13-2-13

Lec 20

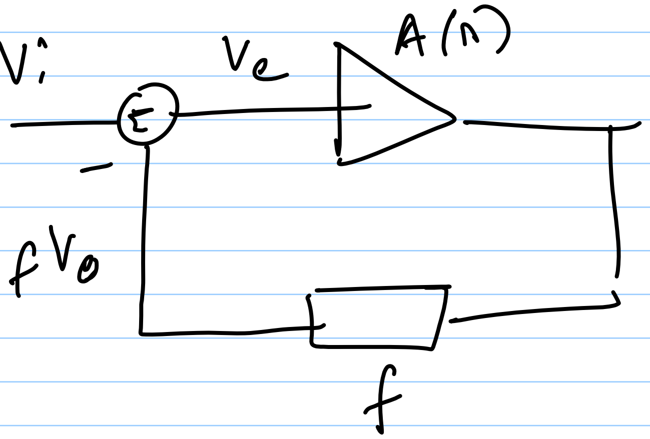
Divide the two equations

$$\frac{\sigma'}{\omega'} = -\cot(\omega'\tau)$$

$$\sigma' = -\omega' \frac{\cos(\omega'\tau)}{\sin(\omega'\tau)} = -\frac{1}{\tau} \frac{\cos(\omega'\tau)}{\operatorname{sinc}(\omega'\tau)}$$

1) for small τ, , σ will be in LHP
   $\Rightarrow$ dying exp. is envelope of ringing
2) $\tau = \tau_a$, $\sigma = 0 \Rightarrow$ sustained ringing
3) $\tau > \tau_a$, $\sigma > 0$ (RHP) $\Rightarrow$ inc. exp.

* assume that a few time constants are dominant



$$\frac{V_o}{V_i} = \frac{A(s)}{1+fA(s)}$$

$$= \frac{1}{f} \cdot \frac{L(s)}{1+L(s)}$$

$L(s) = fA(s)$ Loop gain

$|L| \geq 1$, $\angle L = -180^\circ \Rightarrow$ oscillations

gain margin – $\frac{1}{\text{gain}}$ when phase $= -180^\circ$

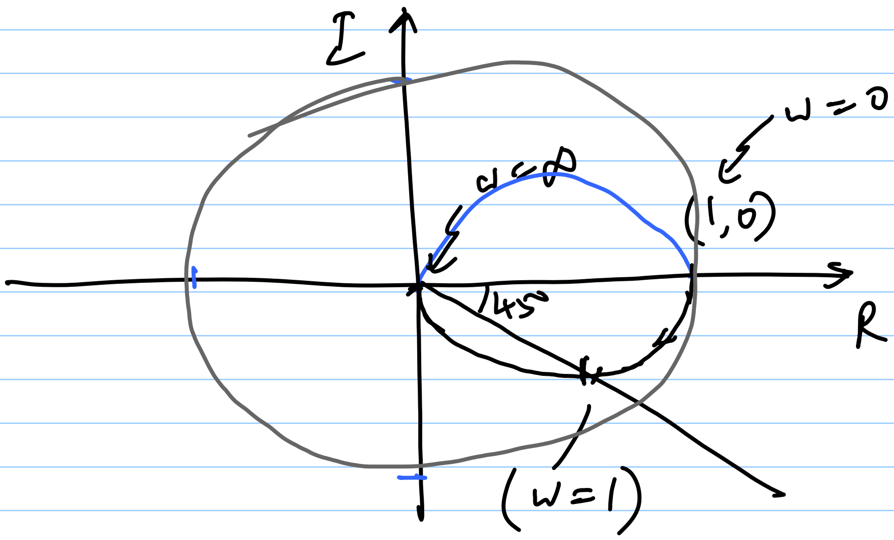
phase margin – $(180^\circ + \text{phase})$ when gain $= 1$

Nyquist Curve

$$H(j\omega) = R(\omega) + j\,I(\omega)$$

* plot $I(\omega)$ vs. $R(\omega)$

$$H(j\omega) = \frac{1}{1+j\omega} = \underbrace{\frac{1}{1+\omega^2}}_{R} - \underbrace{\frac{j\omega}{1+\omega^2}}_{I}$$



