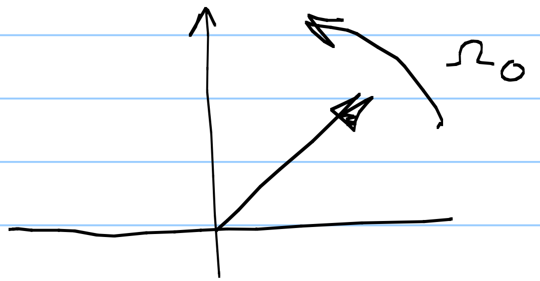
Note Title 04-08-2011

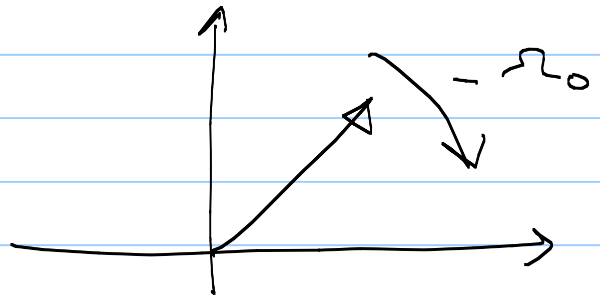
EC 5142 4^{th} Aug. 2011

$e^{j\Omega_0 t}$ is periodic with period $\frac{2\pi}{|\Omega_0|}$

1) Always periodic for any Ω_0 w/ period $\frac{2\pi}{|\Omega_0|}$

2) Oscillates more & more rapidly as $\Omega_0 \uparrow$





$$e^{st} = e^{(\sigma + j\Omega)t} \quad e^{\sigma t} \cdot e^{j\Omega t}$$

$\sigma + j\Omega$: complex frequency

Continuous-Time Impulse

Dirac delta function

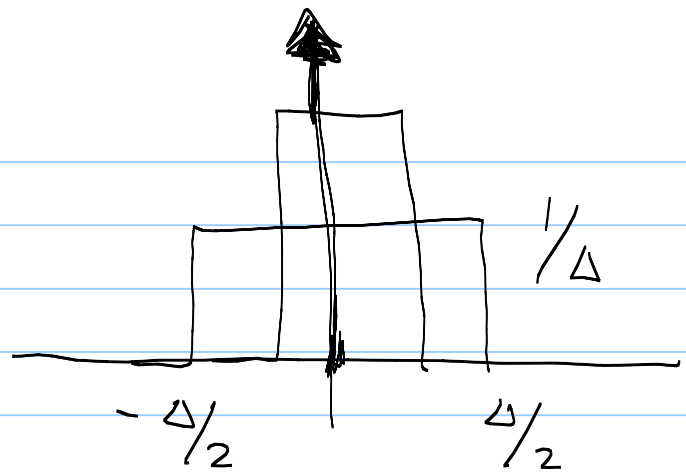
$$\delta(t) = 0 \qquad t \neq 0$$

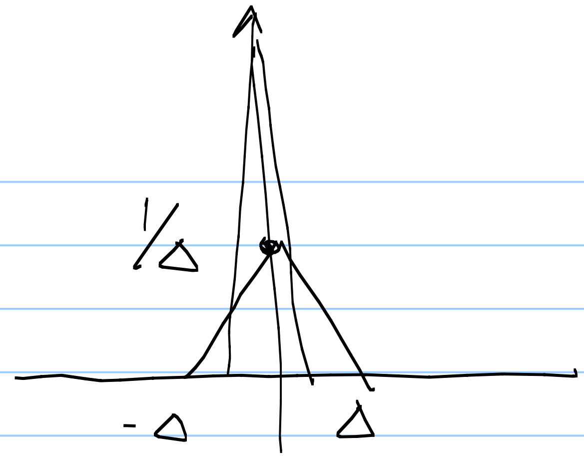
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Not a function in the usual sense.

