12-2-13

Lec 19

Boundary conditions

@ $t = T_d$

$$V_o = k - w_n T_d = k(1-\zeta)$$

@ $t = 0^+$, $V_o = k$

$$\alpha_1 + \alpha_2 = k \quad \checkmark$$

@ $t = T_d$, $\frac{dV_o}{dt} = -w_o \Rightarrow \alpha_1 \sigma_1 e^{\sigma_1 T_d} + \alpha_2 \sigma_2 e^{\sigma_2 T_d} = -w_n$

each value of $T_d \Rightarrow$ unique σ_1 & σ_2

$\Rightarrow$ unique α_1 & α_2

Settling speed

original case: single pole system

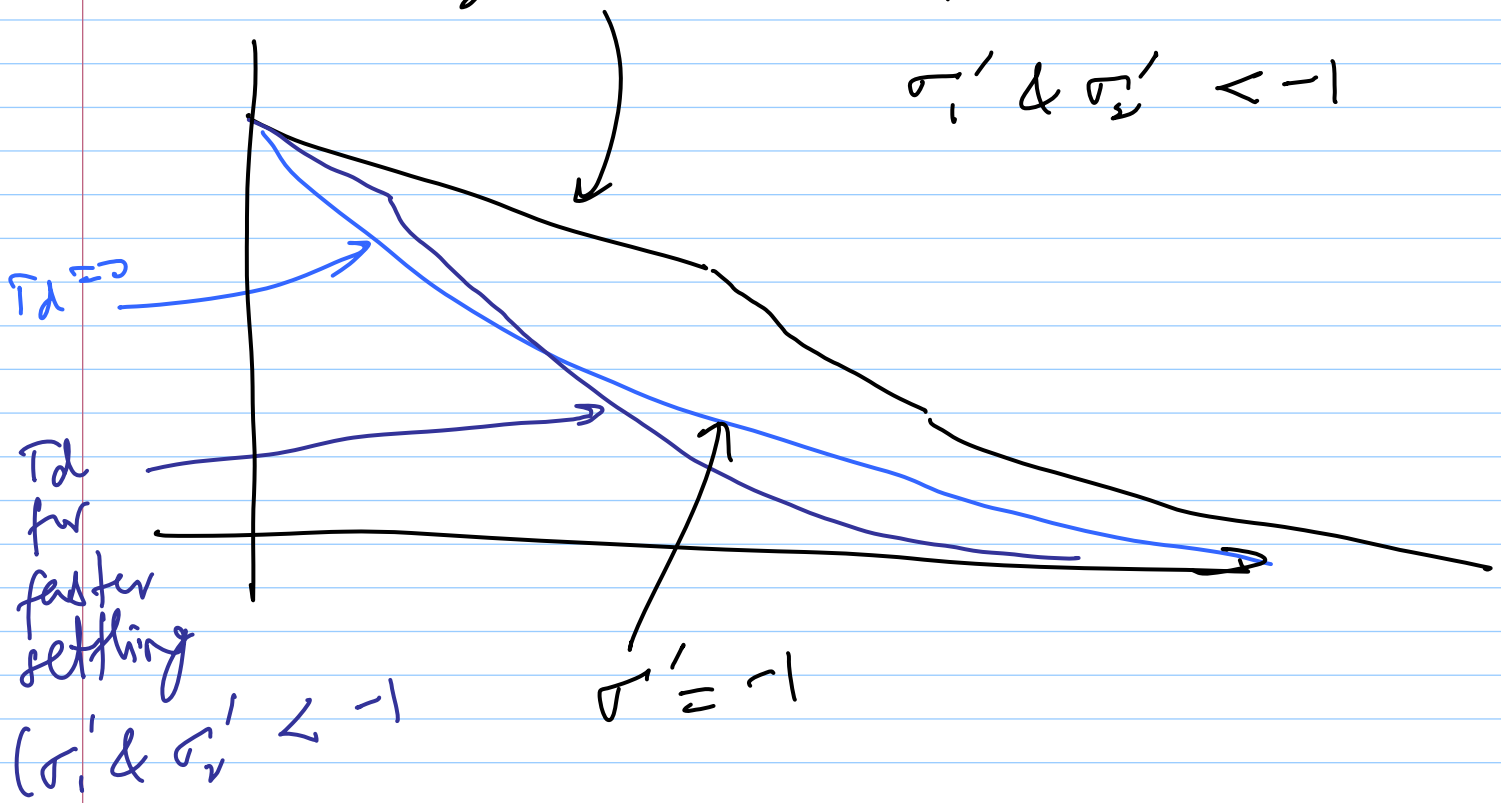
$$e^{-\frac{w_n t}{k}}$$

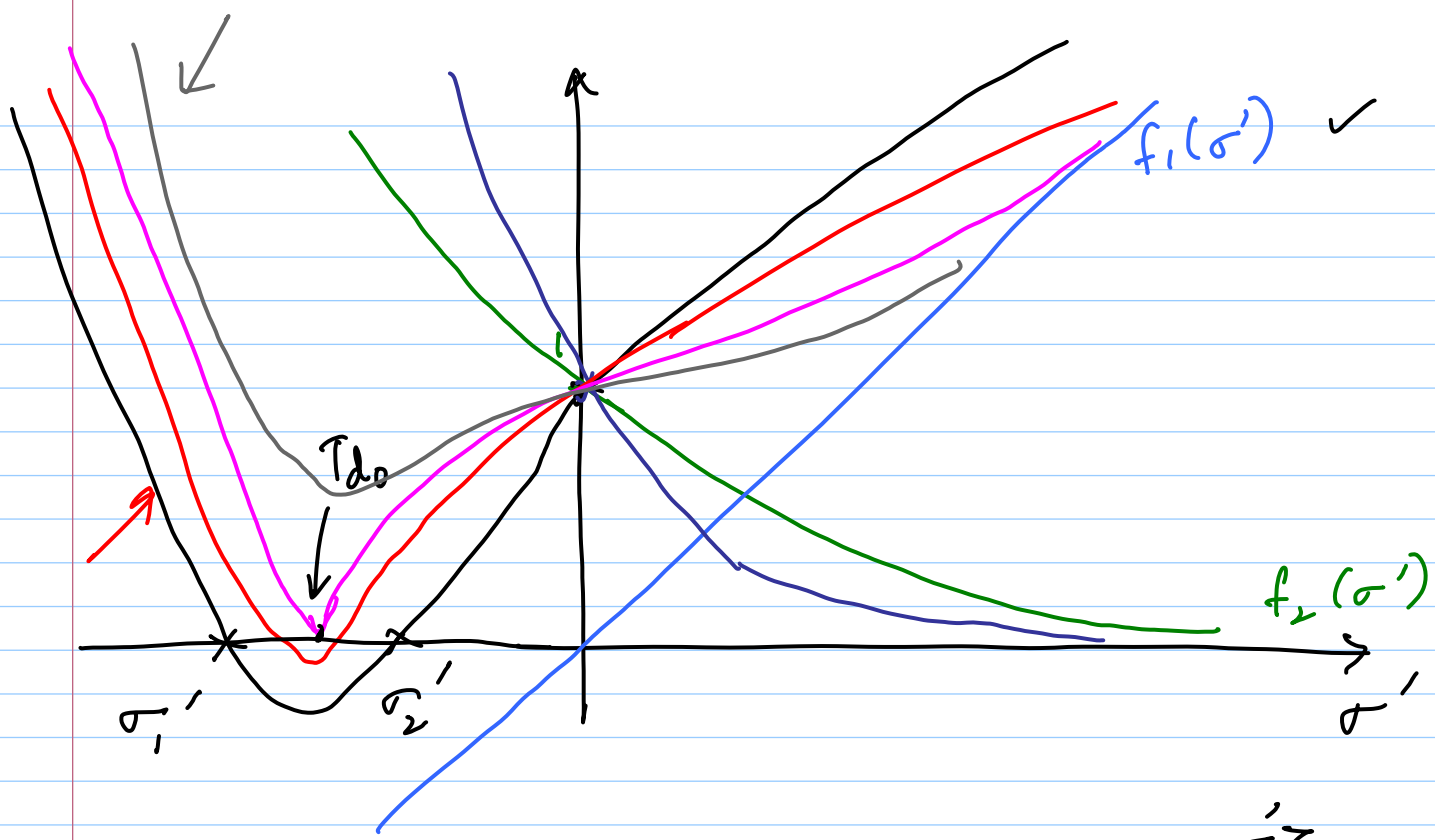
w/ T_d : $\alpha_1 e^{\sigma_1 t} + \alpha_2 e^{\sigma_2 t}$

