Principles of Spread Spectrum and CDMA

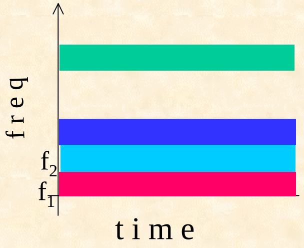
Dr Bhaskar Ramamurthi

Professor

Department of Electrical Engineering

Indian Institute of Technology Madras.

Separation of Overlapping Signals

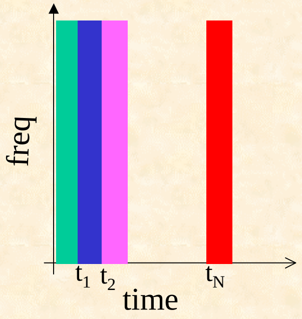


- **F**requency **D**ivision **M**ultiplexing
  - signals non-overlapping in frequency
  - signals overlap in time
  - separation achieved by *filtering*
    - "window" in f-domain
    - convolution with filter impulse response in t-domain
- if $H_1(f) \leftrightarrow h_1(t)$ filters out signal in $[f_1, f_2]$

$$desired\_signal = \int_{-\infty}^{\infty} sum\_signal(\tau) h_1(t-\tau) d\tau$$

$\Rightarrow$ *sliding correlation* with $h_1(-t)$

Separation (contd…)



- **T**ime **D**ivision **M**ultiplexing
  - signals non-overlapping in time
  - signals overlap in frequency
  - separation achieved by *windowing*
    - multiplication in t-domain
    - convolution in f-domain
  - if $u(t; t_1, t_2)$ is non-zero in $[t_1, t_2]$

    $desired\_signal = sum\_signal \,\mathbf{.}\, u(t; t_1, t_2)$

Orthogonality and Separation of Signals

- in FDM, signals are *orthogonal in frequency*

  $\Rightarrow$ Signal_1(f) **.** Signal_2(f) = 0
- in TDM, signal are *orthogonal in time*

  $\Rightarrow$ signal_1(t) **.** signal_2(t) = 0
- signals can be continuous-time or discrete-time
  - however in TDM, signal is usual a d-t digital signal or digitally-modulated c-t signal
- Is any other form of orthogonality possible?

Orthogonality: Any Other Way?

- Yes, if we consider only discrete-time signals
- Let $c^1(t)$ and $c^2(t)$ both of duration T be such that their cross-correlation = 0 *when time-aligned*

$$\int_0^T c^1(t)c^2(t)dt = 0$$

- Let signal_1 be x_1 and signal_2 be x_2 in $[0,T]$
- Consider sum_signal(t)= $x_1c^1(t)+ x_2c^2(t)$

$\Rightarrow$ x_i can be extracted by correlating with $c^i(t)$

$$\int_0^T sum\_signal(t).c^i(t)dt = x_i[\int_0^T c^{i^2}(t)dt]$$

Orthogonality (contd..)

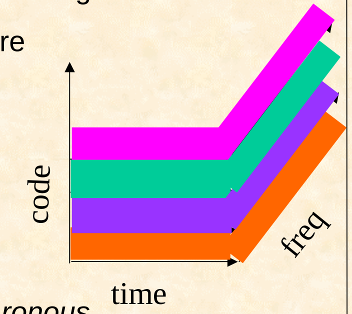
- How many orthogonal functions $c^i(t)$ can we get?
- Landau-Pollak Theorem: $N \sim 2WT$ where *W* is the bandwidth available

$\Rightarrow$ given symbol duration T,

$$N \sim W/(1/T)$$

$\uparrow$

bandwidth expansion factor



- $c^i(t)$ have to be time-aligned, i.e. *synchronous*
- signals overlap in time *and* frequency

