15-2-12

Lec 18

* Critical damping : T_{d_0}

Solution is of the form $(c_1 + c_2 t)e^{\sigma t}$

$T_d > T_{d_0} \Rightarrow \quad \sigma' + e^{\sigma'\tau} > 0 \quad \forall \sigma$ (real)

$s = \sigma + j\omega$

$s' = \dfrac{(\sigma + j\omega)}{(\omega_n/k)}$ (normalised s)

$= \sigma' + j\omega'$

$$s' + e^{-s'\tau} = 0$$

$$(\sigma' + j\omega') + e^{-(\sigma' + j\omega')\tau} = 0$$

$$\sigma' + j\omega' + e^{-\sigma'\tau} e^{-j\omega'\tau} = 0$$

$$\sigma' + j\omega' + e^{-\sigma'\tau}(\cos\omega'\tau - j\sin\omega'\tau) = 0$$

$$\Rightarrow \left.\begin{aligned} \sigma' &= -e^{-\sigma'\tau}\cos\omega'\tau \\ \omega' &= e^{-\sigma'\tau}\sin\omega'\tau \end{aligned}\right)$$

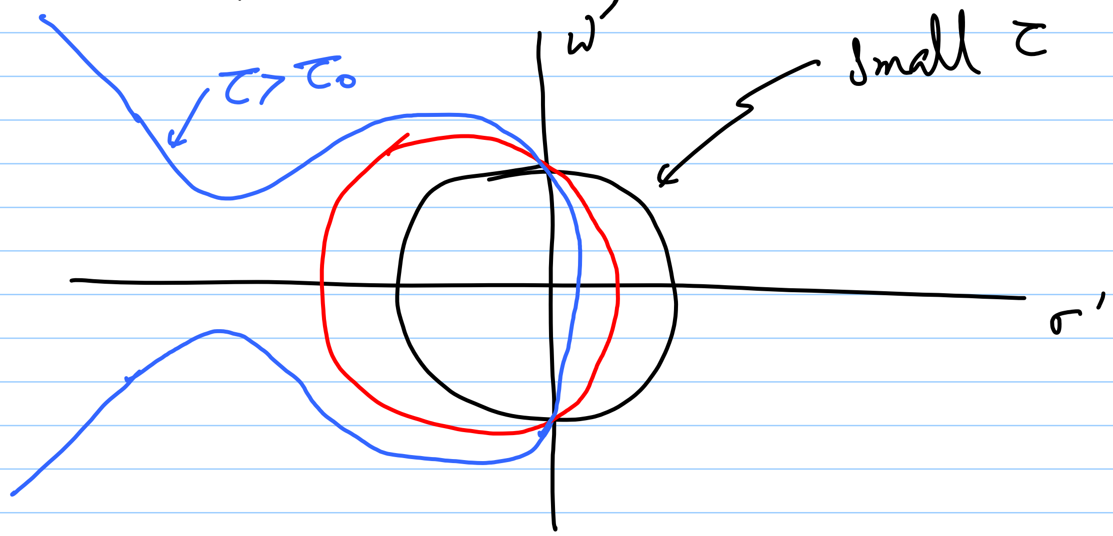
can be solved numerically, graphically etc.

for $\tau = 0$,

$$\sigma' = -1 \quad , \quad \omega' = 0 \qquad \text{(sanity check)}$$

square & add

$$\sigma'^2 + \omega'^2 = e^{-2\sigma'\tau}$$



divide the two eqns:

$$\frac{\sigma'}{\omega'} = -\cot(\omega'\tau)$$

$$\sigma' = -\omega' \frac{\cos(\omega'\tau)}{\sin(\omega'\tau)}$$

$$= -\frac{1}{\tau} \frac{\cos(\omega'\tau)}{\operatorname{sinc}(\omega'\tau/\pi)} \quad \leftarrow \text{can be plotted}$$

* for small τ, σ's are in LHP
   $\Rightarrow$ dying exp.s

* for certain τ (say τ_a)

