14-2-12

Lec-17

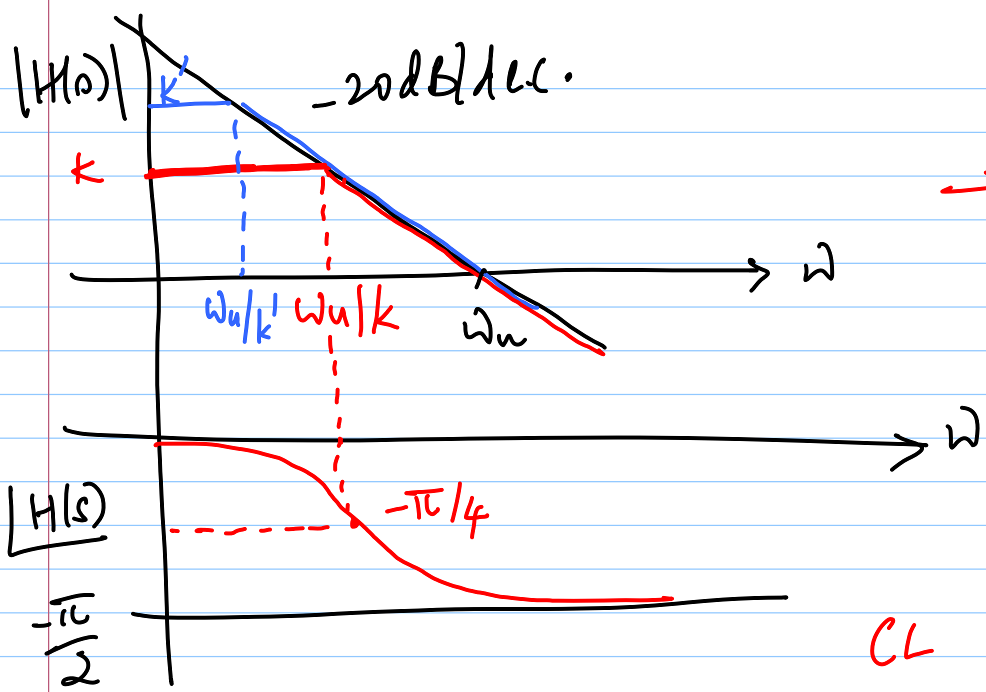
$V_i = V_i(t) \Rightarrow$ output settles close to $V_i(t)$ in a few τ's

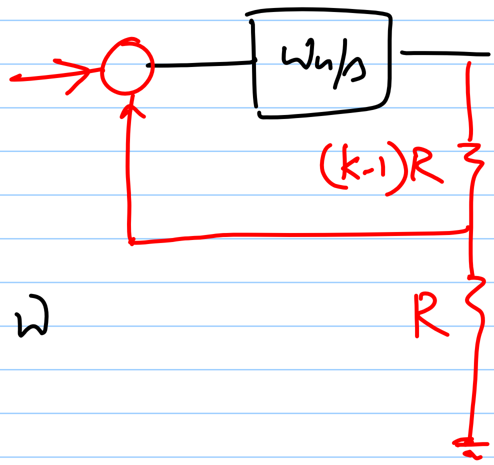
$\omega << \frac{\omega_u}{k}$ — approx. is valid

Last class: $BW = \frac{\omega_u}{k}$

Given amplification (k), how to increase BW? increase ω_u (faster integrator)

$\Rightarrow$ more current consumption





CL $H(s) = \dfrac{k}{1 + \frac{s}{(\omega_u/k)}}$

CL $|H(s)| = 1 \Rightarrow \dfrac{k}{\sqrt{1 + \frac{\omega^2}{(\omega_u/k)^2}}} = 1$

$$\Rightarrow \quad \omega = \frac{\omega_n}{k}\sqrt{k^2-1} = \omega_n\sqrt{1-(1/k^2)}$$

