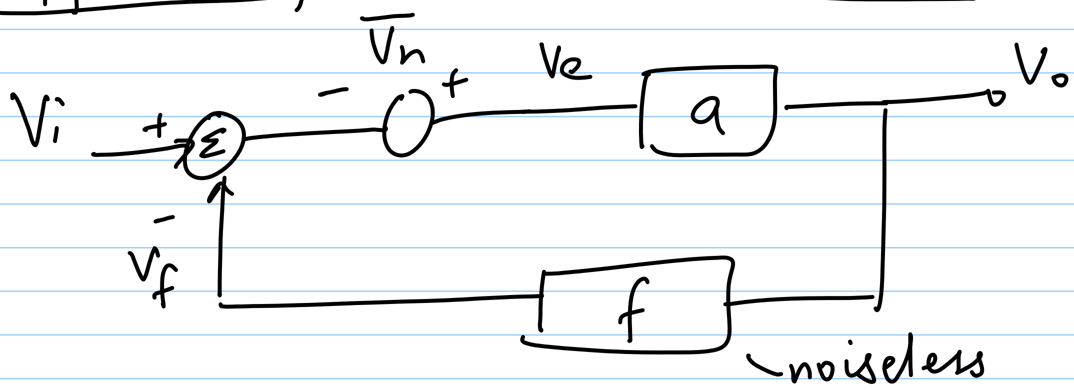
9-2-12

Lec 16

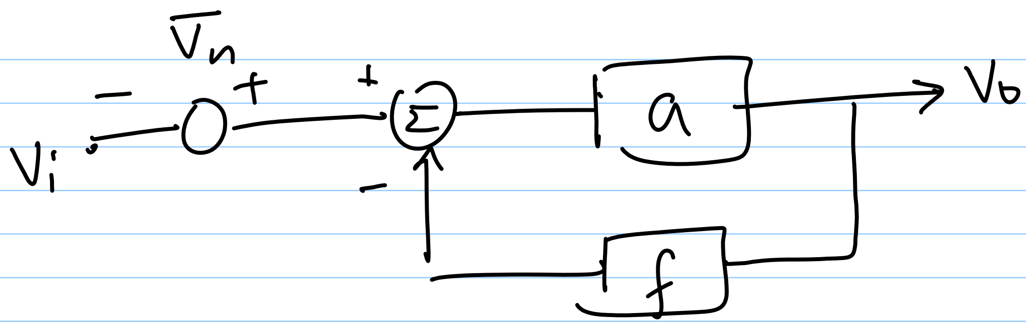
Note Title 08-02-2012

Effect of f.b. on noise



$$\left[(V_i - f V_o) + V_n\right] \cdot a = V_o$$

$$\Rightarrow V_o = (V_i + V_n) \cdot \frac{a}{1 + af}$$



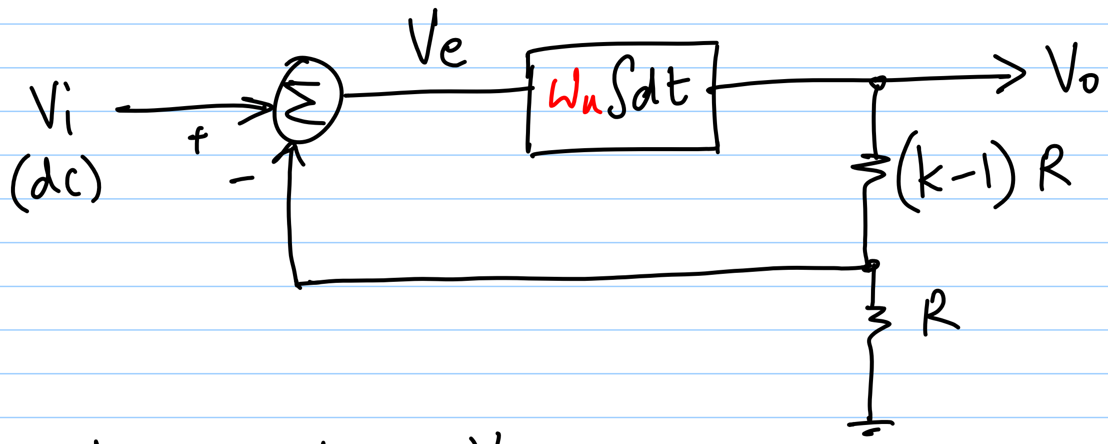
$\Rightarrow$ input-referred noise stays the same even w/ f.b.