$\sigma' = 0$, roots lie on $j\omega$ axis

$\Rightarrow$ Sinusoid of constant amplitude

* for $\tau > \tau_a$,

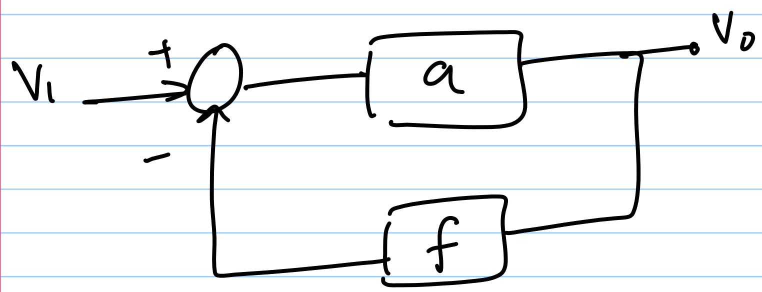
solutions blow up (unstable)

* There may be several solutions (not just 2)

$\Rightarrow$ dominant time constants matter

Loop gain (break loop & find gain around loop)

$\rightarrow$ quantifies strength of f.b.



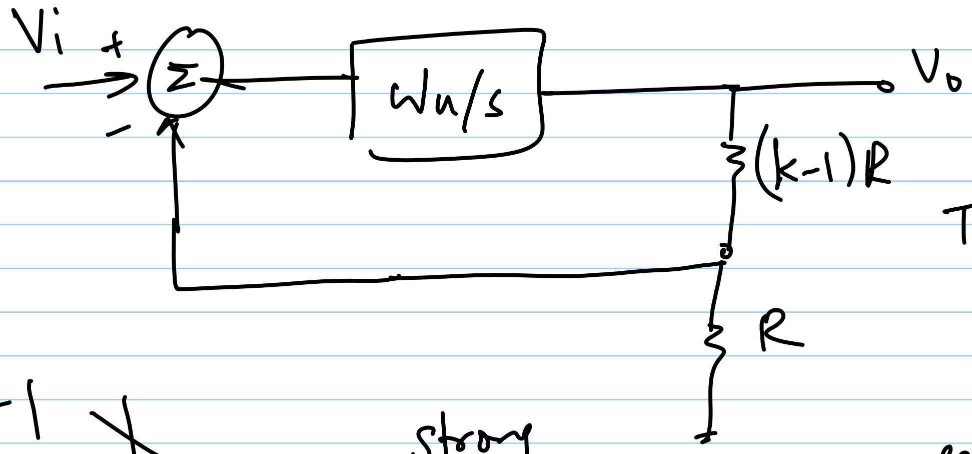
