Throughput Optimal Multi-Slot Sensing Procedure for a Cognitive Radio

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***Abstract*—We consider a cognitive radio system with M primary channels where users' data transmissions follow a slotted structure. We consider the scenario where the channel availability statistics are *correlated* across channels as well as across time slots. In each slot, the cognitive user (CU) senses channels one by one until a suitable channel is found for data transmission. CU adapts the data rate based on the channel's fading gain. We employ a Markov chain model to capture the channel availability statistics across the slots. We address the problem of finding optimal sensing order/procedure in every slot based on the history of sensed channels (including the previous slots), in order to maximize the total CU throughput over N-slots. Using theory of optimal stopping, we derive recursive expressions for the optimal multi-slot CU throughput and find the optimal multi-slot sensing order/procedure using dynamic programming. We also study few sub-optimal sensing procedures. Using numerical results, we illustrate the gains in exploiting the correlation of channel availability statistics.**

***Index Terms*—Spectrum sensing order, cognitive radio throughput, dynamic programming, Markov channel, optimal stopping rule.**

I. INTRODUCTION

COGNITIVE users perform spectrum sensing to find a free licensed channel and use it for data transmission. Note that channel in this paper refers to a slice of the frequency spectrum. Due to hardware limitations, we consider a practical constraint that a cognitive user (CU) can not sense more than one channel simultaneously. We consider the scenario where primary users have a slotted structure for data transmissions. CU needs to have an order in which it senses the channels until it finds a suitable channel available (free from primary users) for data transmission. This sensing order problem was first analyzed in [1] where the channel availability statistics are assumed to be known as well as unknown but random. Here optimization is performed to maximize CU throughput under the assumption that the channel availability statistics are independent across slots. However, measurement studies have shown that there is correlation in the spectrum occupancy across time and frequency [2], [3]. In our paper, we find the optimal sensing procedure to maximize the CU throughput over multiple time slots with the channel availability statistics being *correlated* across time slots as well as across channels. We also propose a low-complexity Greedy sensing approach. Using numerical results, we show the throughput gains of proposed approaches over existing procedures in the literature.

II. SYSTEM MODEL

Primary users' data transmission follow a slotted structure as shown in Figure 1. Let M be the number of primary channels and their fading SNR values are assumed to be i.i.d. across channels and slots, with exponential distribution of mean parameter Γ. In each slot, CU keeps sensing channels one by one until it finds a suitable channel for its data transmission. We assume that CU can neither go back to transmit on a previously skipped channel nor proceed to transmit on an un-sensed channel. Suppose, in a given slot, CU starts data transmission after sensing k channels, then the data rate (reward) obtained for that slot is given by $r_k = \frac{D-k\tilde{D}}{D}\log(1+\gamma)$ where D is slot duration, $\tilde{D}$ is per-channel sensing duration and γ is the SNR of the channel CU chose for data transmission. For convenience, defining $\tau = \frac{\tilde{D}}{D}$ and $c_n = 1 - n\tau$, we have $r_k = c_k \log(1+\gamma)$.

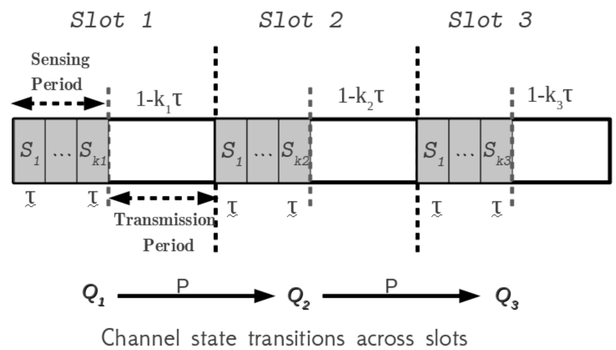

Fig. 1. Slot structure for N = 3 slots.