Orthogonal Direct-Sequence Code Division Multiplexing

- $c^i(t)$ are *binary-valued sequences* such that

$$\int_0^T c^i(t)c^j(t)dt = 0$$

– i.e., $c^i(t) = \sum_{k=0}^{L_c-1} (2c_k^i - 1)h(t - T_c)$ where c_i^k = {0,1},

and T_c is the *chip* period = T/L_c

- equivalently $\sum_{k=0}^{L_c-1} c_k^i \ XOR \ c_k^j = L_c$
- $\{ c_k^i \}$ are called **codes**, hence the terms *CDMA* and *synchronous* CDM

Orthogonal CDM (contd…)

- Example : Walsh codes

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

H_2

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$H_2{}^2$

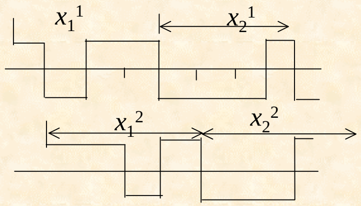
$$\begin{bmatrix} H_2^{n-1} & H_2^{n-1} \\ H^{2n-1} & \overline{H^{2n-1}} \end{bmatrix}$$

$H_2{}^n$

- $H_2{}^6$ used in IS-95

Asynchronous CDMA

- in orthogonal CDM, codes are synchronised
  - ⇒ difficult to ensure between independent transmitters at variable distances from receiver (i.e, multiple access)
- synchronous orthogonal codes can have large correlation when not synchronised
  - ⇒ not used in CDM**A**
- employ pseudo-random or pseudo-noise (PN) sequences
  - ⇒ typically, low but non-zero correlation between any two sequences
  - ⇒ **quasi-orthogonal** codes



Asynchronous CDMA (contd..)

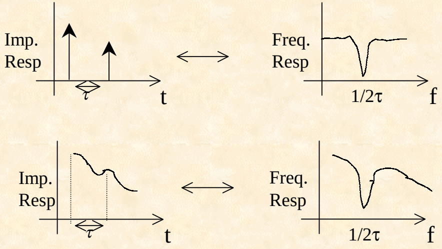
- even chip alignment will not be there
- if $\{b_k\}$ and $\{c_k\}$ are two truly-random binary sequences of length L_c, and cross-correlation $\beta = \sum_{k=0}^{L_c-1}(2b_k-1)(2c_k-1)$
- E[β] = 0 and E[β^2] = L_c
  - ⇒ interference power from (N-1) signals ∝ $(N\text{-}1)\ L_c$
- For the signal we desire to extract,

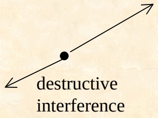
desired_signal power = $[\sum_{k=0}^{L_c-1}(2c_k-1)(2c_k-1)]^2 = L_c^2$

- ⇒ undesired *per-user* interference suppressed by factor 1/L_c on the average
- – for large N, variation around average will be less

Signalling over Fading Channels

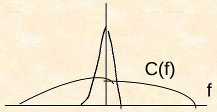
- A fading channel has *deep nulls* in its frequency response
  - ≡ *multiple paths* taken by signals from Tx to Rx





- Only a small part of a *wideband* signal's spectrum is severely distorted by the nulls
  - – in FDM or OFDM, a (narrowband) signal located around f= 1/2 τ will be lost
- multipath leads to **I**nter **S**ymbol **I**nterference if τ ~T (symbol duration)
  - – in TDM, T is small ⇒ tolerable *delay spread* is less