stability : # of encirclements of $(-1, 0)$

$$1+L(s)=0$$

2 zeros, 3 poles

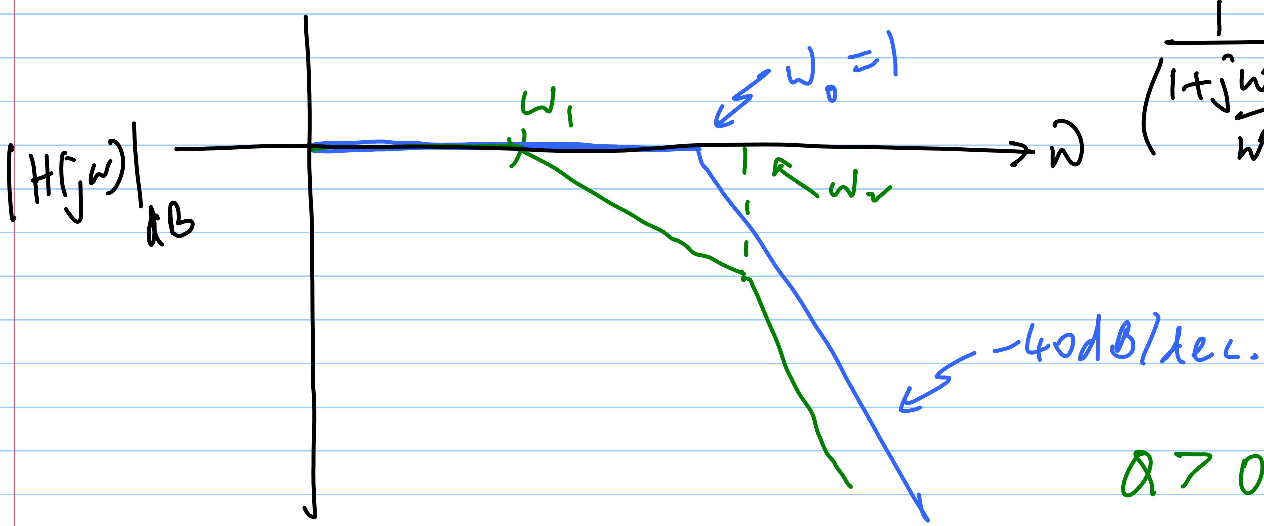
asymptotic to -90° axis

zeros > # poles

$$H(s)=\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$$

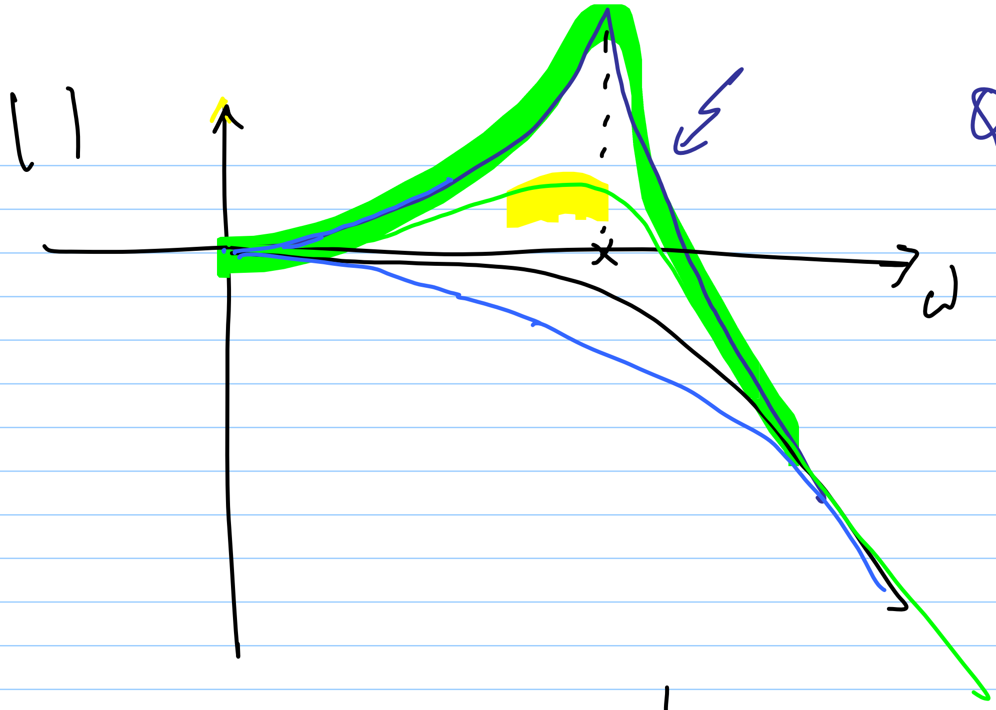
¿ $\frac{1}{(1+j\omega)^2}$

$\frac{1}{\left(1+\frac{j\omega}{\omega_1}\right)\left(1+\frac{j\omega}{\omega_2}\right)}$