We consider the scenario where the spectral occupancy statistics are correlated across both channels and time slots. Channels' busy-free status is represented using a M-length binary string which we refer as the availability state of the channels. Let Q_n denote the channels' availability state at the n^{th} time slot where $Q_n = q$ corresponds to the binary string of $2^M - q$ with $q \in \{1, \cdots, 2^M\}$. Let $\boldsymbol{\mu}_0$ denote the initial probability vector of size 2^M, with its q^{th} entry being the probability $\Pr\{Q_0 = q\}$. Since $\boldsymbol{\mu}_0$ specifies the initial joint probability mass function, it provides the complete characterization of initial channel availability statistics. We use Markov chain model as in [3], [4] to capture the time correlation of channel availability statistics. State transitions across slots happen in Markovian manner with $p_{m,j}$ denoting the conditional probability of state in next slot $Q_{n+1} = j$ given current state $Q_n = m$ where $m, j \in \{1, \cdots, 2^M\}$. We are interested in maximizing the total CU throughput over N slots, by finding the order of channels to be sensed in every slot and the corresponding SNR thresholds to stop sensing when a free channel is found. Our multi-slot throughput optimization problem faces two major challenges, namely *partial observability* and *tradeoff* between present slot reward (rates) and future slots' rewards. Addressing these issues, we find the optimal sensing procedure using dynamic programming. For

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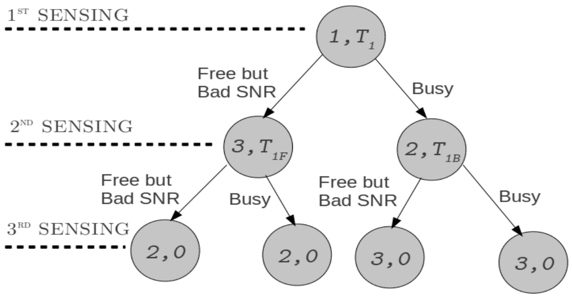

Fig. 2. Sensing Procedure Tree for single slot, 3 channel case.

unslotted systems, with random sensing order, [5] solved the problem of when a CU should access/release/skip a channel based on fading conditions.

III. Optimal Sensing Procedure

A. Preliminaries

Consider the sensing procedure optimization over a single slot (N=1). In this case, the sensing procedure can be denoted by a tree as in Figure 2. Each node in the tree is denoted by the pair (channel index, reward threshold). The channel index specifies the channel to be sensed when we arrive at that node. If the sensed channel is free and the instantaneous reward (data rate based on SNR of that channel) is above the threshold specified for that node, sensing stops. Otherwise, we skip and proceed to the next node depending on the outcome of the sensing. Clearly, when we reach the last node in the sensing tree, we have to use that channel if found free (i.e., the reward threshold is zero). Our goal is to maximize the expected throughput of the cognitive user. Let X_P denote the expected rate we get when we find a channel to be free (after k sensings) and stop if $r_k \geq T$ for some threshold T. To compute X_P, we also need to know the expected rate (denote by X_S) we get when we *skip* the currently sensed free channel when $r_k < T$. We have, $X_P = \int_{r_k<T} X_S \frac{e^{-\frac{\gamma}{\Gamma}}}{\Gamma} d\gamma + \int_{r_k>T} c_k log(1+\gamma) \frac{e^{-\frac{\gamma}{\Gamma}}}{\Gamma} d\gamma$, which can be rewritten as

$$X_P \;=\; X_S Pr(r_k < T) + T Pr(r_k > T) + c_k e^{\frac{1}{\Gamma}} \psi(\frac{e^{\frac{T}{c_k}}}{\Gamma}) \quad (1)$$

where $\psi(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt$. According to the optimal stopping theory [1], the stopping threshold should be equal to the expected future reward, i.e., $T = X_S$, in which case,

$$X_P = T + f_k(T), \quad (2)$$

and the function $f_k(T) := c_k e^{\frac{1}{\Gamma}} \psi(\frac{e^{\frac{T}{c_k}}}{\Gamma})$ will be used later in the throughput recursions.