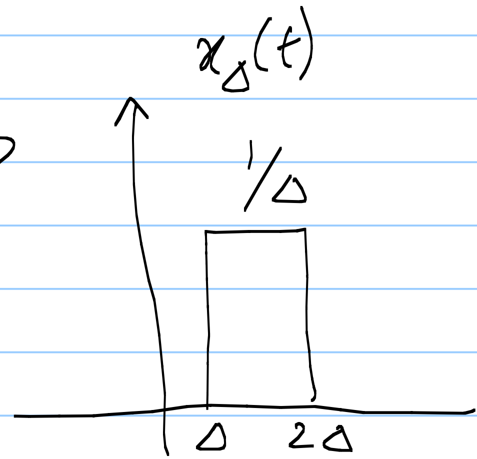
Called functional or generalized function

Also called distribution





$$x_\sigma(t) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-t/2\sigma^2}$$



$x_\sigma(t) \to \delta(t)$ as $\sigma \to 0$

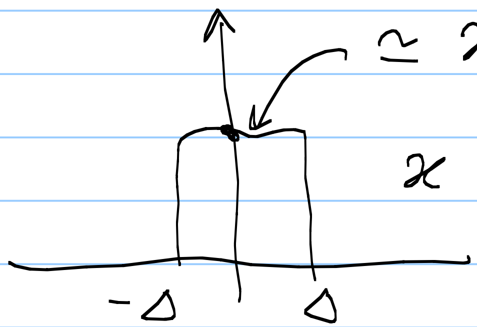
$\delta(0) = \infty$ wrong

$$x_\Delta(t) \rightarrow \delta(t) \quad \text{as} \quad \Delta \rightarrow 0$$

<u>Sifting property</u>

If $x(t)$ is continuous at $t = 0$, then

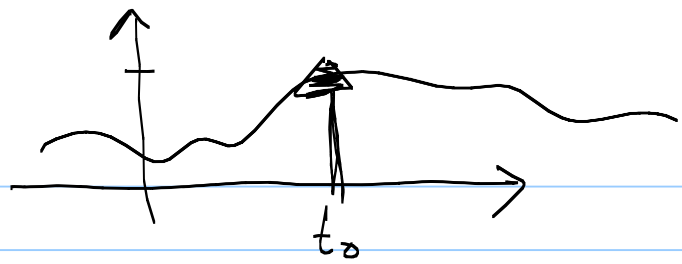
$$x(t)\,\delta(t) = x(0)\,\delta(t)$$



$\simeq x(0)\,\delta_\Delta(t)$

$x \rightarrow x(0)\,\delta(t)$

$\delta(t - t_0)$

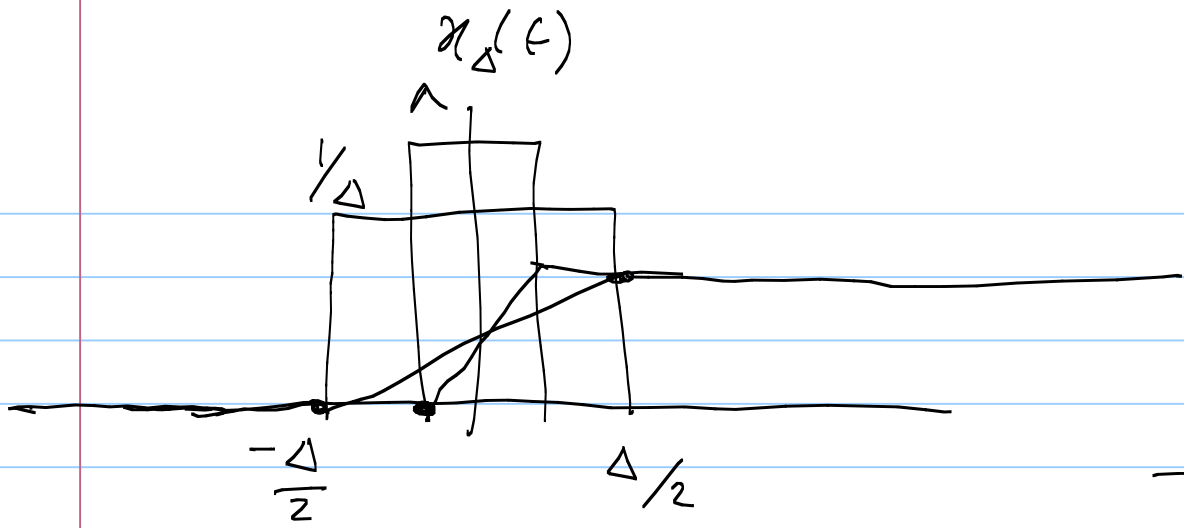


