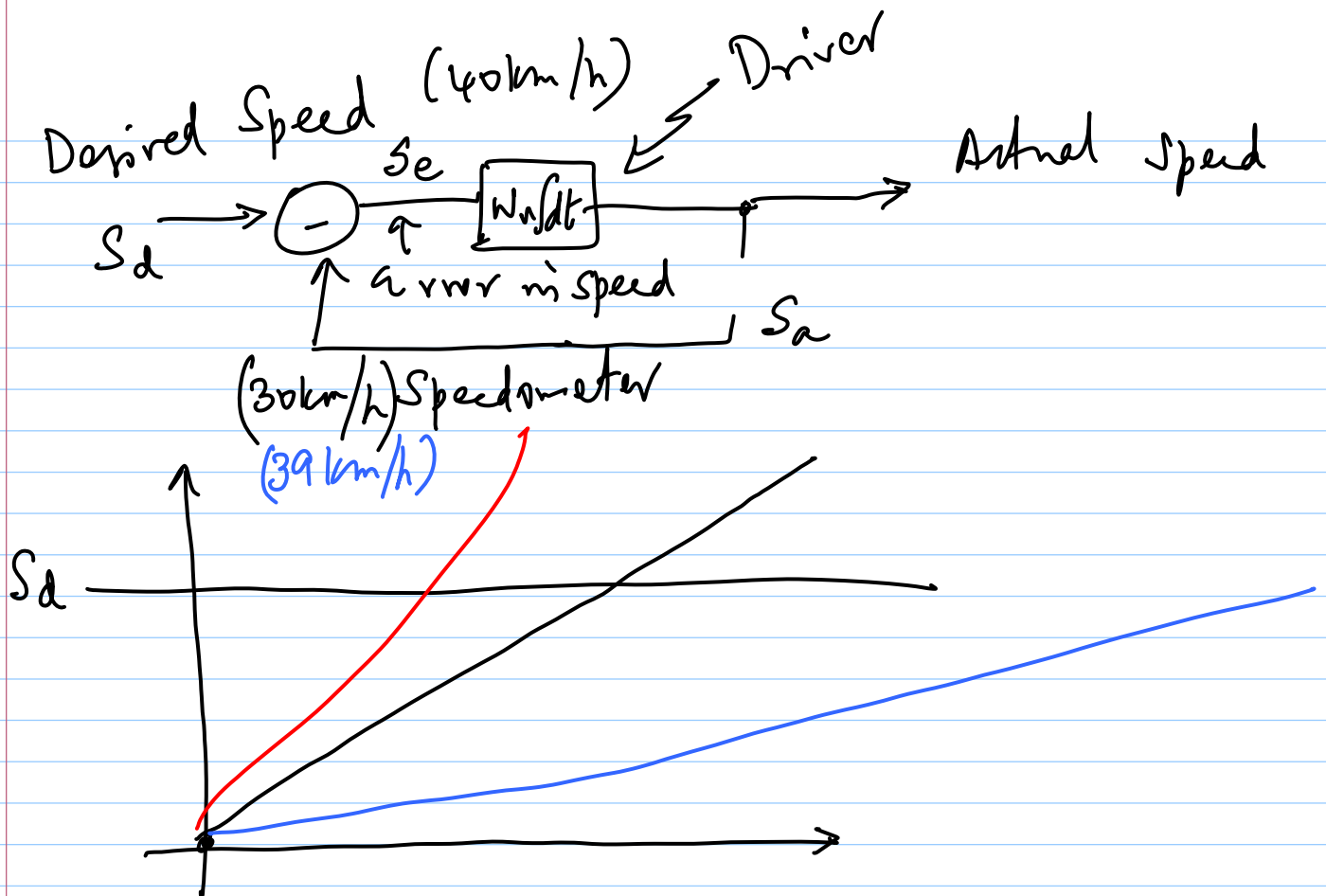


8-2-13

Lec 17

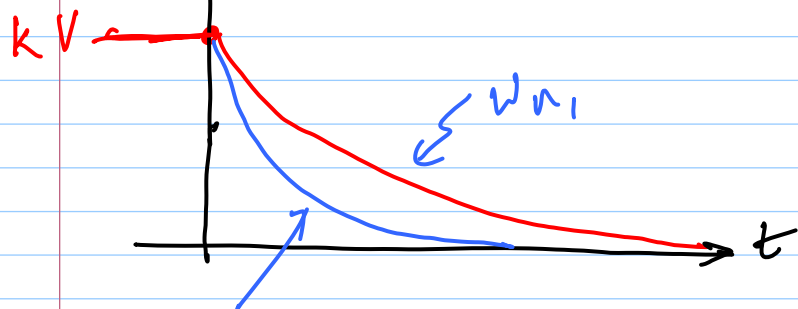