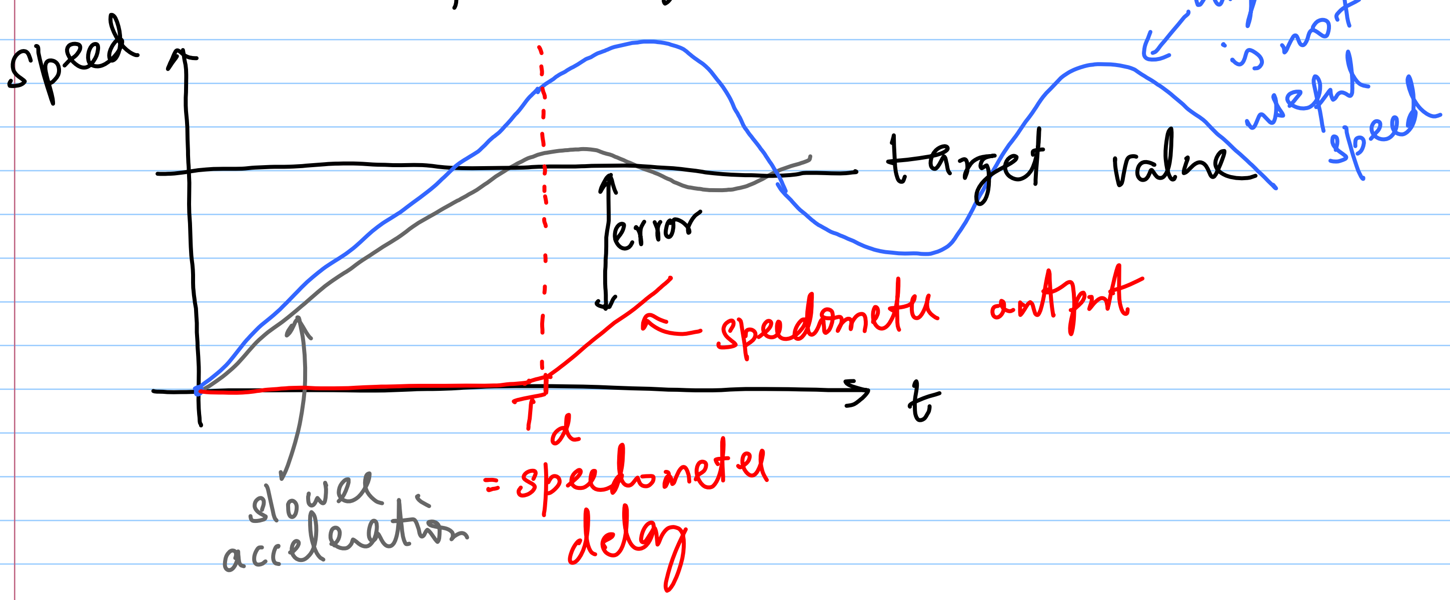
typical $\omega \sim \omega_n$ if k is large enough

$$\tau = \frac{k}{\omega_n}$$

$$BW = \frac{\omega_n}{k}$$

$$\text{pole} = \frac{-\omega_n}{k}$$

Effects of delay in f.b.



large T_d → oscillations may never stop ⇒ instability

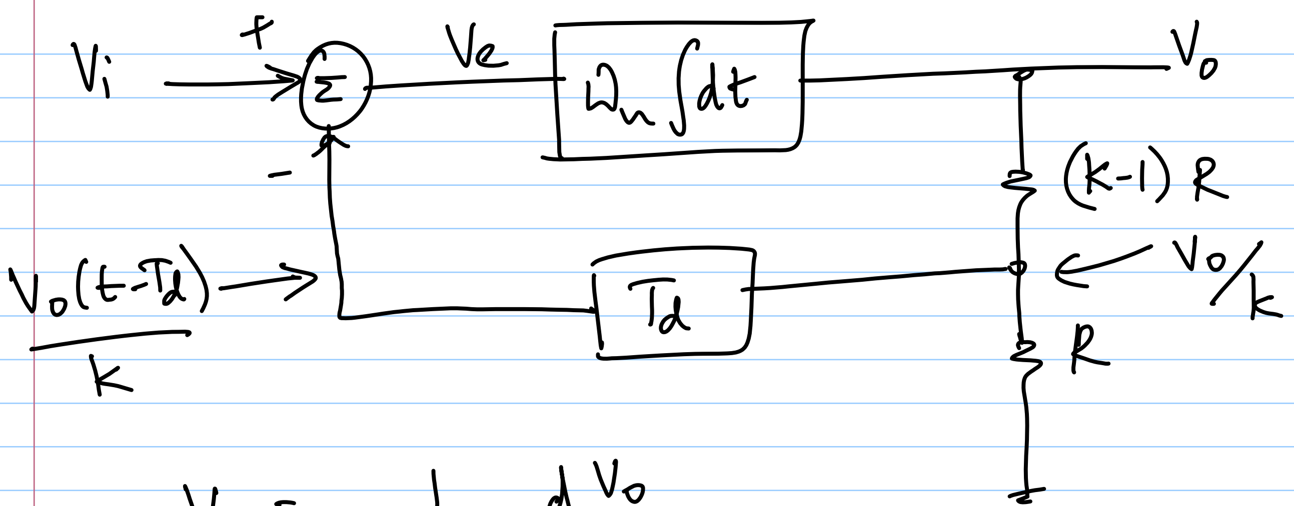
Simple way to fix issue: accelerate slowly

