EE 5142 8^{th} Aug. 2011

Note Title 08-08-2011

Consider $x(t) * h(t)$

Suppose $h(t) = 0 \quad t < 0$

$h(t)$ can be written as $h(t)u(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, u(t-\tau)\, d\tau$$

For $\tau > t \quad u(t-\tau) = 0$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, \underbrace{u(t-\tau)}_{1}\, d\tau$$

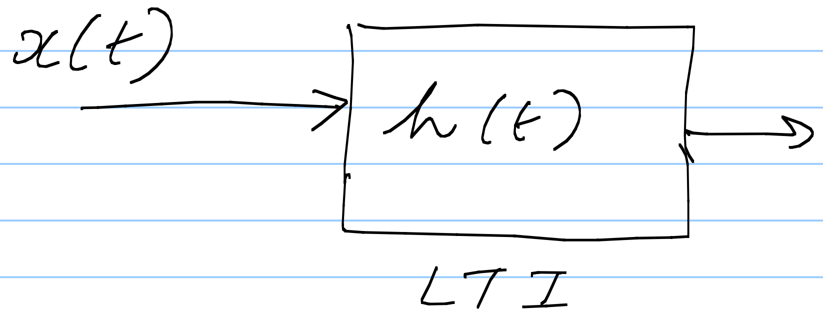
Now suppose $x(t) = 0 \quad t < 0$

i.e. $x(t)$ is written as $x(t)\,u(t)$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,u(\tau)\,h(t-\tau)\,u(t-\tau)\,d\tau$$

$$= \int_{0}^{t} x(\tau)\,\underbrace{u(\tau)}_{1}\,h(t-\tau)\,\underbrace{u(t-\tau)}_{1}\,d\tau$$

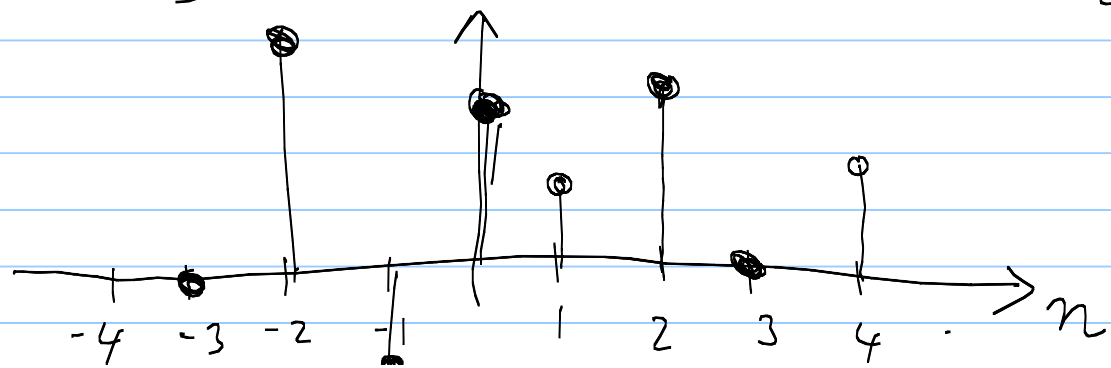
$$= \int_{0}^{t} x(\tau)\,h(t-\tau)\,d\tau$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

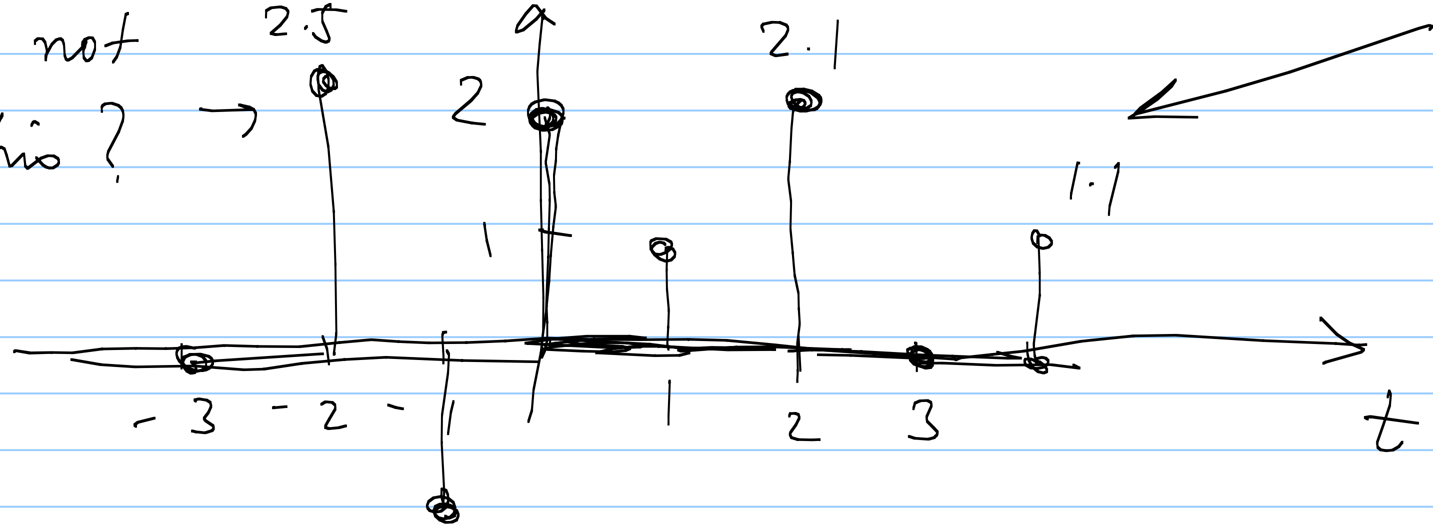
Discrete-time sequence

$x[n] \in \mathbb{C}$ $\qquad n \in \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$



$x[1.5]$ = undefined, NOT zero " "

Why not this?