$$x(t)\,\delta(t - t_0) = x(t_0)\,\delta(t - t_0)$$

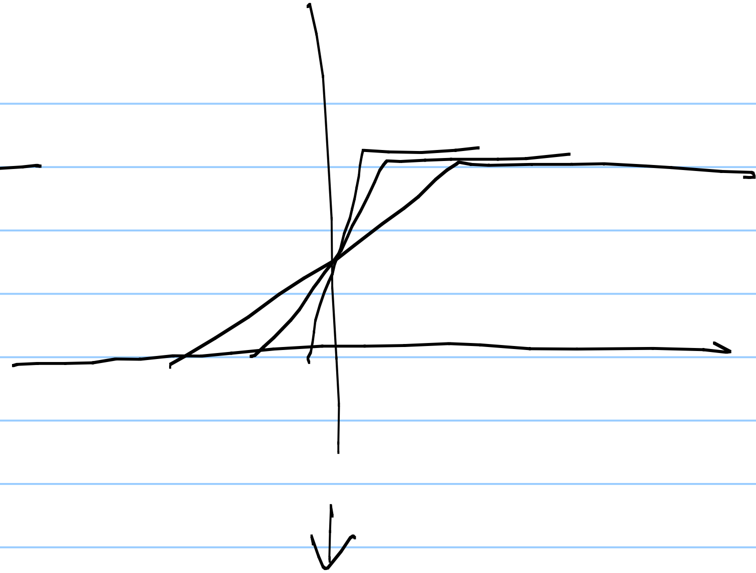
$$\delta(at) = \frac{1}{|a|}\,\delta(t) \quad \text{(verify!)}$$

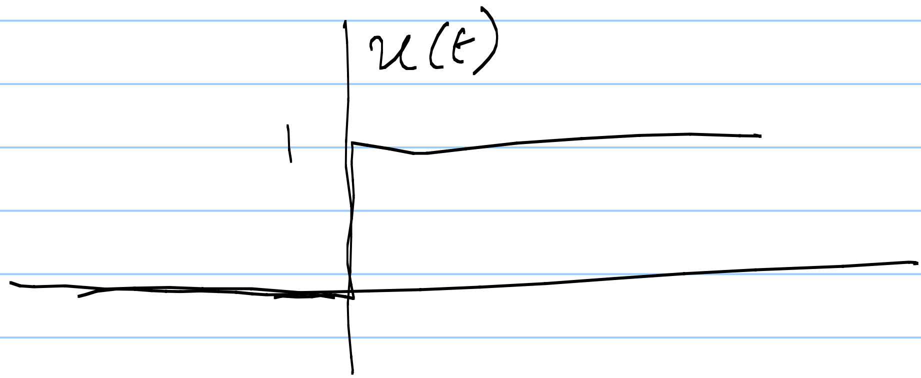
$$\delta(-t) = \delta(t) \quad \text{even function}$$

$$\delta(at + b) = \frac{1}{|a|}\,\delta(t + b/a) \quad \text{(verify)}$$



$$\int_{-\infty}^{t} x_\Delta(\tau)\,d\tau$$





$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

$$\frac{d}{dt} \int_{-\infty}^{t} \delta(\tau)\, d\tau = \delta(t) = \frac{d}{dt} u(t)$$

$$\delta(t) = \frac{d}{dt} u(t)$$

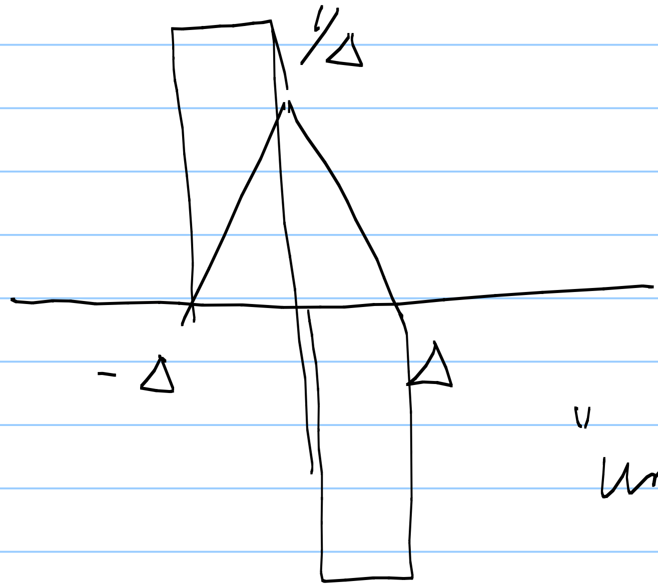
$$\frac{d}{dt}\left[f(t)\, u(t) \right] = u(t)\, f'(t) + f(t)\, u'(t)$$