$$\frac{V_o}{V_i} = \frac{a}{1+af}$$

$$= \frac{1}{f} \cdot \frac{af}{1+af}$$

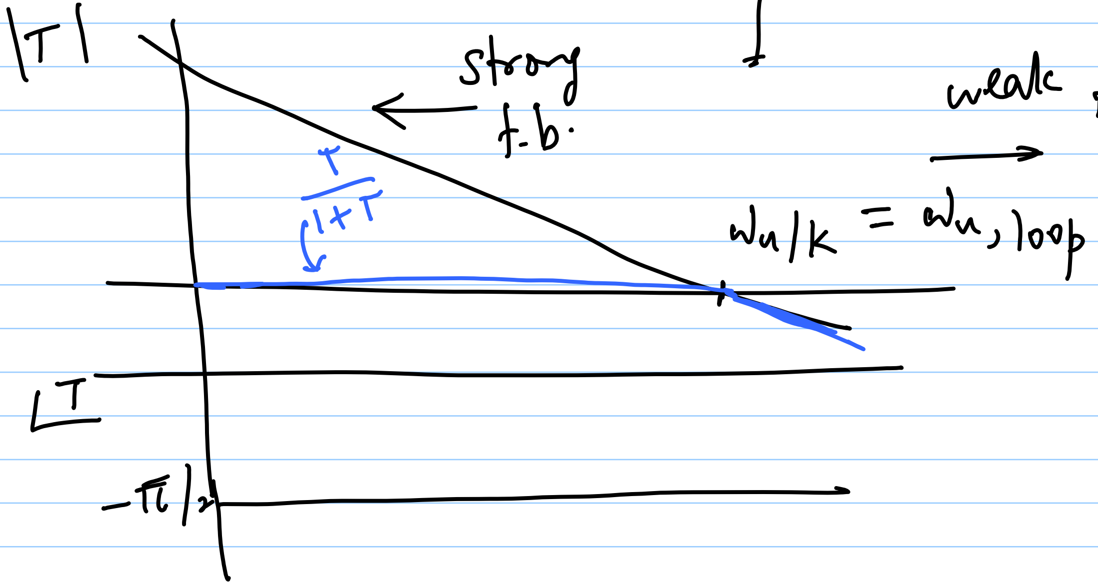
$$= \frac{1}{f} \cdot \frac{T}{1+T}$$

$|T| >> 1 \Rightarrow \frac{V_o}{V_i} = \frac{1}{f}$

$|T| << 1 \Rightarrow \frac{V_o}{V_i} = a$ (i.e. no f.b.)



$$T(s) = \frac{\omega_u/k}{s} = \omega_{u,loop}/s$$



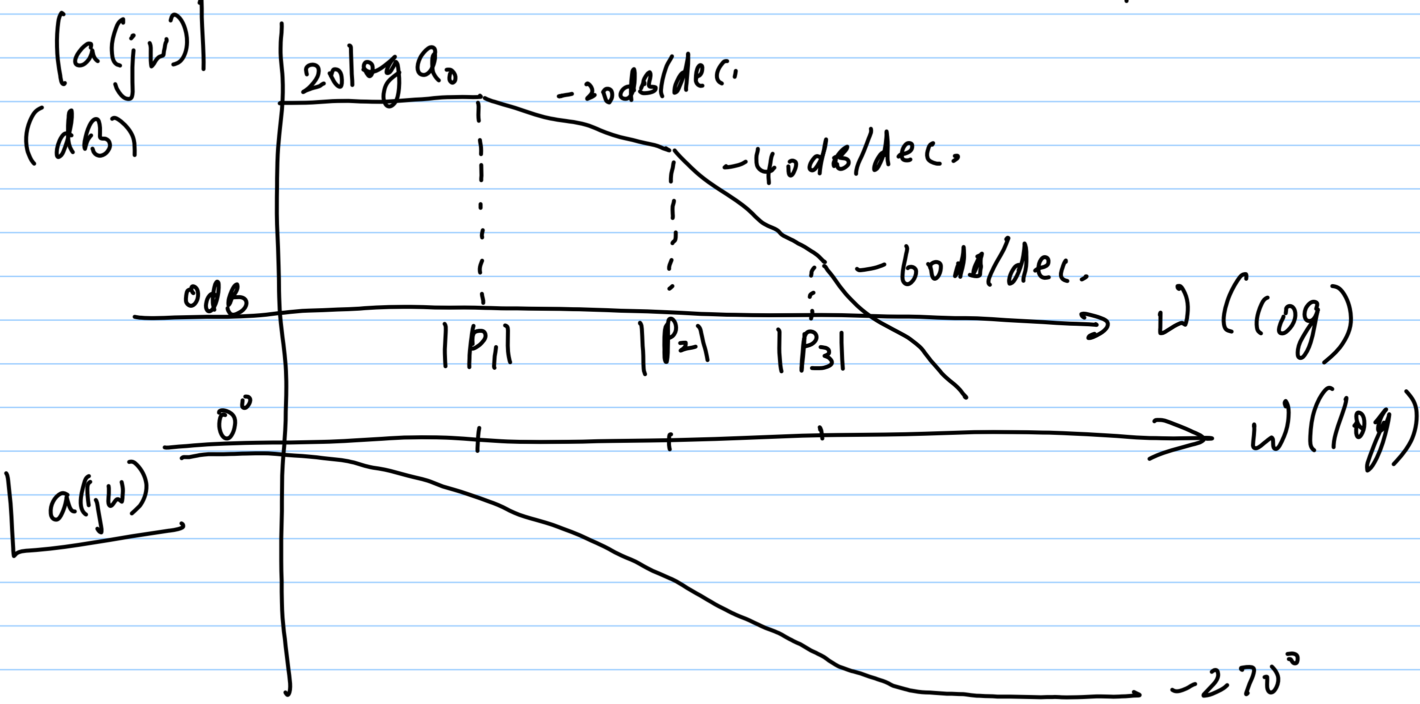
What kind of delays are possible?