$Q > 0.707$

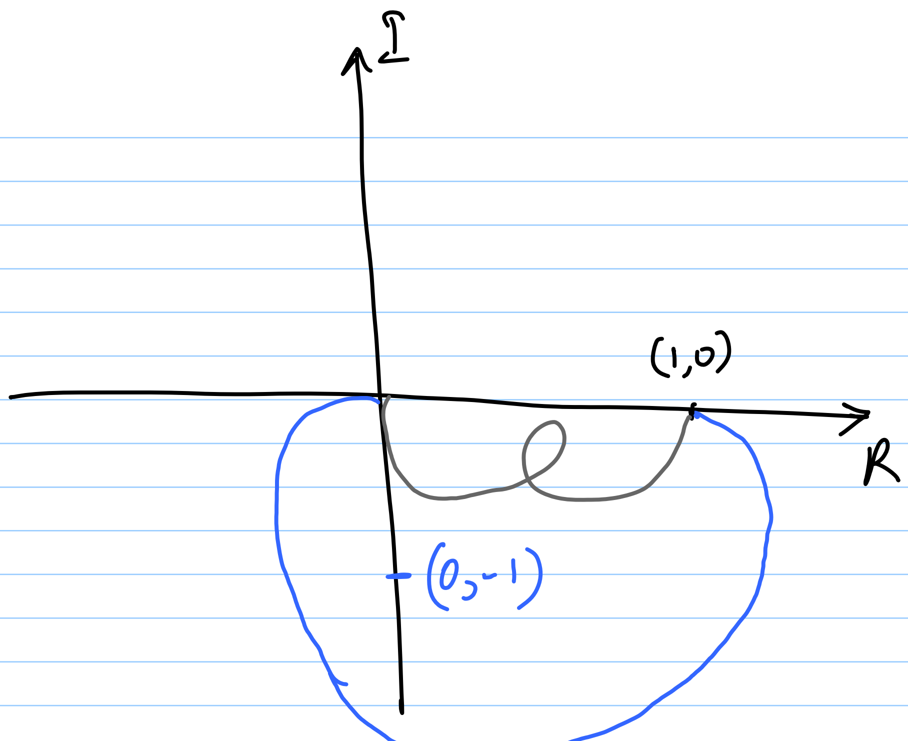
3 >



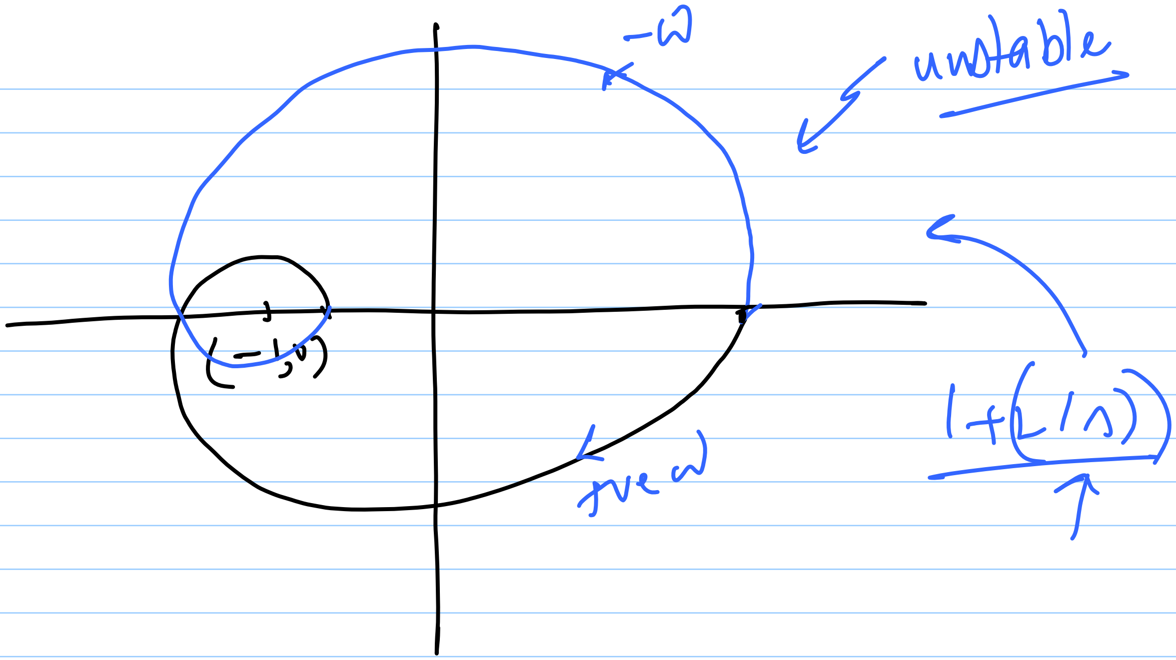
Q = quality factor.

(Resonance)

$\zeta = 0 \Rightarrow \dfrac{1}{s^2 + \omega_n^2}$



$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



* Nyquist plot → delays
→ non-rational terms