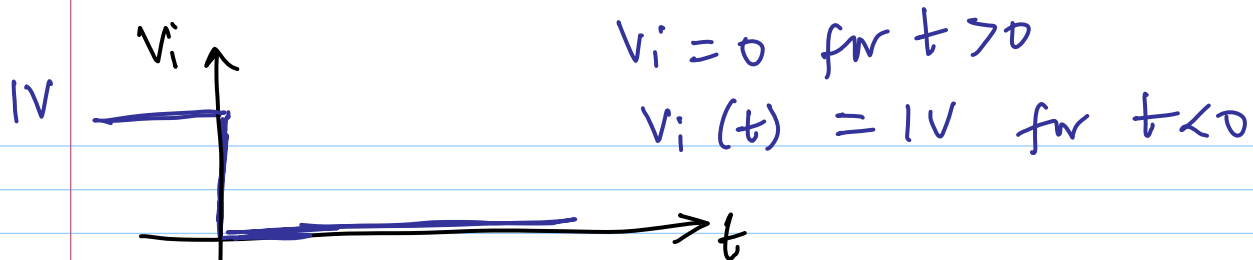
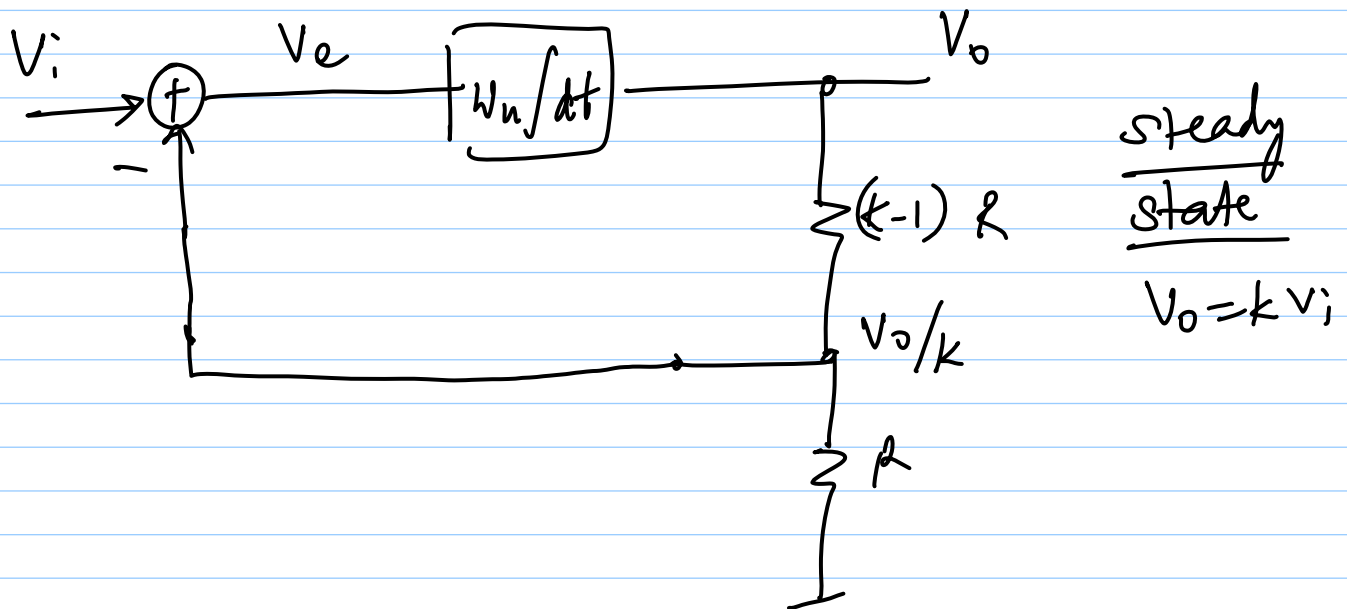
Feedback