The history of sensed channels is maintained using set $\mathcal{R}$ and set $\mathcal{S}$ denote the list of yet unsensed channels, in a given slot. For example if $\mathcal{S} = \{1, 2\}$ and $\mathcal{R} = \{(3, F), (4, B)\}$ it means that we have sensed channel 3 and found it to be free and also sensed channel 4 and found it busy and the channels remaining to be sensed are 1 and 2. The pair $(\mathcal{S}, \mathcal{R})$ denotes a *situation* in our sensing process. Let $X(\mathcal{S}, \mathcal{R})$ denote the maximum expected throughput we get in that slot using the optimal procedure *after* having reached the sensing situation $(\mathcal{S}, \mathcal{R})$. Let $X^i(\mathcal{S}, \mathcal{R})$ denote the maximum expected reward we get if we *choose* to sense channel i from the set $\mathcal{S}$ and let $T^i(\mathcal{S}, \mathcal{R})$ be the corresponding optimal stopping threshold. As discussed earlier, optimal stopping threshold is equal to the maximum expected future reward, i.e., $T^i(\mathcal{S}, \mathcal{R}) = X(\mathcal{S}\backslash\{i\}, \mathcal{R} \cup \{(i, F)\})$. Using (2) and denoting the size of $\mathcal{R}$ by $|\mathcal{R}|$, we have the recursion, $X^i(\mathcal{S}, \mathcal{R}) = \theta^i \Big(f_{|\mathcal{R}|+1}(X(\mathcal{S}\backslash\{i\}, \mathcal{R} \cup \{(i, F)\})) + X(\mathcal{S}\backslash\{i\}, \mathcal{R} \cup \{(i, F)\}) \Big) + (1-\theta^i) X(\mathcal{S}\backslash\{i\}, \mathcal{R} \cup \{(i, B)\})$ where θ^i is the probability that channel i is free, taking the history $\mathcal{R}$ into account. Now the required reward becomes $X(\mathcal{S}, \mathcal{R}) = \max_{i\in\mathcal{S}} X^i(\mathcal{S}, \mathcal{R})$. Note that setting optimal thresholds $T^i(\mathcal{S}, \mathcal{R})$ requires computation of maximum expected reward in a subsequent sensing situation $X(\mathcal{S}\backslash\{i\}, \mathcal{R} \cup \{(i, F)\})$. For each possible sensing tree, we find the maximum throughput with the computation of stopping thresholds done by backward recursion (from the final nodes for which stopping thresholds are zero). Single-slot optimal sensing procedure maximizing $X(\{1, \cdots, M\}, \varnothing)$ is found by searching over all the possible sensing trees using a dynamic program (similar to [1]).

B. Partial Observability

In the multi-slot sensing problem $N > 1$, we encounter the partial observability situation. Since sensing in a slot stops as soon as a free channel with suitable reward is found, we may not know the *exact* channel availability state of the current slot, resulting in a phenomenon known as partial observability (PO). Nature of this PO depends on the sensing history. Let $\mathcal{H}_n$ denote sensing history , up to time slot n (including the sensings done in previous slots). PO class denoted by $\mathcal{C}(\mathcal{H}_n) \subset \{1, \cdots, 2^M\}$ is a collection of all the possibilities of the current channel availability state Q_n for the given sensing history $\mathcal{H}_n$. For instance, with $M = 3$ and $\mathcal{H}_1 = \{(1, F), (3, F)\}$ then we have (in binary format) $Q_1 \in \mathcal{C}(\mathcal{H}_1) = \{111, 101\}$. We track the sensing history using a *belief* vector. For the present slot n, belief vector $\boldsymbol{\mu}$ is composed of terms $\mu_k^n = \Pr\{Q_n = k|\mathcal{H}_n\}$, $k = 1, \cdots, 2^M$. Some comments: 1) Belief vector needs to be updated after each sensing. 2) Belief vector needs to be updated when we move onto next slot using Markov process's state transition matrix. 3) Primary free probability for each channel (θ^i) has to be computed using belief vector.

Denoting the entries in $\mathcal{C}(\mathcal{H}_n)$ which correspond to channel i being free by $\mathcal{C}(\mathcal{H}_n) \cap \{(i, F)\}$, we have $\theta^i(\boldsymbol{\mu}) = \Pr\{\text{channel } i \text{ is free in slot } n \mid \mathcal{H}_n\} = \frac{\sum_{k\in\mathcal{C}(\mathcal{H}_n)\cap\{(i,F)\}} \mu_k^n}{\sum_{k\in\mathcal{C}(\mathcal{H}_n)} \mu_k^n}$.

Now, let us see how to update belief vector across slots. Let μ_k^{n+1} denote the probability that the channel availability state $Q_{n+1} = k$ given sensing history $\mathcal{H}_n$. Given that current state is in PO class $\mathcal{C}(\mathcal{H}_n)$, we have, due to Markovity, $\mu_k^{n+1} = \Pr\{Q_{n+1} = k|Q_n \in \mathcal{C}(\mathcal{H}_n)\} = \frac{\sum_{j\in\mathcal{C}(\mathcal{H}_n)} p_{j,k} \mu_j^n}{\sum_{j\in\mathcal{C}(\mathcal{H}_n)} \mu_j^n}$. For notational convenience, we will use $\boldsymbol{\mu}_u$ to denote $\boldsymbol{\mu}^{n+1}$ - the across-slot belief vector update obtained above.