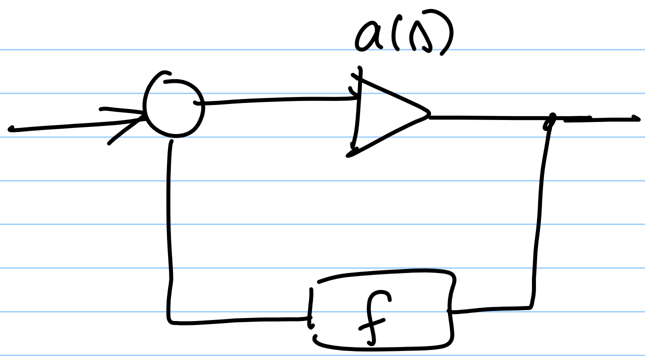
* multiple poles & zeroes due to parasitic caps. in forward amp or f.b network

* we want poles to be in LHP (stability)
  → too complicated

Use <u>Nyquist</u> <u>criterion</u>

let $a(s) = \dfrac{a_0}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\left(1-\frac{s}{p_3}\right)}$

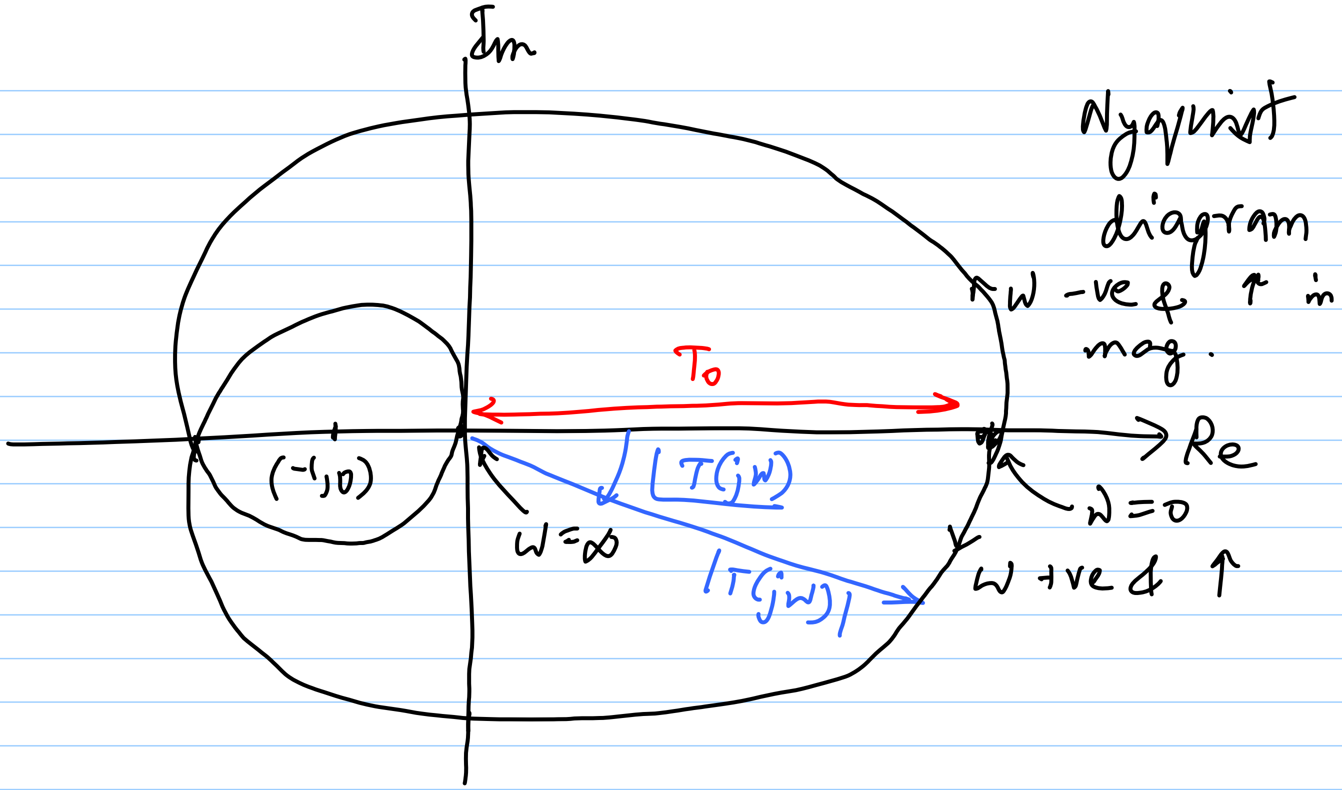




f = constant

$T(j\omega) = f \cdot a(j\omega)$; $A(j\omega) = \dfrac{a(j\omega)}{1+T(j\omega)}$