$\Rightarrow$ $\boxed{\text{less } \omega_n}$ for lower overshoot

For f.b. system w/ delay, reduce ω_n to keep it stable

$\rightarrow$ conflicts with BW

* typically T_d = depends on process
* T_d originates from charging of parasitic cap.



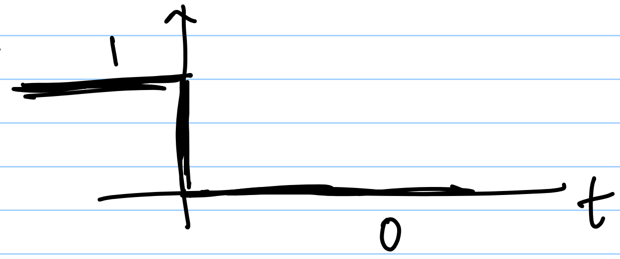
$$V_e = \frac{1}{\omega_n}\frac{dV_o}{dt}$$

New diff. eq^n:

$$\frac{1}{\omega_n}\frac{dV_o}{dt} = V_i(t) - \frac{V_o(t-T_d)}{k}$$

* No problem w/ steady state (as long as stability is ok)

e.g. $V_i(t) =$



i.e. $V_i(t) = 0$ for $t > 0$

$$\Rightarrow \quad \frac{1}{\omega_n}\frac{dV_o(t)}{dt} = -\frac{V_o(t-T_d)}{k}$$

What are the solutions?

Without T_d:

$$\frac{1}{\omega_n}\frac{dV_o}{dt} = -\frac{V_o}{k} \leftarrow \exp\left(-\frac{\omega_n}{k}t\right)$$

With T_d:

$$\frac{1}{\omega_n}\frac{dV_o(t)}{dt} = -\frac{V_o(t-T_d)}{k}$$

exp solution?

* delayed exp = scaled exp!
  ∴ exp. soln is possible
* sine or cosine can also be a solution

e.g. delayed cos = sin

deriv (cos) = sin

1) Solutions of the form $V_0 = e^{\sigma t}$

substitute :

$$\frac{1}{\omega_n} \cdot \sigma e^{\sigma t} = - \frac{e^{\sigma t} e^{-\sigma T_d}}{k}$$

$$\Rightarrow \quad \sigma + \frac{\omega_n}{k} e^{-\sigma T_d} = 0$$

Order of this system = ?

$$\sigma/(\omega_n/k) + e^{-\sigma/(\omega_n/k)\left(\frac{\omega_n}{k} \cdot T_d\right)} = 0$$

$$\sigma' = \frac{\sigma}{(\omega_n/k)} \; ; \; \tau = \frac{\omega_n}{k} \cdot T_d$$