- 1) Desired value
- 2) Compare $\downarrow$ w/ actual value
- 3) Drive actual value towards desired value



$$V_i \rightarrow \boxed{} \rightarrow V_o = k V_i$$

$$\Rightarrow V_i = V_o / k$$



$$V_e = \frac{1}{w_n} \frac{dV_o}{dt}$$

$$V_e = V_i - \frac{V_o}{k}$$

$$V_i - \frac{V_o}{k} = \frac{1}{\omega_n} \frac{dV_o}{dt}$$

$$\frac{dV_o}{V_i - \frac{V_o}{k}} = \omega_n dt$$

$$\underline{V_i = 0}$$

$$\frac{dV_o}{(-V_o/k)} = \omega_n \cdot dt$$

$$V_o = e^{(-\omega_n/k) \cdot t}$$

$$\underline{V_i \neq 0}$$

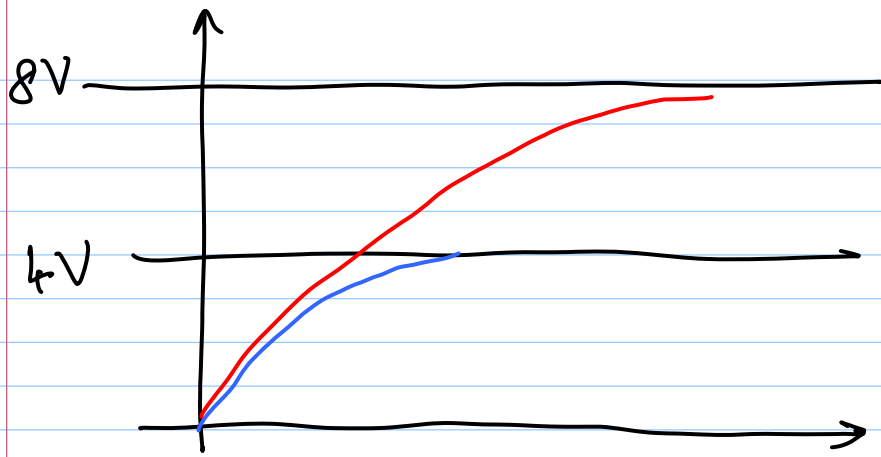
$$\frac{dV_o}{V_i - \frac{V_o}{k}} = \omega_n dt$$

$$-k \ln \left(V_i - \frac{V_o}{k} \right) \bigg|_{V_o(0)}^{V_o(t)} = \omega_n t \bigg|_0^t$$

$$V_o(t) = V_o(0) \exp \left(-\frac{\omega_n}{k} \cdot t \right) \leftarrow \text{effect of } V_o(0)$$