$x(t)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = 0$$

Unit impulse: $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

Unit step: $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \underbrace{n < 0}_{} \end{cases}$

$= n \leq -1$

Analogous to complex exponential e^{st}, in discrete time we have

$$x[n] = z^n$$

$$z = r e^{j\omega} \qquad r > 0$$

$$x(n) = r^n e^{j\omega n}$$

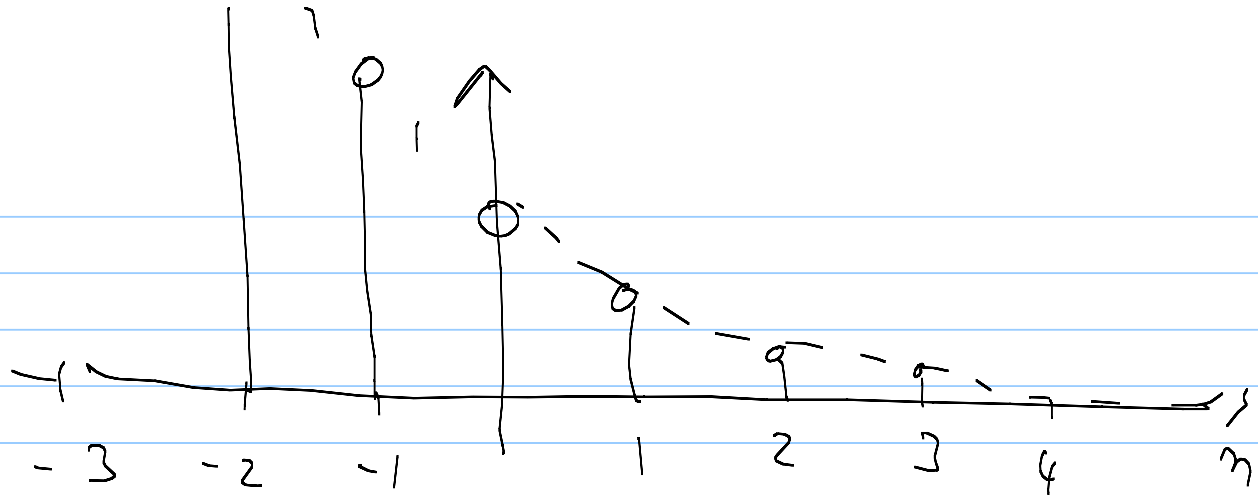
"DC": $x(n) = 1 \ \forall n$ $\quad r = 1, \quad \omega = 0$

real exponential: r^n, $\omega = 0$

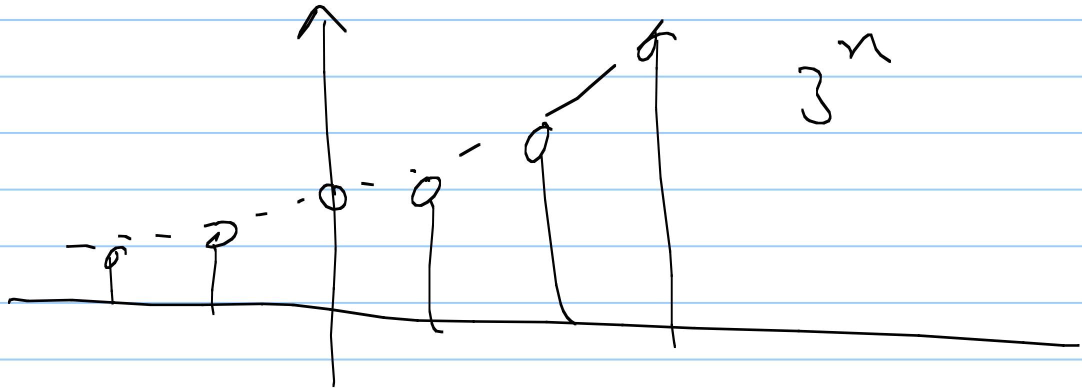
$|r| > 1$ growing exponential to the right

$|r| < 1$ " " to the left

$x[n] = (1/2)^n$



$x[n] = 3^n$



If $-1 < r < 0$, then signs alternate

$x[n] = \left(-\frac{1}{2}\right)^n = (-1)^n \left(\frac{1}{2}\right)^n$

Let's look at $e^{j\omega_0 n}$ & compare it with $\underbrace{e^{j\Omega_0 t}}$

→ 1. always periodic w/ period $T = \frac{2\pi}{|\Omega_0|}$

2. Oscillates more & more rapidly as $\Omega_0 \uparrow$

$$x[n] = e^{j\omega_0 n}$$

If $x[n+N] = x[n]$, then $e^{j\omega_0 (n+N)} = e^{j\omega_0 n}$

$$\Rightarrow e^{j\omega_0 N} = 1$$

$$\omega_0 N = 2\pi k$$

$$2\pi f_0 N = 2\pi k$$

$$\boxed{f_0 = k/N}$$

Hence $e^{j\omega_0 n}$ is periodic with period N

if and only if $f_0 = \frac{k}{N}$ i.e. a rational number

Now if $\omega_1 = \omega_0 + 2\pi$

$$e^{j\omega_1 n} = e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n} \cdot e^{j2\pi n}$$

$$= e^{j\omega_0 n}$$