C. Throughput Recursion and Optimization

Let $(\mathcal{S}, \mathcal{R})$ be a sensing situation in the first slot where $\mathcal{S}$ denotes set of channels available for sensing in the first slot and the history of the sensing process is $\mathcal{R}$ (With notational changes, the derivations can be carried out to a sensing situation in any slot). The number of sensings already done in the present slot is $M-|\mathcal{S}|$ where $|\mathcal{S}|$ denotes the size of $\mathcal{S}$. Let $X_j(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ be the maximum total expected reward we get over the next j slots from the given sensing situation $(\mathcal{S},\mathcal{R})$ with the channel availability statistics given by the present belief vector $\boldsymbol{\mu}$. We have $X_0(\mathcal{S},\mathcal{R},\boldsymbol{\mu})=0$ and $X_1(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ is the single slot reward discussed in Section III-A. Note that $X_1(\varnothing,\mathcal{R},\boldsymbol{\mu})=0$ and we are interested in maximizing $X_N(\mathcal{D},\varnothing,\boldsymbol{\mu}_0)$ where $\mathcal{D}=\{1,2,..,M\}$ and $\boldsymbol{\mu}_0$ denote the initial channel availability statistics. For $j>1$, the expected optimal reward can be written as

$$X_j(\mathcal{S},\mathcal{R},\boldsymbol{\mu})=X_1(\mathcal{S},\mathcal{R},\boldsymbol{\mu})+F_{j-1}(\mathcal{S},\mathcal{R},\boldsymbol{\mu}) \quad (3)$$

where $X_1(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ is the reward in the current slot and the second term $F_{j-1}(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ is the total reward in the *future* $j-1$ slots. The optimal value of $X_j(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ is obtained when the total sum is maximized. Here is where tradeoff between present slot reward and future slot rewards arise, as maximizing one term may not lead to the maximization of other term. We have to maximize the sum together.

Let $X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ be the be the maximum expected reward over next j slots *if* we sense the channel $i\in\mathcal{S}$ in the sensing situation $(\mathcal{S},\mathcal{R})$ and let $T_j^i(\mathcal{S},\mathcal{R})$ be the corresponding optimal threshold to be used (for notational convenience, we have suppressed the dependence on $\boldsymbol{\mu}$). Clearly, we have

$$i^* = \arg\max_{i\in\mathcal{S}} X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu}), \quad (4)$$

$$X_j(\mathcal{S},\mathcal{R},\boldsymbol{\mu}) = X_j^{i^*}(\mathcal{S},\mathcal{R},\boldsymbol{\mu}). \quad (5)$$

Note that we need to find the optimal thresholds $T_j^i(\mathcal{S},\mathcal{R})$. Towards that, we define $F_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ as the reward in the future j slots *if* we sense channel $i\in\mathcal{S}$ in situation $(\mathcal{S},\mathcal{R})$ of the current slot. Let us first derive the recursion for the future reward term $F_{j-1}^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$. When we sense the channel $i\in\mathcal{S}$, three possible events can happen. (1) Channel i is free with probability $\theta^i(\boldsymbol{\mu})$ and instantaneous rate $\tilde{r}=c_{M-|\mathcal{S}|+1}\log(1+\gamma)$ is above threshold $T_j^i(\mathcal{S},\mathcal{R})$. In this case we stop sensing in that particular slot and use channel i. For convenience, we use the shorthand notation $T_j^i(\mathcal{S},\mathcal{R}):=T$ and $\Pr\{\tilde{r}>T_j^i(\mathcal{S},\mathcal{R})\}:=\tilde{g}(T)$. (2) Channel i is free but reward is below threshold $T_j^i(\mathcal{S},\mathcal{R})$. In this case, we continue sensing in that slot. (3) Channel i is not free and continue sensing. So we have the recursion,