$$= u(t)\, f'(t) + f(t)\, \delta(t)$$

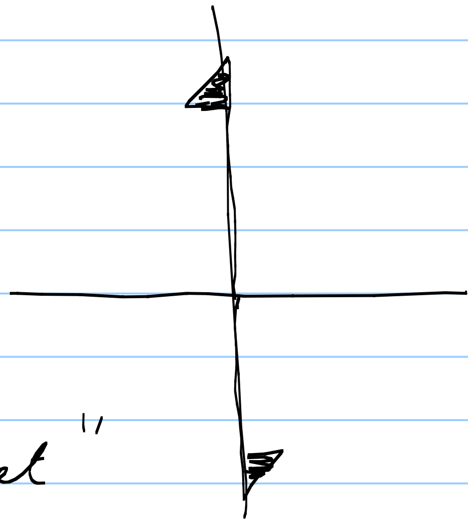
$$= u(t) f'(t) + f(0)\,\delta(t)$$

$f(t)\,u(t) =$ "right sided signal"

$f(t)\,u(-t) =$ "left sided signal"



$\longrightarrow$



"unit doublet"

$$\int_{-\infty}^{\infty} \delta'(t)\,dt = 0$$

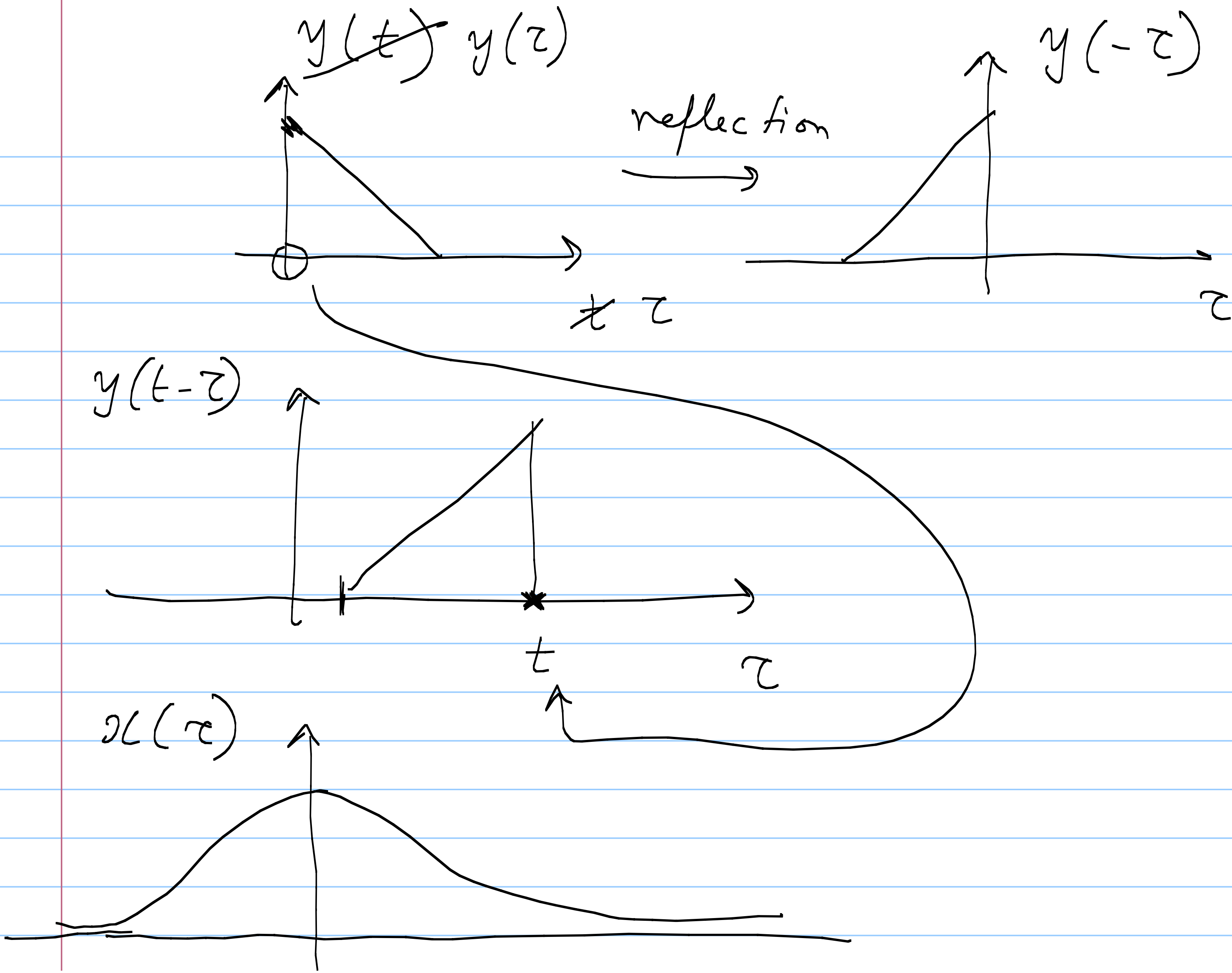
<u>Convolution</u>

$$x(t) * y(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\,d\tau = z(t)$$

$$y(t) \longrightarrow y(\tau)$$

$w(\tau) = y(-\tau)$ reflection

$w(\tau - t) = y(-(\tau - t)) = y(t - \tau)$ translation


~~y(t)~~ y(τ)
reflection
y(−τ)