T_d large

for $T_d > 0$,

$\sigma_1' \& \sigma_2' < -1$





$$f_2(\sigma') = e^{-\sigma' \tau}$$

$$\tau = T_d \cdot \frac{\omega_n}{K}$$

* Critical T_{do}

solution is of the form $(C_1 + C_2 t)e^{\sigma t}$

* $T_d > T_{do}$

$$s = \sigma + j\omega$$

$$s' = \frac{\sigma + j\omega}{\omega_u/k} = \sigma' + j\omega'$$

$$s' + e^{-s'\tau} = 0 \Rightarrow \sigma' + j\omega' + e^{-\sigma'\tau} e^{-j\omega'\tau} = 0$$

$$\Rightarrow \quad \sigma' = -e^{-\sigma'\tau} \cos\omega'\tau \qquad -(1)$$

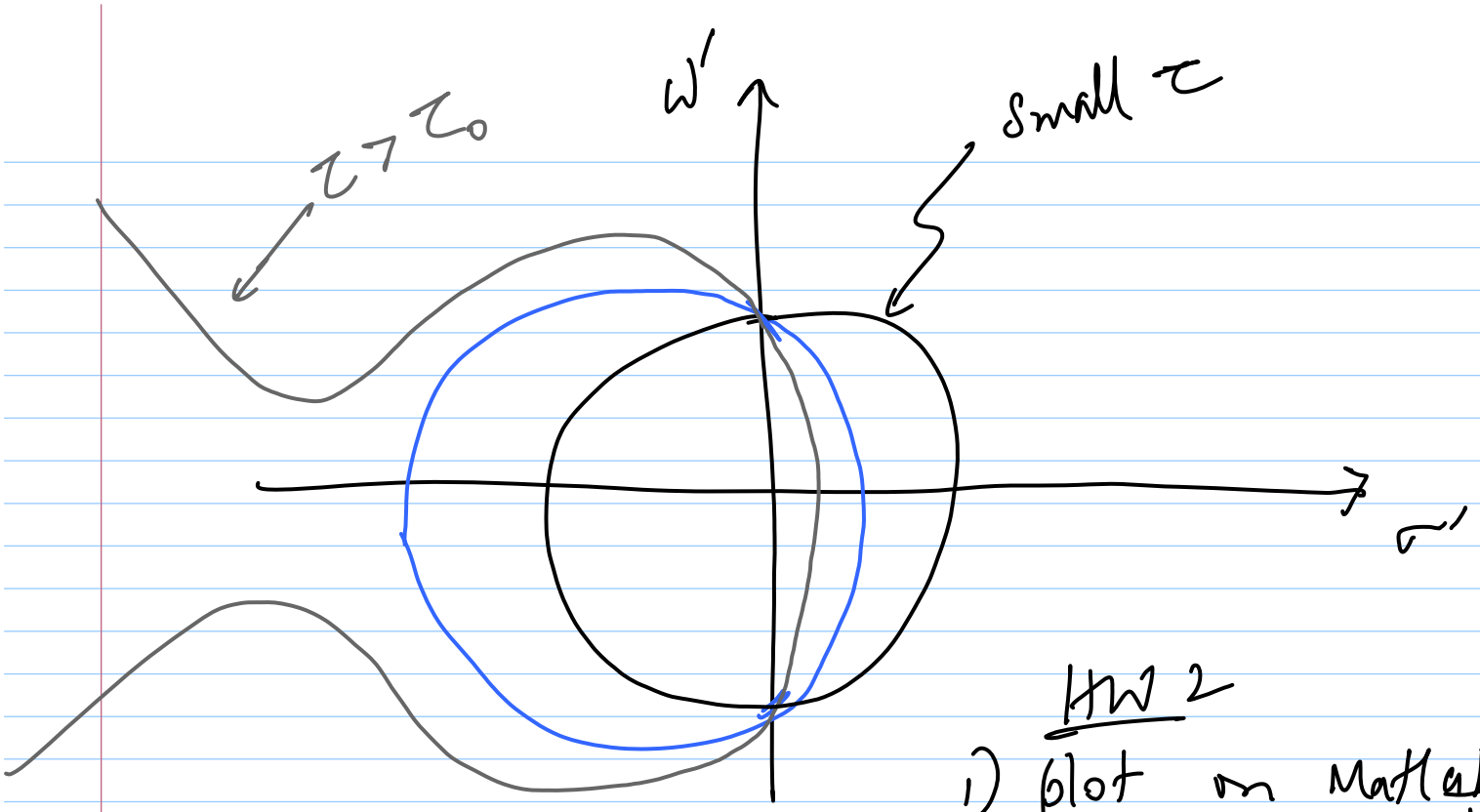
$$\omega' = e^{-\sigma'\tau} \cdot \sin\omega'\tau \qquad -(2)$$

$\tau = 0 \quad (T_d = 0)$

$$\omega' = 0 \; ; \quad \sigma' = -1$$

square & add (1) & (2) :

$$\sigma'^2 + \omega'^2 = e^{-2\sigma'\tau}$$



HW 2

1) plot in Matlab

a) $\alpha_1 e^{\sigma_1 t} + \alpha_2 e^{\sigma_2 t}$

b) $\sigma'^2 + \omega'^2 = e^{-2\sigma\tau}$