$$\begin{aligned}F_{j-1}^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu}) \;=\; &\theta^i\tilde{g}(T)X_{j-1}(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)+\\ &\theta^i[1-\tilde{g}(T)]F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)+\\ &(1-\theta^i)F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)\end{aligned} \quad (6)$$

where $\boldsymbol{\mu}_u$ in the first term denotes the across-the-slot belief vector update, $\boldsymbol{\mu}_f$ in the second term is the within-slot-update based on sensing history $\mathcal{R}\cup\{(i,F)\}$, and $\boldsymbol{\mu}_b$ in the third term is the within-slot-update based on sensing history $\mathcal{R}\cup\{(i,B)\}$. In the following, belief vector update should be implicitly understood from the context. Now, the boundary conditions are given by $F_j(\mathcal{S},\mathcal{R},\boldsymbol{\mu})=X_{j-1}(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)$, if $\mathcal{S}=\varnothing$ and $j>1$ and $F_j(\mathcal{S},\mathcal{R},\boldsymbol{\mu})=0$, if $j\le 1$. Using (1), the single slot reward is computed as

$$\begin{aligned}&X_1^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu})=\\ &\theta^i\Big\{[1-\tilde{g}(T)](X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)+T\tilde{g}(T)\\ &+f_{M-|\mathcal{S}|+1}(T)\Big\}+(1-\theta^i)X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)\end{aligned} \quad (7)$$

Using (6) and (7), we get $X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu}) = (1-\theta^i)\Big\{X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)+F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)\Big\}+\theta^i\Big\{\tilde{g}(T)[T+X_{j-1}(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)]+f_{M-|\mathcal{S}|+1}(T)\Big\}+\theta^i[1-\tilde{g}(T)]\times\Big\{X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)+F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)\Big\}$. Differentiating w.r.t T and setting the term to zero we get the optimal threshold

$$\begin{aligned}T_j^i(\mathcal{S},\mathcal{R}) &= X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)-X_{j-1}(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)\\ &\quad+F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f) \quad (8)\\ &= X_j(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)-X_{j-1}(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)\end{aligned}$$

With this optimal threshold, we have $X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu}) = \theta^i\Big\{X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)+f_{M-|\mathcal{S}|+1}(T_j^i)+F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)\Big\}+(1-\theta^i)\Big\{X_1(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)+F_{j-1}(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)\Big\}$. Setting $Y_1=X_j(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)$, $Y_2=X_{j-1}(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)$ and $Y_3=X_j(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)$, we have $X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu})=\theta^i(Y_1+f_{M-|\mathcal{S}|+1}(Y_1-Y_2))+(1-\theta^i)Y_3$. Using elementary calculus, it can be shown that $X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu})$ is an increasing function of Y_1,Y_2 and Y_3. Hence maximizing the terms Y_1, Y_2 and Y_3 independently, we have

$$\begin{aligned}X_j^i(\mathcal{S},\mathcal{R},\boldsymbol{\mu}) \;=\; &(1-\theta^i)X_j(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,B)\},\boldsymbol{\mu}_b)+\\ \theta^i\Big\{&f_{M-|\mathcal{S}|+1}(T_j^i(\mathcal{S},\mathcal{R}))+X_j(\mathcal{S}\backslash\{i\},\mathcal{R}\cup\{(i,F)\},\boldsymbol{\mu}_f)\Big\}.\end{aligned} \quad (9)$$

To summarize, for a given sensing situation in the present slot, we have derived the maximum expected total throughput over next j slots given by (5) using recursion given in (9) and optimal stopping thresholds given in (8). In order to find the multi-slot optimal sensing procedure, we need to carry-out throughput optimization for each and every possible sensing situation and search for the procedure which results in maximal throughput over N slots. This search can be accomplished using dynamic programming as an extension of the single-slot optimization discussed earlier and is similar to the travelling salesman problem. It is worth noting that once we have *reached* a given sensing situation/history, the *order* in which we *had* sensed the channels to reach that situation does not affect the future reward from that situation. The search complexity for $M-$ channel $N-$ slot optimization grows exponentially with MN.

D. Suboptimal procedures

Greedy algorithm: We propose a greedy algorithm, where in each slot, we maximize the expected rewards for only the *immediate* next slot. Specifically, given the belief vector $\boldsymbol{\mu}_u$ based on partial observability after sensing in n slots, we maximize $X_1(\mathcal{D},\varnothing,\boldsymbol{\mu}_u)$ for $(n+1)^{th}$ slot. Here, the search complexity grows exponentially with M.