~~t~~ τ
τ
y(t−τ)
t
τ
x(τ)

$$\int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau = z(t)$$

http://www.jhu.edu/~signals/

$$\int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$t - \tau = \lambda$$

$$\int_{-\infty}^{\infty} y(\lambda)\, x(t-\lambda)\, d\lambda$$ Commutative

Under certain conditions,

$$x * (y * z) = (x * y) * z \quad \text{associative}$$

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

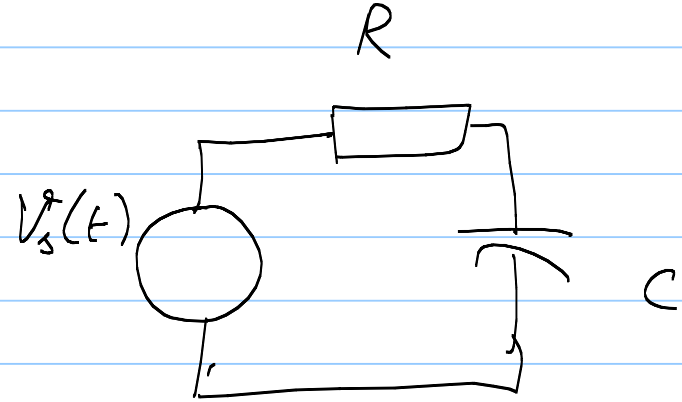
$$= x(t)$$

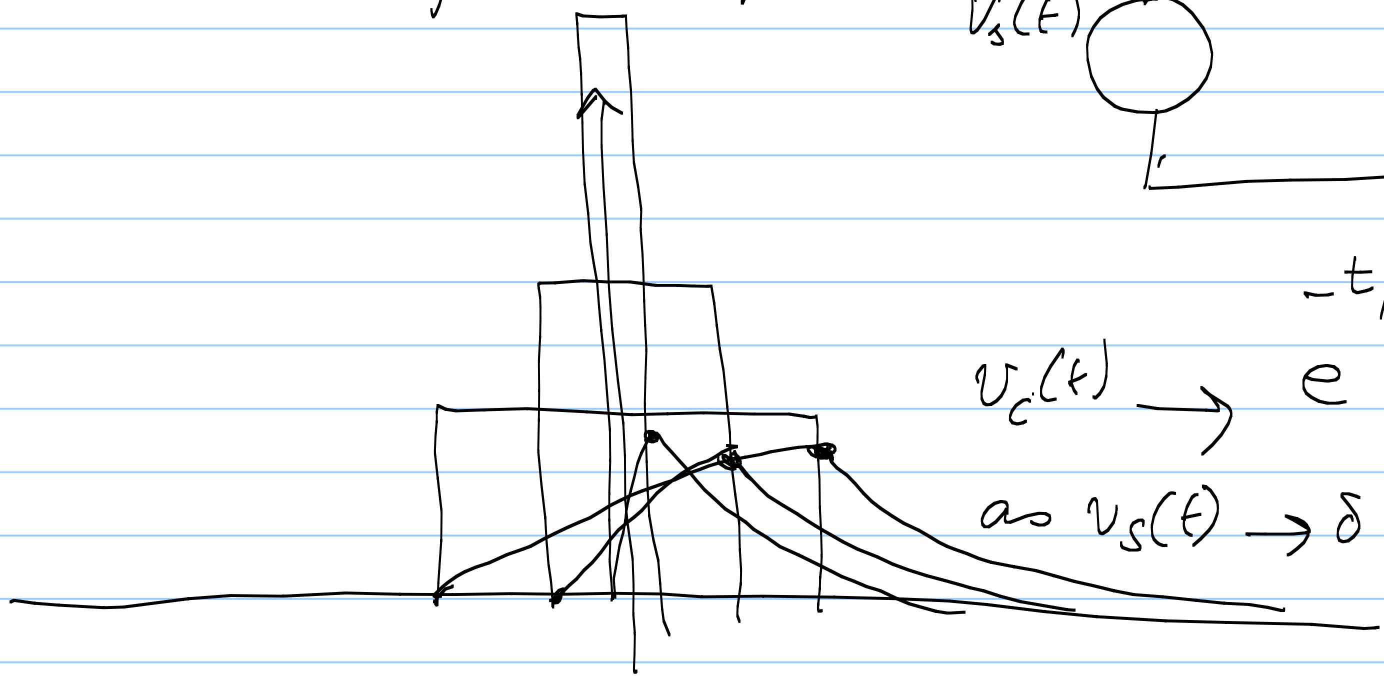
$$x(t) * \delta(t - t_0) = x(t - t_0) \quad \text{(verify)}$$