* If f is noisy, overall noise gets worse

* If f.b. is not applied between input-output, $\overline{v_{i,n}^2}$ changes

Negative feedback theory

* from control system point of view

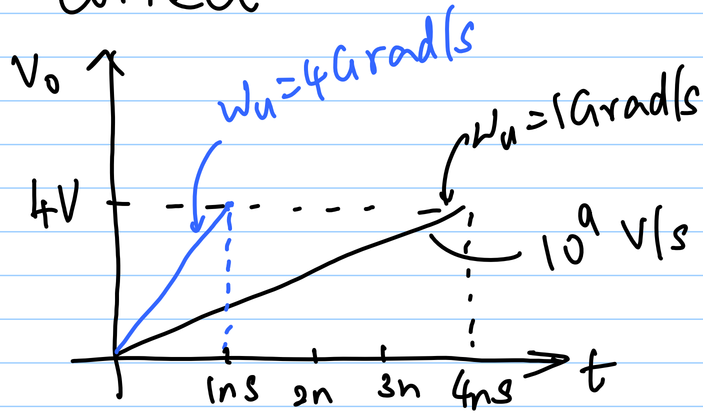


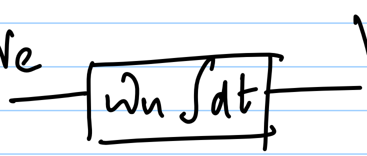
$$V_e = V_i - \frac{V_o}{k}$$

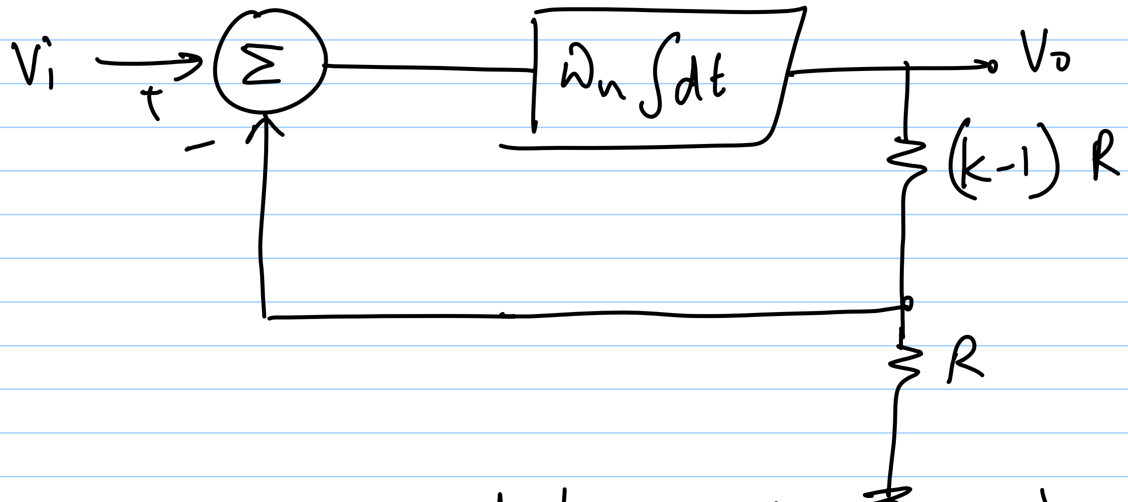
* Steady state : $V_o = k V_i$ or $V_e = 0$ for DC (otherwise integrator o/p ↑ or ↓)

* W_u ⇒ needed to keep dimensions correct

*







$V_i = 1V$, $V_o = kV$ (steady state)

if $V_o(0) = 0$, what does o/p look like?

write equations in differential form:

$$V_e = \frac{1}{\omega_n}\frac{dV_o}{dt}$$

also, $V_e = V_i - \dfrac{V_o}{k}$

$$V_i - \frac{V_o}{k} = \frac{1}{\omega_n}\frac{dV_o}{dt}$$

$$\frac{dV_o}{V_i - \frac{V_o}{k}} = \omega_n\,dt$$

easy case: $V_i = 0$

$$\frac{dV_o}{dt} = \frac{-\omega_u}{K} V_o$$

$$\Rightarrow \quad V_o = e^{-(\omega_u/k)t}$$

If $V_i \neq 0$

$$\frac{dV_o}{V_i - \frac{V_o}{K}} = \omega_u \, dt$$

$$-k \ln\left(V_i - \frac{V_o}{K}\right)\Bigg|_{V_o(0)}^{V_o(t)} = \omega_u t \Big|_0^t$$

$$V_o(t) = V_o(0) \exp\left(-\frac{\omega_u}{K} t\right) + k V_i \left[1 - \exp\left(-\frac{\omega_u}{K} t\right)\right]$$

