want $x[n+N] = x[n]$

$$\Rightarrow f_0 = \frac{\omega_0}{2\pi} = \frac{k}{N} \Rightarrow \quad k = 0, 1, \ldots, N-1$$

Discrete-Time Convolution:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= x[n] * h[n]$$

$$y(t) = x(t) \cdot h(t)$$
$$y(ct) = x(ct) \cdot h(ct)$$

$$y(t) = x(t) * h(t)$$
$$y(ct) = x(ct) * h(ct) \quad \text{WRONG}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(ct) = \int_{-\infty}^{\infty} x(\tau)\, h(ct-\tau)\, d\tau$$

find the correct expression

Let $\left.\begin{array}{l} x[n+N] = x[n] \\ h[n+N] = h[n] \end{array}\right\}$ same period N

$$\underbrace{\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]}_{\text{periodic w/ period } N}$$

$= \quad 0, \ \infty, \text{ or } -\infty$

Suppose, $$\sum_{k=0}^{N-1} x[k]\, h[n-k] \overset{\text{def}}{=} x[n] \circledast_N h[n]$$

circular convolution — optional suffix N

periodic convolution

$$x(t+T) = x(t) \quad ; \quad h(t+T) = h(t)$$

$$x(t) \circledast h(t) = \int_0^T x(\tau)\, h(t-\tau)\, d\tau$$

For linear convolution, if $x[n]$ is of length M, & $h[n]$ is of length N, then

$x[n] * h[n] = y[n] \rightarrow$ is of length $M+N-1$

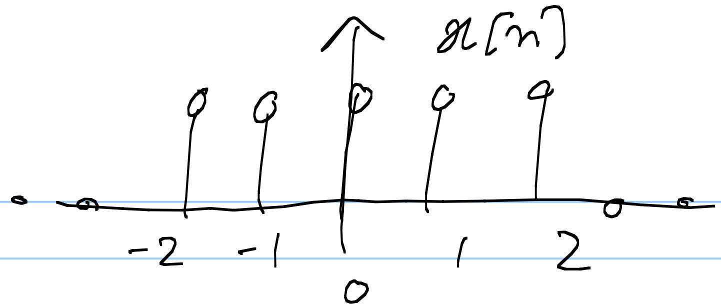
(Verify)

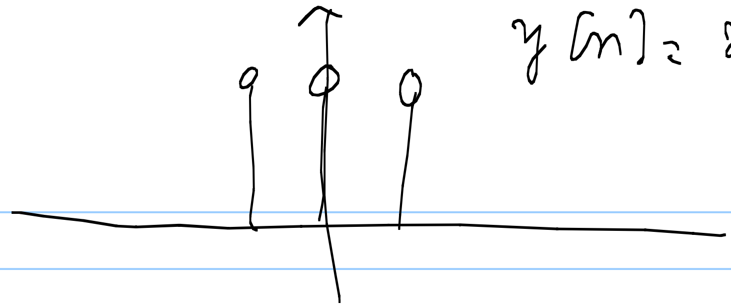
Scaling in Discrete Time

$x[an]$ $\qquad an \in \mathbb{Z}$

Eg: $y[n] = x[2n]$

$y[0] = x[0]$, $y[1] = x[2]$, $y[-1] = x[-2]$, ...





$y[n] = x[2n]$

$y[n] = x[n/2]$

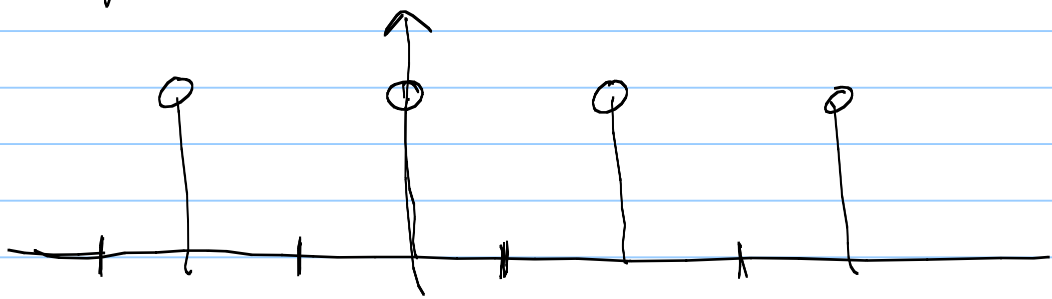
$y[0] = x[0]$

$y[2] = x[1]$

$y[-2] = x[-1]$

$y[1] = x[1/2]$ = undefined

$y[-1] = x[-1/2]$ = undefined



It is common to set $y[n] = 0$ when
$n = 2k+1$

$$y[n] = x[mn + p]$$

$$w[n] = x[n+p]$$
$$w[mn] = x[mn+p]$$

Is the other way possible? i.e., scaling first & then shifting?

Think about this

Systems

Takes an input, processes it, & produces an output.

$$x \longrightarrow \boxed{T} \longrightarrow y = Tx$$

(1) Linear

(a) additivity: If $x_1 \longrightarrow y_1$

& $x_2 \longrightarrow y_2$

and $x_1 + x_2 \longrightarrow y_1 + y_2$

then additivity holds

(b) Homogeneity:

$a \in \mathbb{C}$

If $x_1 \to y_1$

$a \cdot x_1 \to a \cdot y_1$

then homogeneity holds

If both additivity & homogeneity hold,
the system is <u>linear</u>

$$a_1 x_1 + a_2 x_2 \to a_1 y_1 + a_2 y_2 \quad \text{(superposition)}$$

$$y(t) = \frac{dx(t)}{dt}$$ verify that this is linear

$$y(t) = x^2(t)$$ verify that this is nonlinear

$$y[n] = b_0\, x[n] + b_1\, x[n-1] + a_1\, y[n-1]$$
(linear)

One important consequence of linearity is
if $x(t) = 0 \;\; \forall t$, $y(t) = 0$ for all t

$$x_1(t) \longrightarrow y(t)$$

$$0 \cdot x(t) \longrightarrow 0 \cdot y(t)$$

$$0 \longrightarrow 0$$