$$\sigma' + e^{-\sigma' \tau} = 0$$

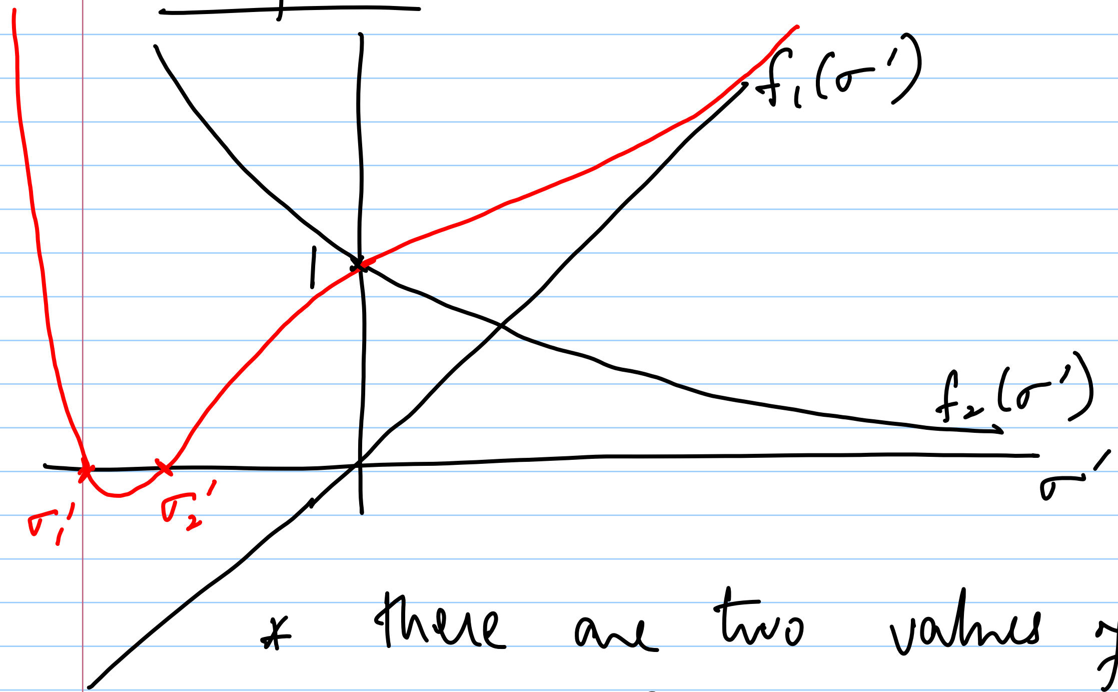
σ' & τ are dimensionless

$$f_1(\sigma') + f_2(\sigma') = 0$$

how to solve this nonlinear eqn?

$\tau = 0 \Rightarrow \sigma' = -1 \Rightarrow \sigma = -\omega_n/k$

Graphical:



* there are two values of σ' (σ_1' & σ_2')

$\Rightarrow$ σ_1 & σ_2

what is the general solution for V_0?