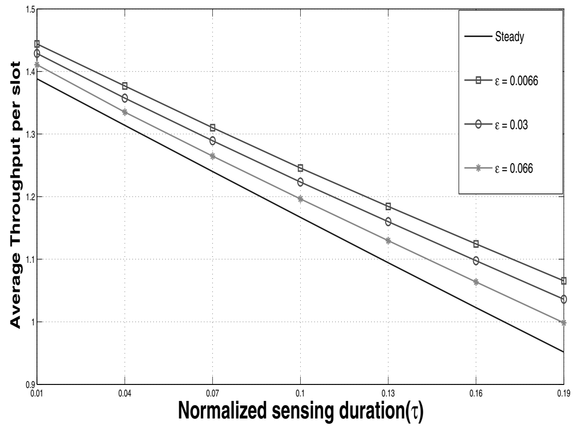

Fig. 3. Effect of Markov transitions uncertainty Parameter ϵ.

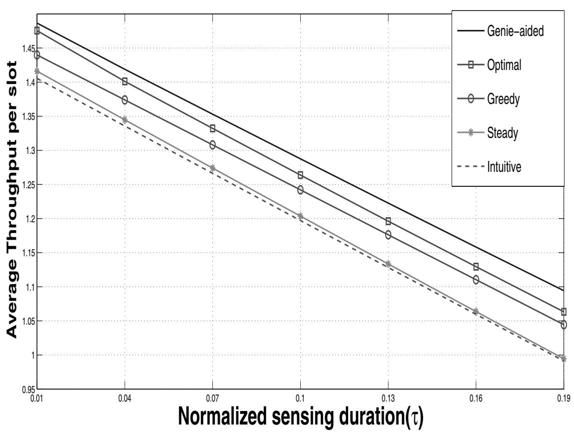

Fig. 4. Comparison for the transition matrix $P(0.05, [2, 2, 2, 3, 4, 5, 7, 7])$.

Steady State sensing: Using the steady state probability vector of the Markov chain, we design single-slot sensing procedure as done in Section III-A and follow the same procedure in all slots. This procedure will be optimal if the channel availability statistics are i.i.d. across slots [1], but sub-optimal, if there is correlation of channel-availability across slots. There is a one-time search whose complexity grows exponentially with M.

Intuitive Sensing: Using the steady state probability vector of the Markov chain, we can find the channels' marginal availability probabilities - θ^i, $i \in \{1, \cdots, M\}$. Intuitive sensing [6] senses channels in the descending order of their availability probabilities. There is one-time complexity in sorting.

IV. Numerical Results

For numerical simulations, we consider the case of $M = 3$, $N = 4$. With $\epsilon \in (0, \frac{1}{7})$ and an ordered set $\mathcal{A}$ of size 8 with entries from $\{1, \cdots, 8\}$, we construct Markov transition matrix $P(\epsilon, \mathcal{A})$ such that transition from state i to state $\mathcal{A}(i)$ happens with probability $1 - 7\epsilon$ and transition from state i to other states happen with probability ϵ. In Fig. 3, we plot the optimal throughput for the transition matrix $P(\epsilon, [8, 1, 2, 3, 4, 5, 6, 7])$ for various values of ϵ and compare the performance with steady state sensing procedure. For small values of ϵ (small uncertainty in state transitions across slots) we see that multi-slot optimal procedure performs significantly better than steady-state sensing. For higher values of ϵ, the correlation of channel availability across slots gets *reduced* and hence the loss in steady-state sensing is small.

We compare the throughput of the various approaches in Fig. 4 along with genie aided throughput in which the state in the previous slot was assumed to be completely known while maximizing the reward for the current slot (and thus eliminating the problem of partial observability). As the steady state sensing [1] and intuitive sensing [6] ignore the correlation of channel availability statistics, they perform poorer compared to our multi-slot optimal approach. We also note that the greedy sensing procedure (whose complexity does not grow with N) performs better than intuitive and steady state sensings.

To conclude, we considered the cognitive radio scenario where channel availability statistics are correlated across time and frequency and developed optimal sensing procedure to maximize the cognitive throughput. We also proposed a low-complexity Greedy sensing procedure. Numerical results show gains of proposed techniques over existing methods.

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