* draw $T(j\omega)$ in a polar plot
  with ω as a variable
  → ω goes from $-\infty$ to ∞



$w = 0 \Rightarrow T(jw) = T_0, \ \angle T = 0$

as $w \uparrow \Rightarrow T(jw) \downarrow, \ \angle T < 0 \Rightarrow 4^{th}$ quadrant

@ $w = w_{180} \Rightarrow T(jw) = -T_{180}$

$w \to \infty, \ \angle T \to -270°, \ |T| \to 0$

$\Rightarrow$ asymptotic to origin, tangential to Im axis

* If $|a(jw_{180}) \cdot f| > 1$ @ w_{180}, diagram will encircle $(-1,0)$

$\boxed{\text{If Nyquist diagram encircles } (-1,0), \text{ amplifier is unstable}}$

* This tests for poles of $A(s)$ in RHP
  → If $(-1, 0)$ is encircled, amplifier will be unstable
* # of encirclements of $(-1,0)$ = # of RHP poles
  → in this example, 2 RHP poles
* Nyquist plot is powerful — non rational functions (e.g. delays) can be accomodated

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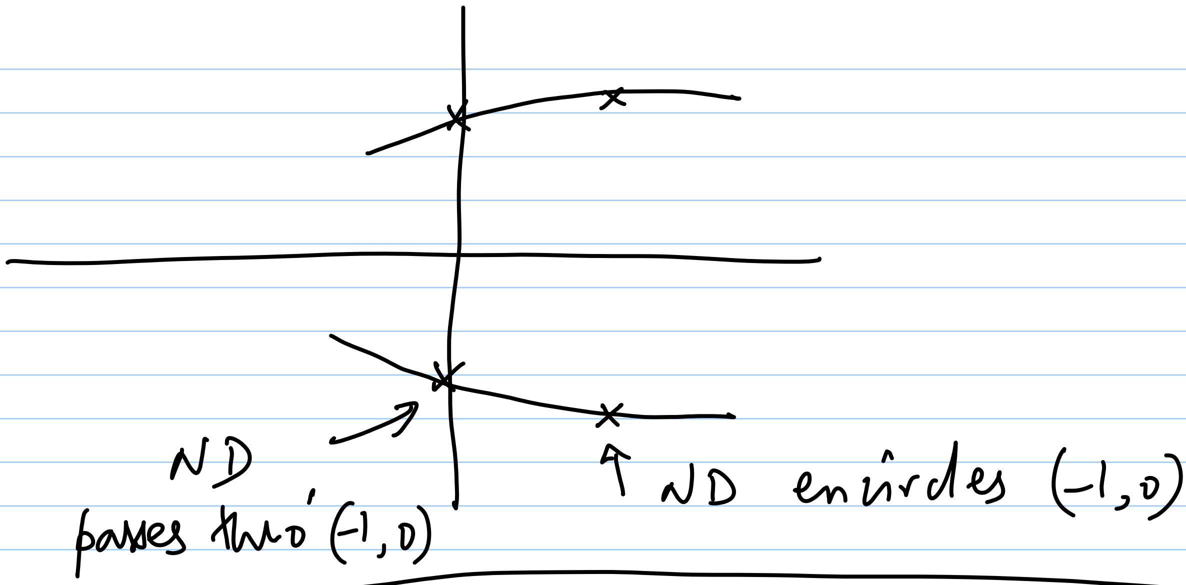
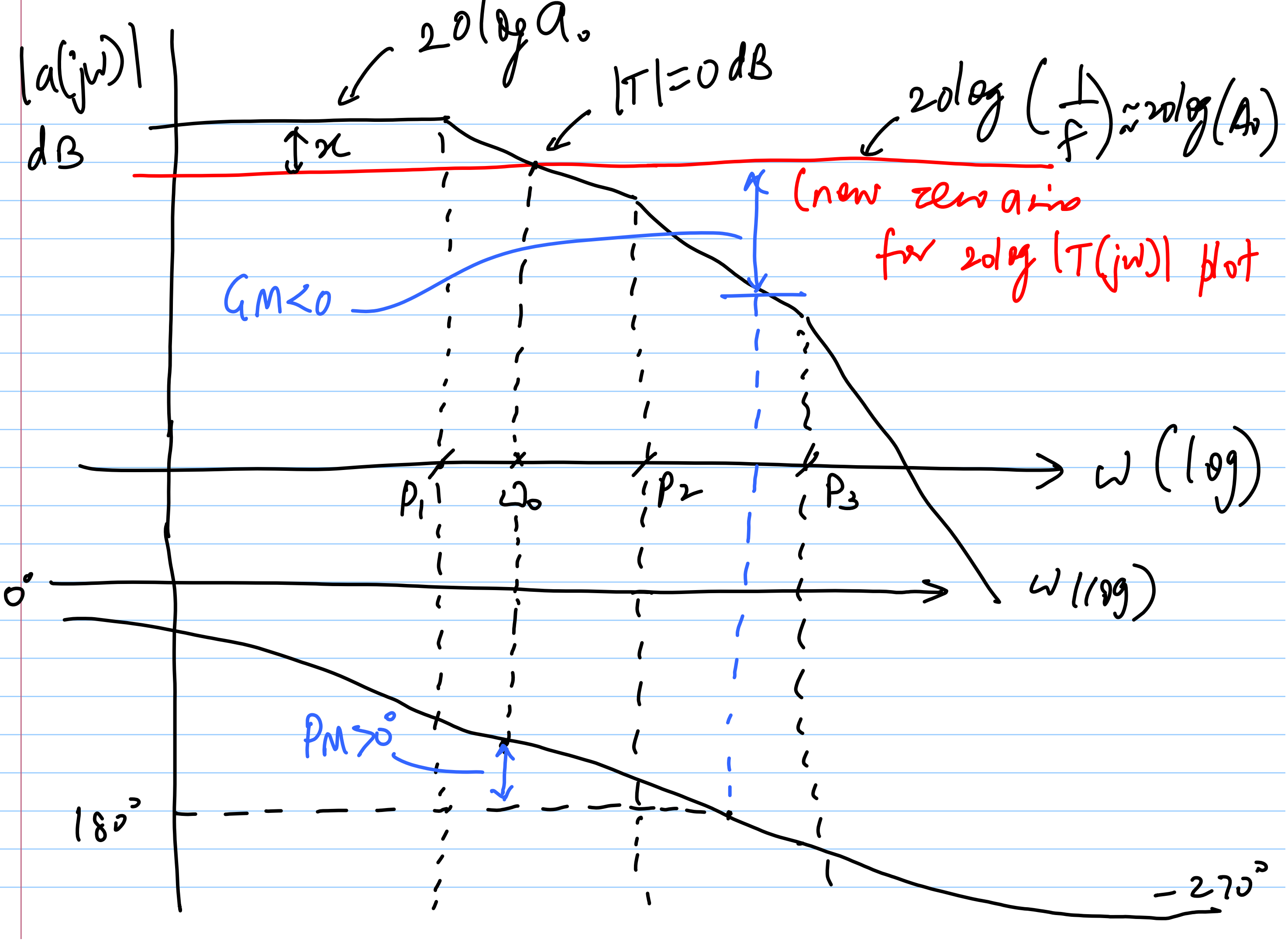
* Significance of $(-1,0)$
  → Say nyquist diagram passes thro' $(-1,0)$
  $\Rightarrow$ @ ω_{180}, $T(j\omega) = a(j\omega) \cdot f = -1$
  $\Rightarrow$ $A(j\omega) = \infty$ (unstable)
  (poles on $j\omega$ axis)
  → If $T_0 \uparrow$, nyquist diagram expands <u>linearly</u> to encircle $(-1,0)$ $\Rightarrow$ poles in RHP