$$+ k V_i \left[1 - \exp \left(-\frac{\omega_n}{k} \cdot t \right) \right]$$

↑ effect of V_i



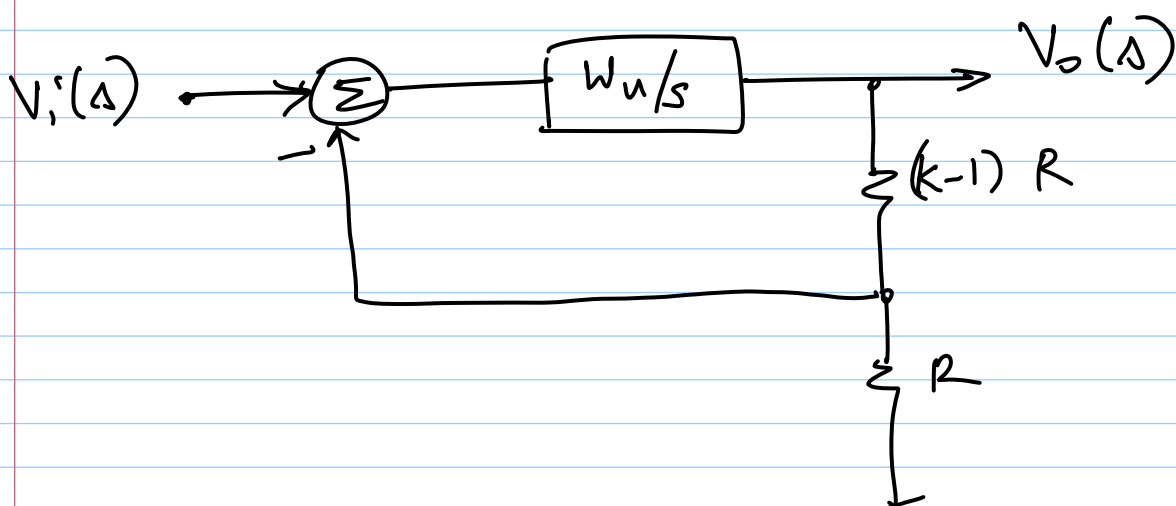
$$k=8, V_o=8V$$

$$k=4, V_o=4V$$

Initial slope = $\omega_n \cdot V_i$

$$\tau \text{ (time constant)} = \frac{k}{\omega_n}$$

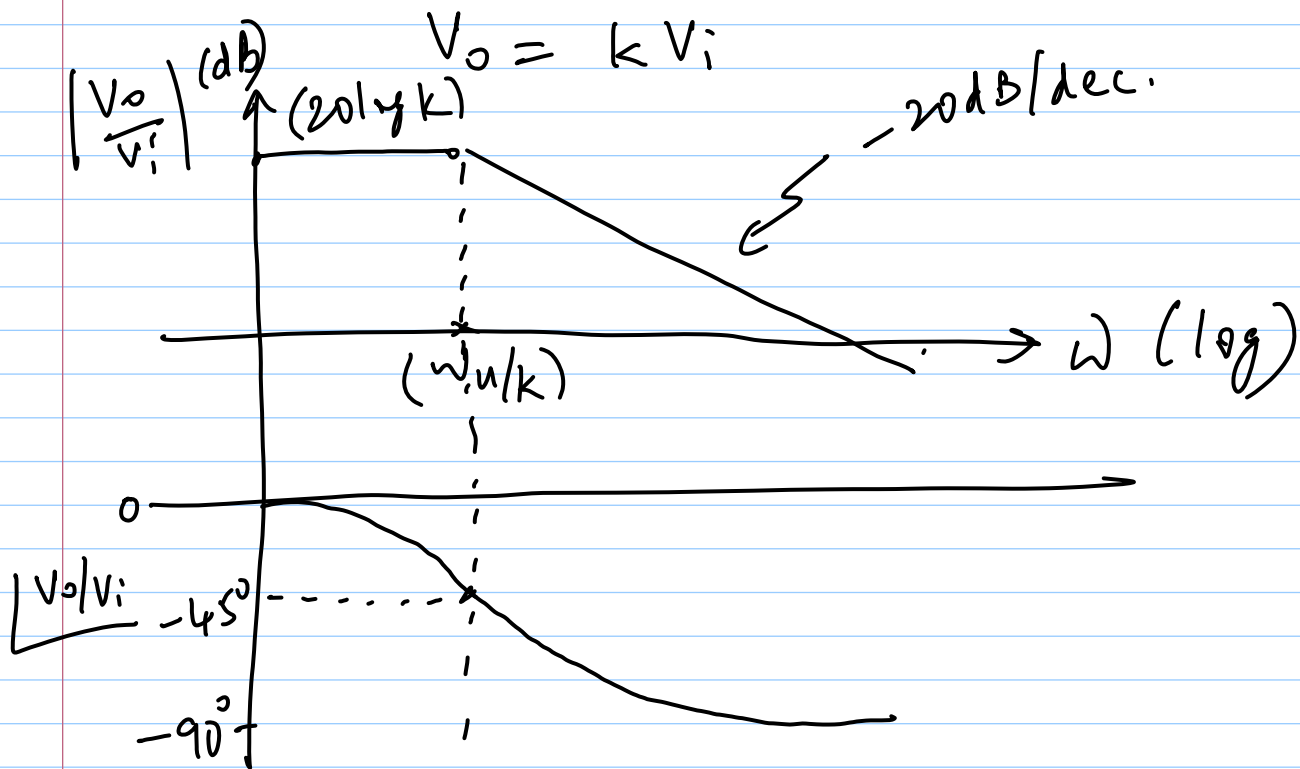
Laplace domain solution



$$V_o(s) = V_i(s) \cdot \frac{\omega_n}{s + \omega_n/k}$$

$$\frac{V_o}{V_i}(s) = k \cdot \frac{1}{1 + s/(\omega_n/k)}$$

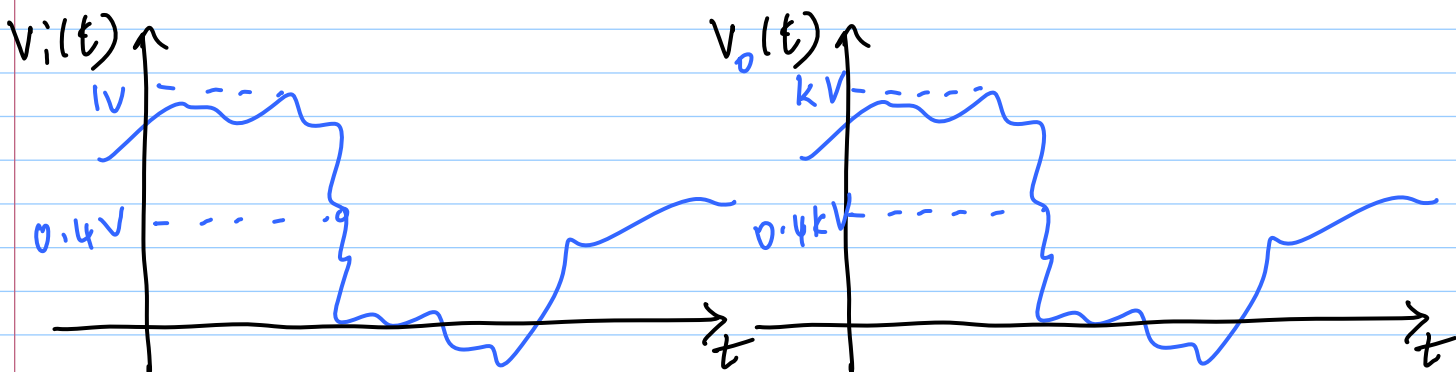
constant $V_i \Rightarrow s=0$



$V_i = \text{sinusoid}$ e.g. Acoswt

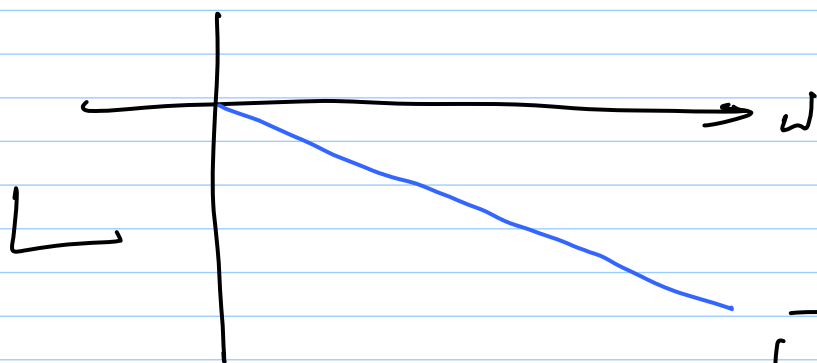
