EC5142 Intro To DSP

Note Title 02-08-2011

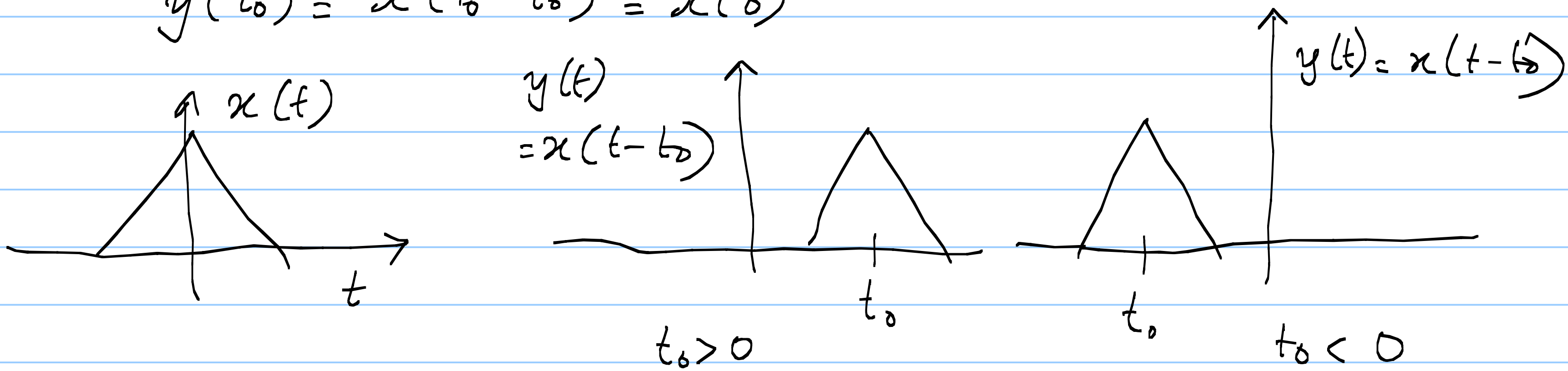
Basic operations on signals

Operation on the independent variable.

<u>Shift</u>:

$$y(t) = x(t - t_0)$$

$$y(t_0) = x(t_0 - t_0) = x(0)$$



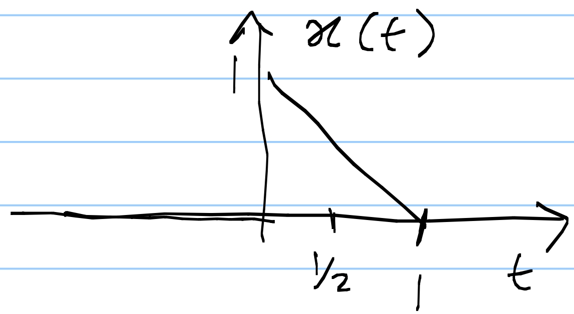
Scaling

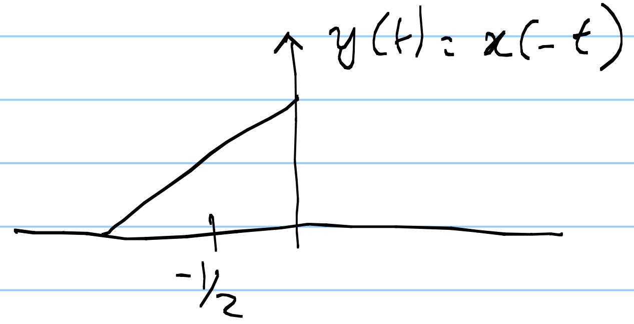
$$y(t) = x(at) \qquad a \in \mathbb{R}$$

Eg:

$$y(t) = x(-t) \qquad a = -1$$

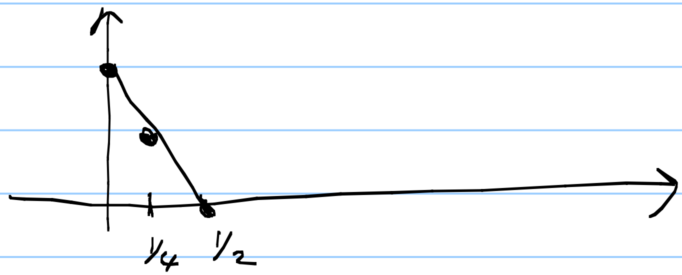
Reflection about the y-axis



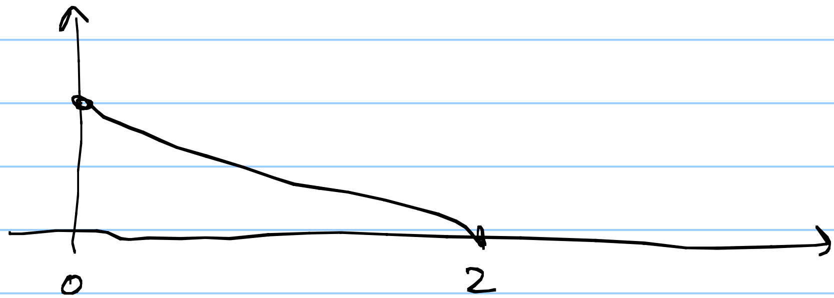


For $|a| \neq 1$:

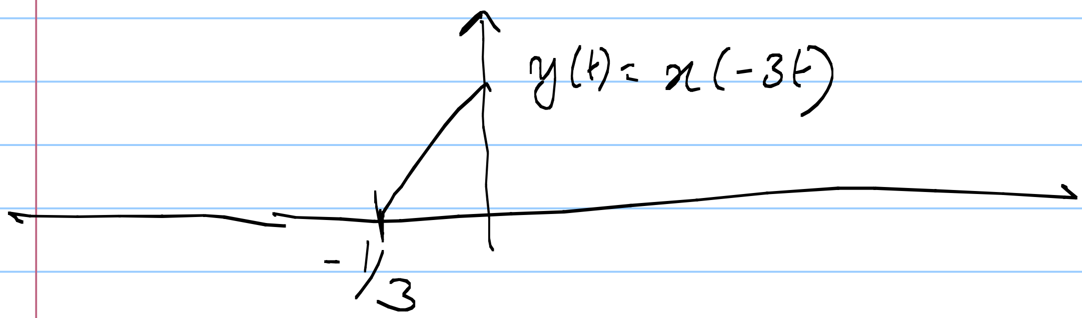
Eg: say $a = 2$, $y(t) = x(2t)$



Eg: $a = 1/2$ $y(t) = x(1/2\, t)$



Eg: $a = -3$ $y(t) = x(-3t)$

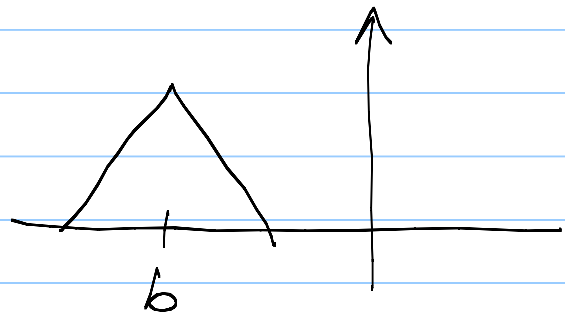


Affine transformation. $at + b$

$$y(t) = x(at + b)$$

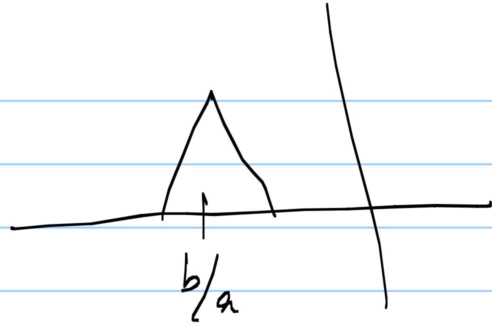
Shift first, then scale

$$w(t) = x(t + b)$$



$$w(at) = x(at+b)$$

<u>not</u> $x(at+ab)$



<u>Scale first, then shift</u>

$$w(t) = x(at)$$

$$w(t+b/a) = x[a(t+b/a)]$$
$$= x(at+b)$$

Non linear transform:

$$X_M(s) = \int_0^\infty x(t)\, t^{s-1}\, dt \qquad \text{Mellin transform}$$

$$= \int_{-\infty}^{\infty} x(e^{-t})\, e^{-st}\, dt$$

$=$ Laplace transform of $x(e^{-t})$

Energy & Power of a Signal

Given $x(t)$, energy is defined as

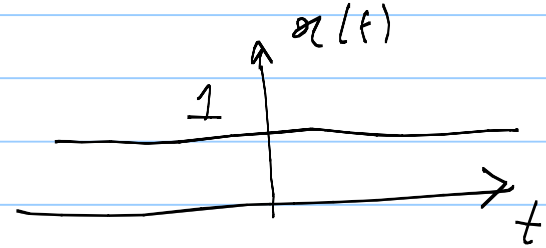
$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \|x\|_2^2 \quad \text{(2-norm squared)}$$