$\Rightarrow$ $\boxed{\text{If } |T(j\omega)| > 1 \text{ @ } \angle T = -180^\circ, \text{ then amplifier is unstable}}$



$$A_0 \approx \frac{1}{f} \quad \text{if } T_0 = a_0 f \gg 1$$

$$x = 20\log|a(j\omega)| - 20\log(1/f)$$

$$= 20\log|T(j\omega)| \quad (\text{LG in dB})$$

when $|T(j\omega)| = 1$, $\angle T(j\omega) < 180^\circ$

$\Rightarrow$ this f.b. loop is stable

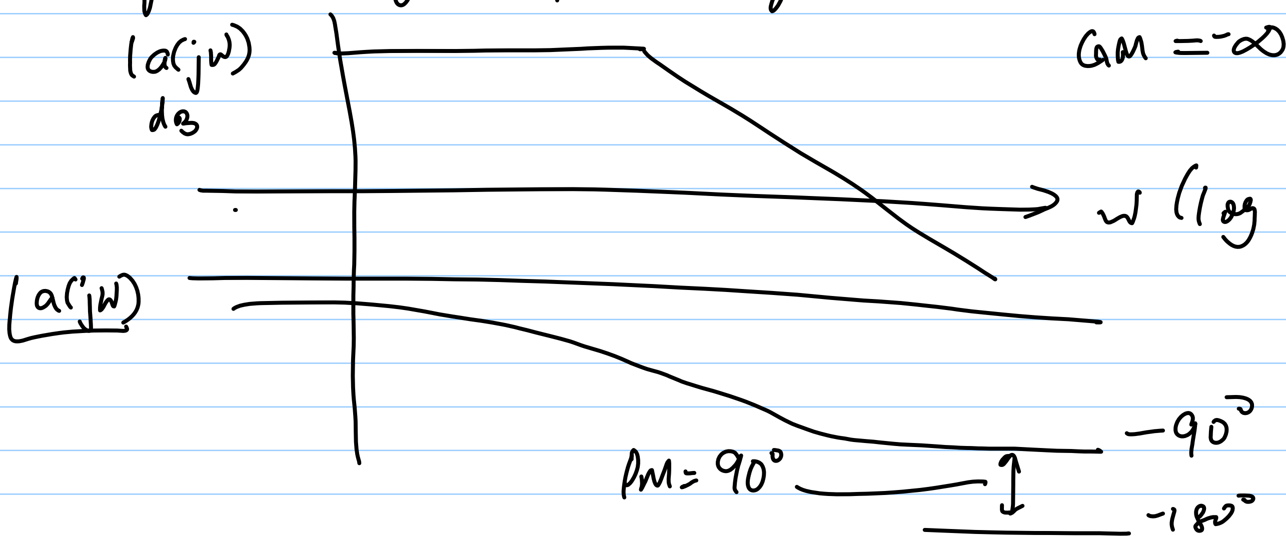
Phase Margin $\doteq 180^\circ + \angle T(j\omega)$ at ω where $|T(j\omega)| = 1$

PM $> 0^\circ$ for stability

Gain Margin $= |T(j\omega)|$ in dB at ω where $\angle T(j\omega) = 180^\circ$

GM < 0 dB for stability

e.g. single pole system



typical PM ~ 60° (lower limit of 45°)

e.g. $\angle T(j\omega_0) = -135°$ @ ω_0 $\Rightarrow$ 45° PM

$$|T(j\omega_0)| = 1 \Rightarrow |a(j\omega_0)| = \frac{1}{f}$$

$$A(j\omega) = \frac{a(j\omega)}{1+T(j\omega)}$$

$$A(j\omega_0) = \frac{a(j\omega_0)}{1+e^{-j135°}} = \frac{a(j\omega_0)}{0.3-0.7j}$$

$$|A(j\omega_0)| = \frac{|a(j\omega_0)|}{0.76} = \frac{1.3}{f}$$

ω_0 = −3dB point for 1-pole amp.

here, we get 1.3X ≡ 2.4dB of peaking above low freq. gain of $1/f$

PM = 60° $\Rightarrow$ $\angle T(j\omega_0) = -120°$

$|T(j\omega)| = 1$

we get $|A(j\omega_0)| = \frac{1}{f}$ ← no peaking @ ω_0

$PM = 90° \Rightarrow \angle T(j\omega_0) = -90°$

$|T(j\omega)| = 1$

we get $|A(j\omega_0)| = \frac{0.7}{f}$ ← 3dB below $\frac{1}{f}$