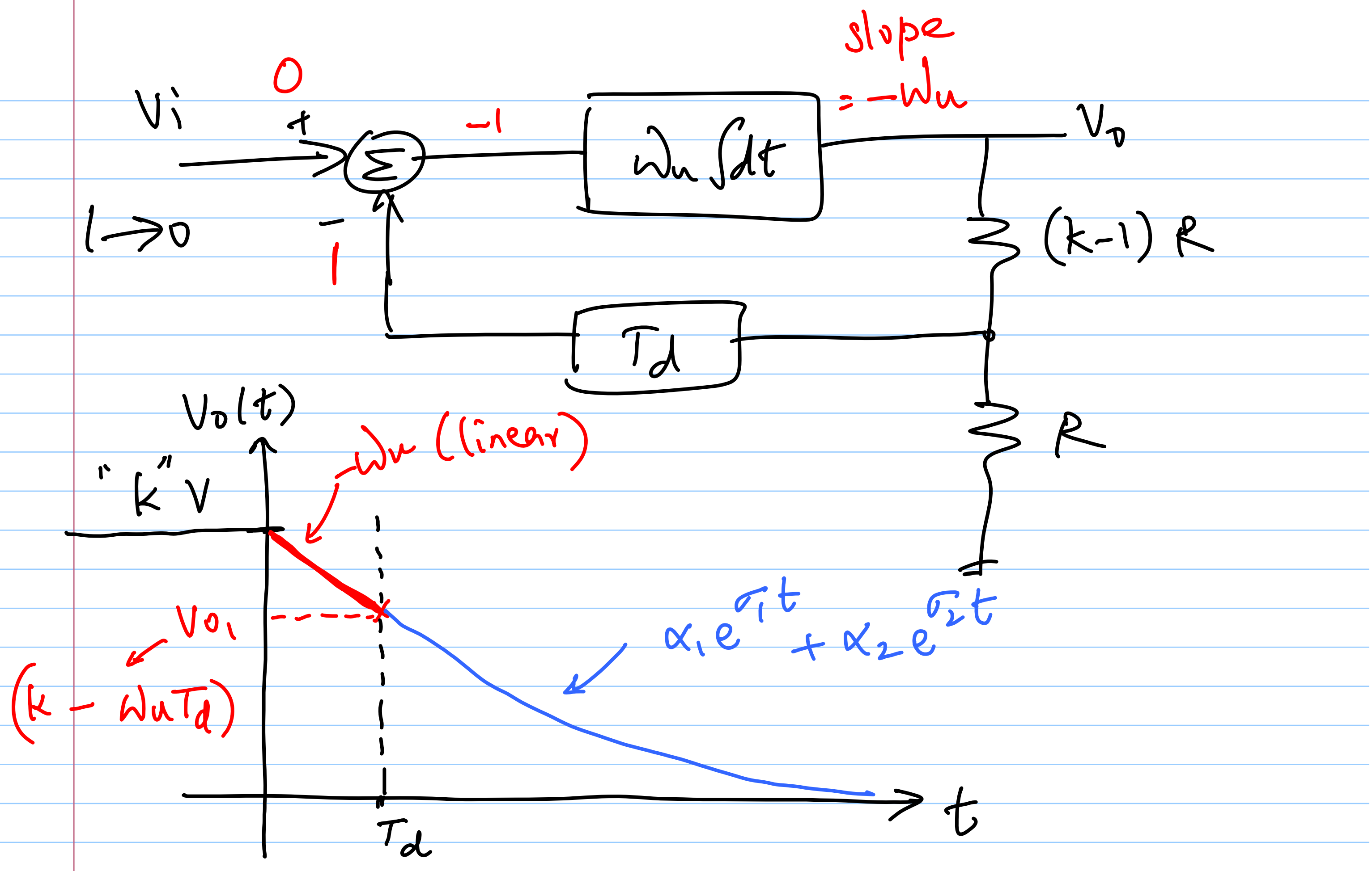
$$\therefore \quad V_0(t) = \alpha_1 e^{\sigma_1 t} + \alpha_2 e^{\sigma_2 t}$$

* σ_1' & σ_2' cannot be positive

$f_1(\sigma')$ & $f_2(\sigma')$ are both +ve

* How to find out α_1 & α_2?

$\Rightarrow$ boundary conditions



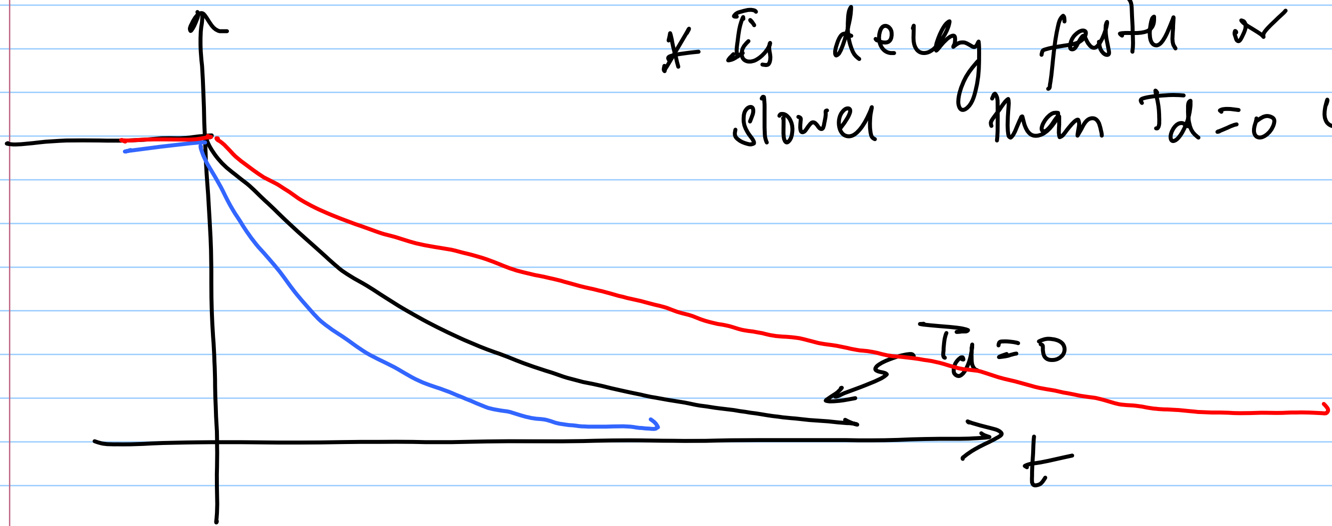
boundary conditions:

@ $t = T_d$

$$\begin{cases} V_o = k - \omega_u T_d = k(1 - \tau) \\ \left.\dfrac{dV_o}{dt}\right|_{t=T_d} = -\omega_n \end{cases} \Rightarrow \alpha_1 \ \& \ \alpha_2$$

each $T_d \Rightarrow \sigma_1 \ \& \ \sigma_2 \ \exists \ \alpha_1 \ \& \ \alpha_2$ (unique)

(more in HW 3)



* Is decay faster or slower than $T_d = 0$ case?

Once you find σ_1 & σ_2 can you tell?

for $T_d = 0$, $\sigma' = -1 \Rightarrow \sigma = -\frac{\omega_n}{k}$

e.g. if you get

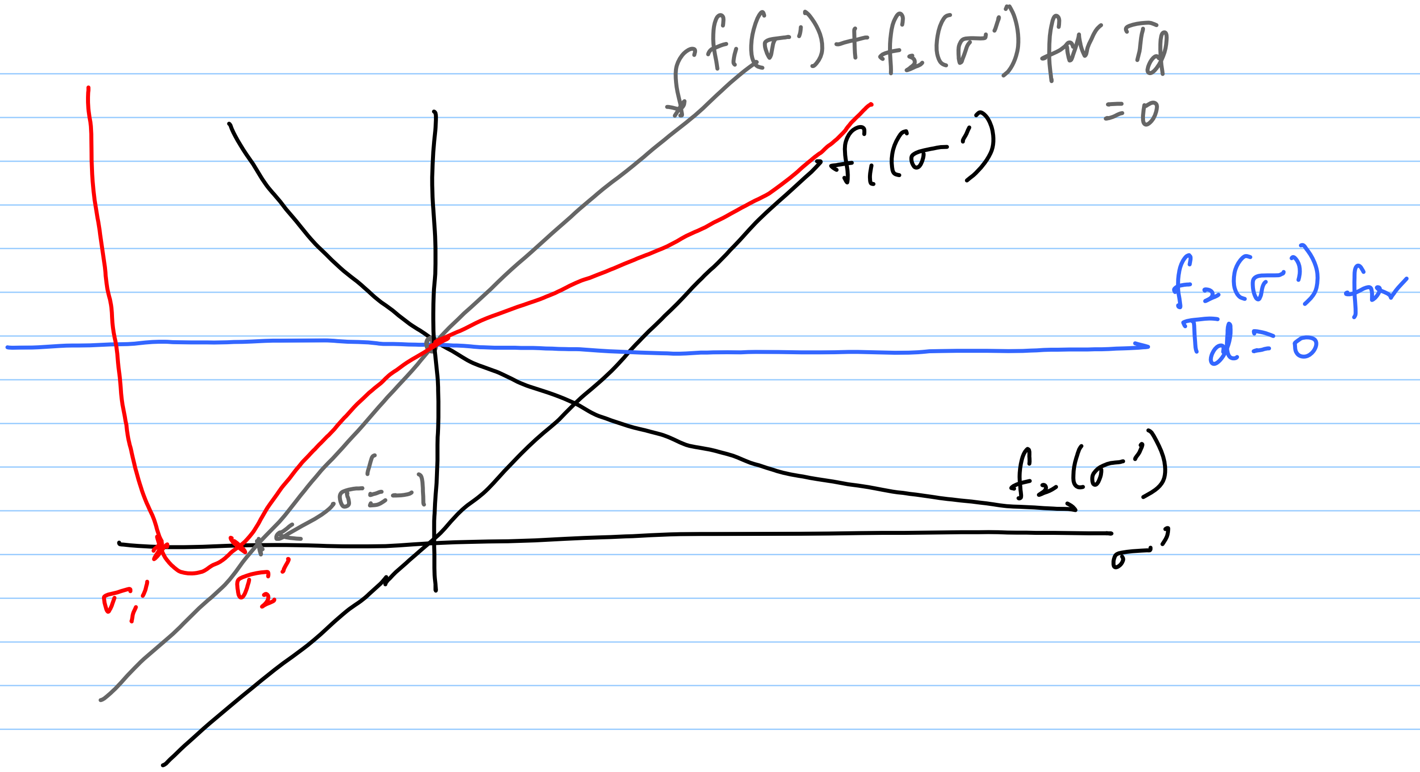
$\sigma_1' = -2$ & $\sigma_2' = -3$