If $x(t) = \delta(t)$, $x(t) * \delta(t) = x(t) \Rightarrow \delta(t) * \delta(t) = \delta(t)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

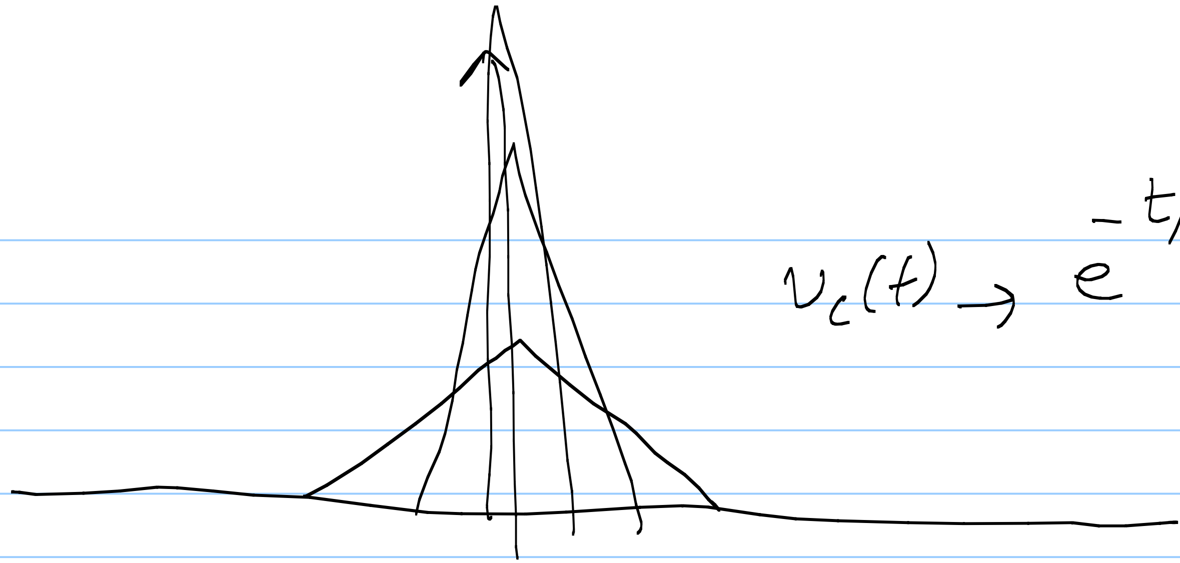
$h(t)$: system's impulse response





$$v_C(t) \longrightarrow e^{-t/RC}\, u(t)$$

as $v_S(t) \to \delta(t)$



$v_c(t) \rightarrow e^{-t/RC} u(t)$

$\delta^2(t)$ = not defined

$\delta(t) = 0 \quad t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

WRONG!

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$