Spread Spectrum Signalling and Fading

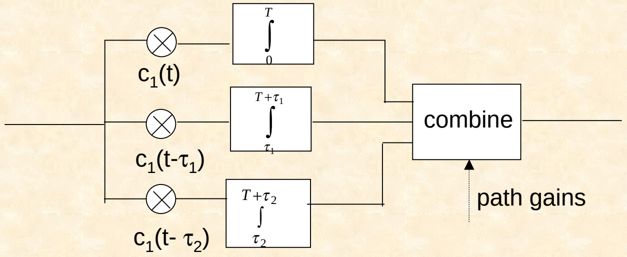


convolution is uniform wideband

- $c(t)$ spreads a narrowband signal uniformly over a band T/T_c larger
  - ⇒ $C(f)$ must be flat and wide
  - ⇒ $R_{cc}(\tau)$ must be impulse like
- multipath signal $\alpha_1 x_1\, c(t) + \alpha_2 x_1 c(t-\tau)$ correlated with $c(t)$
  - ⇒ $\alpha_1 x_1 R_{cc}(0) + \alpha_2 x_1 R_{cc}(\tau) \approx \alpha_1 x_1 R_{cc}(0)$ if $\tau > T_c$
  - ⇒ delayed signal is suppressed for delay > T_c
- for *slow* frequency hopping, some *bursts* of symbols falling in *spectral nulls* are affected
  - ⇒ employ coding across bursts to overcome this

Spread Spectrum Diversity

- estimate path delays greater than T_c in a multipath channel
- implement one correlator for every significant path : RAKE receiver



RAKE RECEIVER

SS Diversity (contd…)

- each *finger* will give $\alpha_i R_{cc}(0) x_1$

  ⇒ *combine* to get better decision on x_1

- ISI has been **resolved** and converted to **diversity gain**

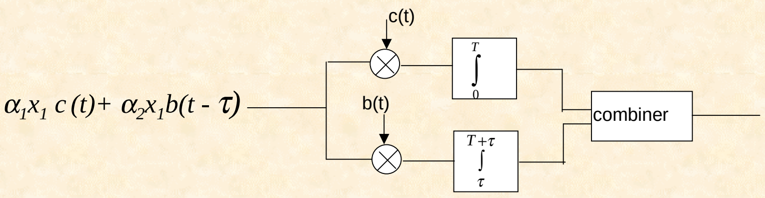
- if path gains $[\alpha_i]$ are also estimated, can weight each finger proportionately

  ⇒ “maximal ratio” combining

  else “equal gain” combining

Multi-Transceiver (Macro) SS Diversity

- multiple transmissions on same carrier with different codes can be combined



⇒ signals from two base stations can be combined during handoff

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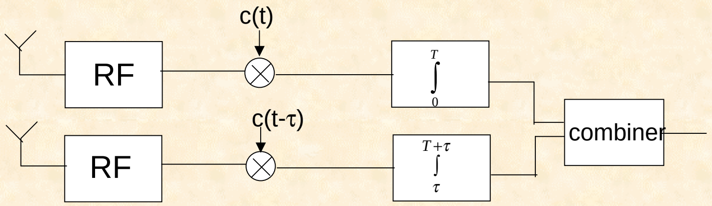
Macro Diversity (contd…)

- The fingers of a RAKE receiver need not be fed by same RF front-end

  ⇒ different transceivers at same base station (e.g., adjacent sectors)

  ⇒ different base stations!



Multi-Access Interference in CDMA

- codes employed have low autocorrelation sidelobes, and typically low cross correlation also
- assume all signals arrive with **equal power**

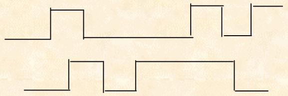
  ⇒ *perfect power control* at Tx to counteract path loss and fading
- desired signal energy at each RAKE finger $\propto L_c^2$
- sum of undesired signal (interference) energies at each RAKE finger $\propto NL_c$
- if **MAI** is approximated as Gaussian for large N,

  ⇒ variance of MAI = NL_c

  ⇒ **C**arrier-to-**I**nterference **R**atio (CIR) = L_c / N

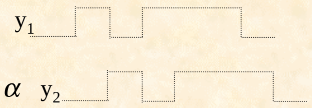
Refinement of MAI Computation

- effect of bandpass transmission
  - since each interferer will have a random phase offset (actually, small frequency offset), MAI will be less