$\Rightarrow$ faster

$\sigma_1' = 0.2$ & $\sigma_2' = -0.3$

$\Rightarrow$ slower

what σ's do you expect to get?



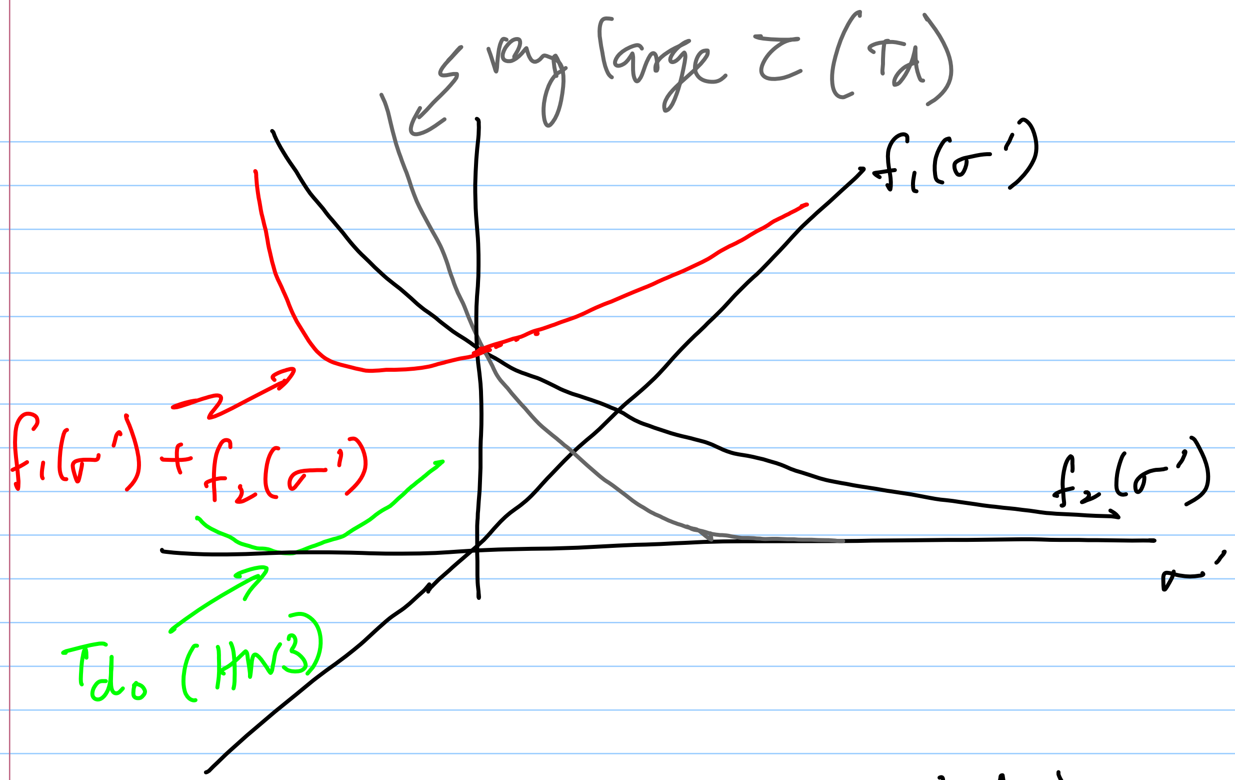
* can show that σ_1' & σ_2' for $T_d > 0$ will be < -1

$\Rightarrow$ exponential decay is faster than $T_d = 0$ case!

* little bit of T_d is good for faster response

* In this case (small T_d) no overshoot

→ similar to damping in a second-order system (critical damping)



large $\tau / T_d \Rightarrow$ no solution

$T_d > T_{d0} \Rightarrow$ no real solution