* group into two terms

  → effect of $V_o(0)$

  → effect of V_i

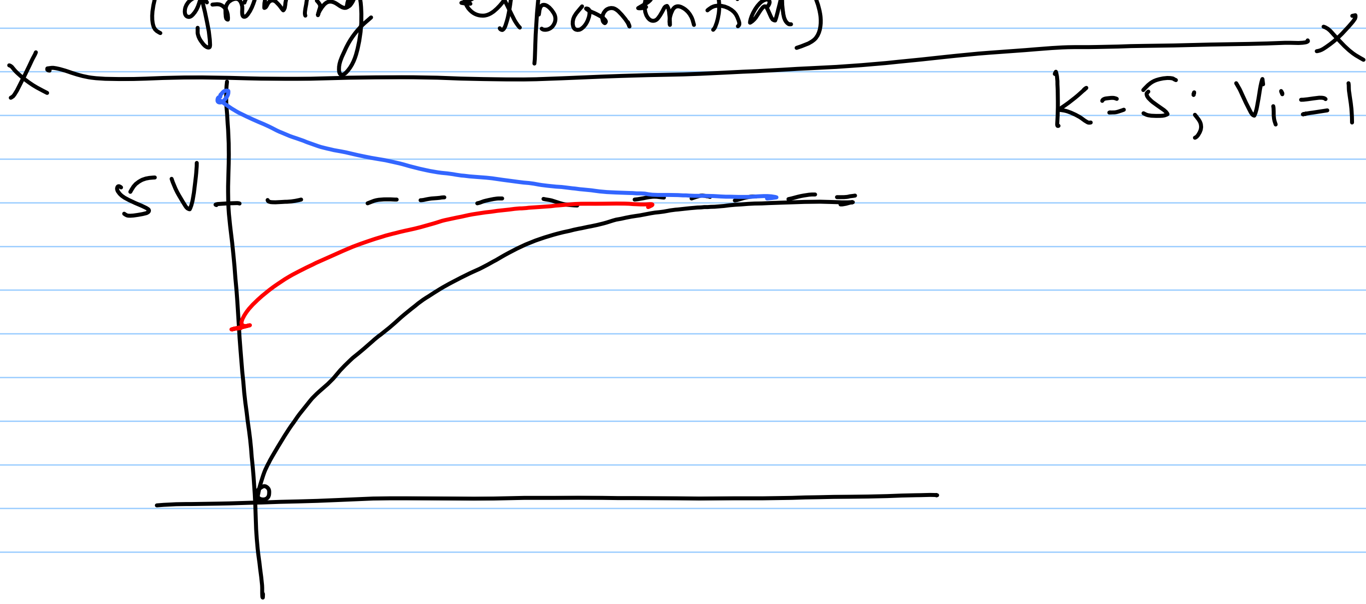
If equation was

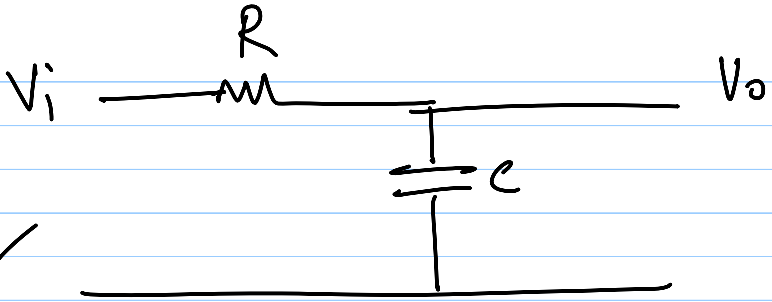
$$\frac{1}{\omega_u}\frac{dV_o}{dt} = V_i + \frac{V_o}{K} \quad \leftarrow \text{what happens?}$$

$V_o = -kV_i$ is a potential solution;

In a real case: voltage blows up with time

(growing exponential)





constant V_i

$$V_o = V_i\left[1 - \exp\left(-\frac{t}{RC}\right)\right] + V_o(0)\exp\left(\frac{-t}{RC}\right)$$

time constant $= RC$

F.B.: $V_o = kV_i\left[1 - \exp\left(-\frac{w_n}{k}t\right)\right] + V_o(0)\exp\left(-\frac{w_n}{k}t\right)$