Power is defined as:

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

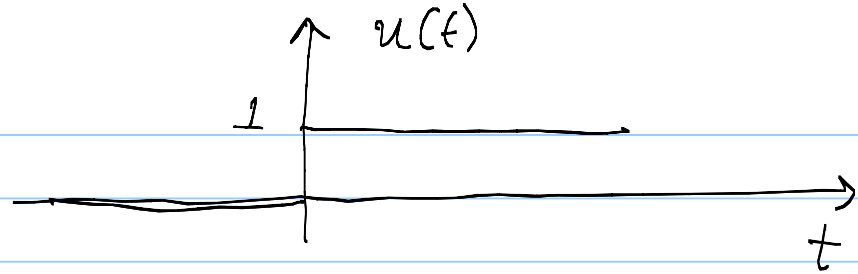
Elementary Signals:

$x(t) = 1 \quad \forall t$ "DC signal"



Unit Step Function

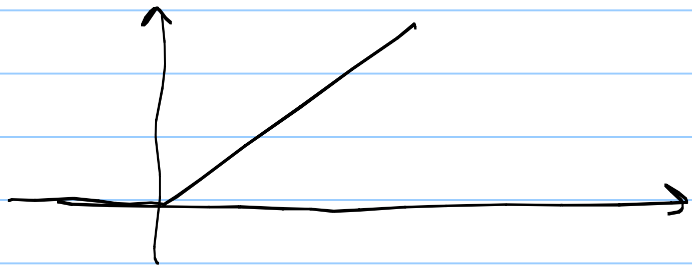
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



Variants:

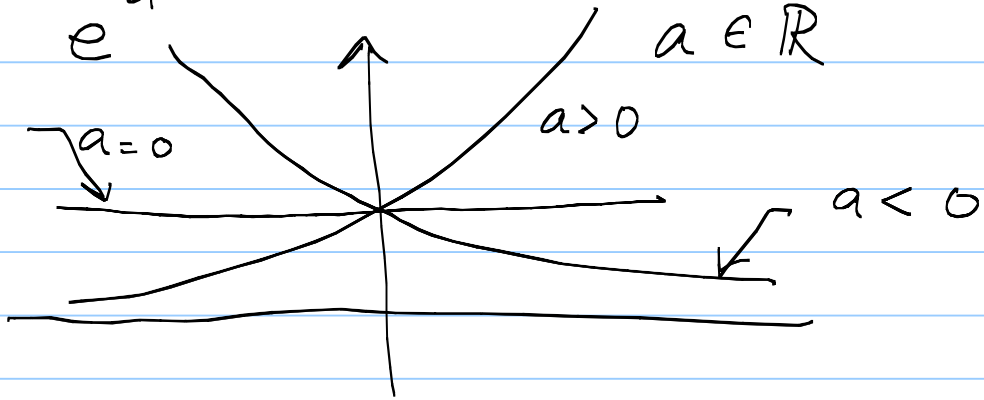
For $t = 0$, $u(t)$ can be defined to be 0, 1, $1/2$, any value, or undefined.

Unit ramp



$t\,u(t)$

e^{at} $\quad a \in \mathbb{R}$



$e^{j\Omega_0 t}$ $\quad \Omega_0 \in \mathbb{R}$

$= \cos \Omega_0 t + j \sin \Omega_0 t$

$$e^{j\Omega_0(t+T_0)} = e^{j\Omega_0 t + j\Omega_0 T_0} = e^{j\Omega_0 t} \cdot \underbrace{e^{j\Omega_0 T_0}}_{1}$$

$$\Omega_0 T_0 = k\, 2\pi$$

$$T_0 = k \frac{2\pi}{\Omega_0}$$

If T_0 is the period (fundamental period)

$$T_0 = \frac{2\pi}{\Omega_0}$$

For $e^{-j\Omega_0 t}$, $T_0 = \frac{2\pi}{\Omega_0}$

$$e^{j\Omega_0 t}, \quad T_0 = \frac{2\pi}{|\Omega_0|} \Rightarrow$$

As $\Omega_0 \to \infty$

$T_0 \to 0$

Signal is periodic $\forall\, \Omega_0$ with period $\frac{2\pi}{|\Omega_0|} = T_0$

Most general exponential:

$$e^{st} = e^{(\sigma + j\Omega)t}$$

$\sigma + j\Omega$: complex frequency