$$\left| \frac{V_o}{V_i} \right| = \frac{k}{\sqrt{1 + \frac{k^2 \omega^2}{\omega_n^2}}}$$

$$\angle \frac{V_o}{V_i} = -\tan^{-1} \left(\frac{k\omega}{\omega_n} \right)$$



* equal delays for all freq. components

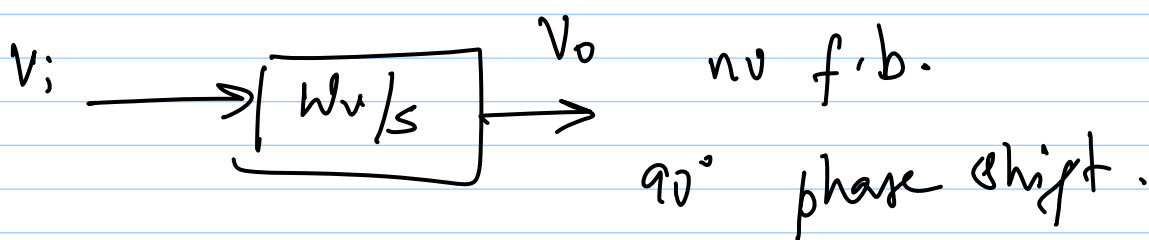
→ phase of the system is linear w/ freq.



phase is linear when:
gain $\approx k$ when:

$$\omega \ll \frac{\omega_n}{k}$$

$$\omega \gg \frac{\omega_n}{k}$$



$$\omega \ll \frac{\omega_n}{k}$$

gain $= k$
delay $\approx k/\omega_n$

$$\omega \gg \frac{\omega_n}{k}$$

gain $\propto 1/\omega$
delay $\approx$

$$\left. \begin{array}{l} \text{BW of system} \approx \frac{\omega_n}{k} \end{array} \right\}$$