$E[\cos^2\varphi] = 0.5$ for uniformly distributed φ

- effect of chip mis-alignment
  - if interference is y_1 for left alignment and y_2 for right alignment, actual interference $\alpha y_1 + (1-\alpha) y_2$ where α is uniformly distributed in [0,1]



$E\{[\alpha y_1 + (1-\alpha)y_2]\}^2 = (1/3)\,[Ey_1^2 + Ey_2^2] = 2L_c/3$

- Net effect : CIR increases by a factor of 3
- For large cells, thermal noise adds to MAI near cell boundaries : CI**N**R drops

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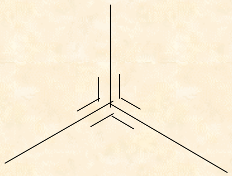
Sectoring Reduces MAI



- BS employ sectoral antenna

  ⇒ downlink adjacent cell interference reduced

  ⇒ uplink uncontrolled interference from adjacent cells reduced
- capacity improvement factor vis-à-vis circular cell = η × 360°/ sectoral angle (η = 2.8 (4.5) for 3 (6) sectors)
- sectoral gain improves link budget in noise-limited situation

  ⇒ no impact in interference-limited case
- uncontrolled adjacent sector interference reduces capacity by 0.6

  ⇒ ~ 0.6 x 60 x η (~35 η) users per cell with variable rate coding

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Power Control in CDMA

- if an interferer's power is higher by, say, 10 times

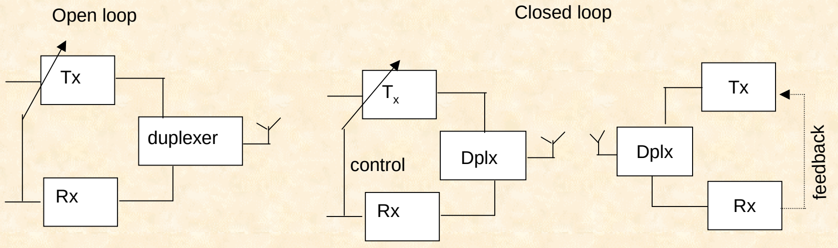
  ⇒ equivalent to 10 interferers of equal power

  ⇒ CIR drops dramatically
- need to control power of each user to ±0.5dB,
  - use combination of **open-loop** and **closed-loop** control
- path loss due to shadowing similar on up and down links even if frequencies are different

  ⇒ open-loop control of Tx power based on local Rx signal strength

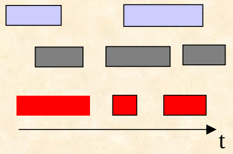
Power Control in CDMA (contd…)

- fading will be different in up and down link (different frequencies)
  - ⇒ closed-loop control of Tx power based on feedback from far-end Rx
    - - usually a low bitrate binary signal :1/0 → up/down by 0.5 dB



Statistical Multiplexing in CDMA

- individual signal is bursty with activity factor γ
  - – bursts occur randomly
- *for large* N, total “traffic” at any time t is equivalent to γN non-bursty signals



- in CDMA, MAI from M bursty signals is not NP_{TX}/L_c, but $\gamma NP_{TX}/L_c$ for large N
- if variable rate voice coder employed
  - ⇒ γ ~ 0.4, N ~20-30 sufficiently large
- for IP packets, γ ~ 0.1 or less, but N has to be much larger due to long-tailed distributions of ON/OFF periods

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Pros and Cons of CDMA

- spread spectrum diversity
  - - multipath, macro
- statistical multiplexing easily exploited
  - - large number of quasi-orthogonal codes
    - ⇒ large number of bursty users
- easy to support a variety of bit-rates

  e.g; if chip rate is 2.048 Mbps service at n kbps, has a spreading factor of 2048/n

  ⇒ n can be any power of 2

- strict power control required
  - - combination of open and closed loop control
- difficult to hand over from one carrier to another
  - - seamless handoff not possible between carriers, similar to FDM