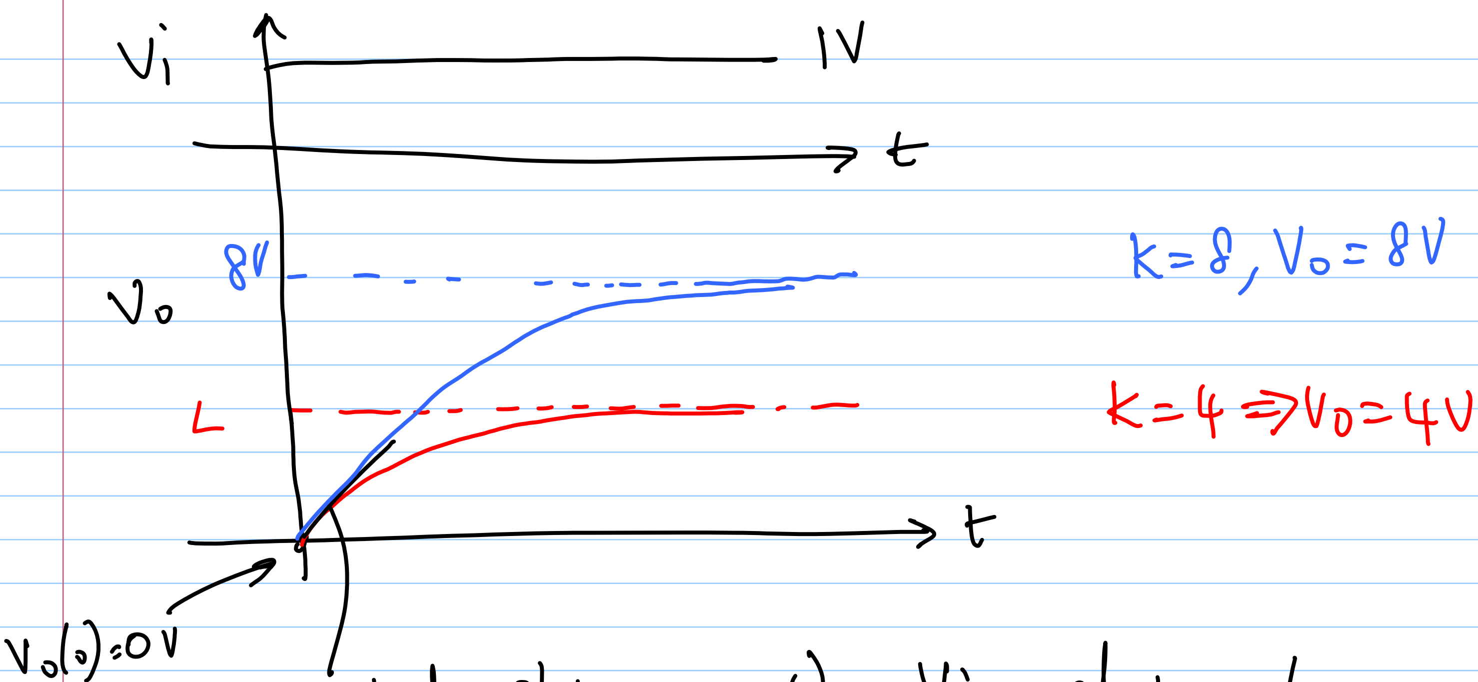
time constant $= \dfrac{k}{w_u}$

1) If $\omega_u \uparrow$, $\tau \downarrow$ $\Rightarrow$ why?

integrating faster, so get to steady state faster!

2) If $k \uparrow$, $\tau \uparrow$ $\Rightarrow$ why?

$V_o = k V_i$ $\leftarrow$



initial slope $= \omega_u \cdot V_i$ always!

why? at $t=0$ $V_e = V_i - 0 = V_i$

$\frac{1}{\omega_u} dV_o/dt \rightarrow \xrightarrow{V_e} \boxed{\omega_u \int dt} \rightarrow V_o$

initial slope $= \omega_n \cdot V_i$ always

final voltage $= k V_i$ (↑ with k)

$\therefore$ τ ↑ with k

$$\boxed{\tau = \frac{k}{\omega_n}}$$

problem — @ higher freq. ω_n is not easy

τ cannot be increased arbitrarily

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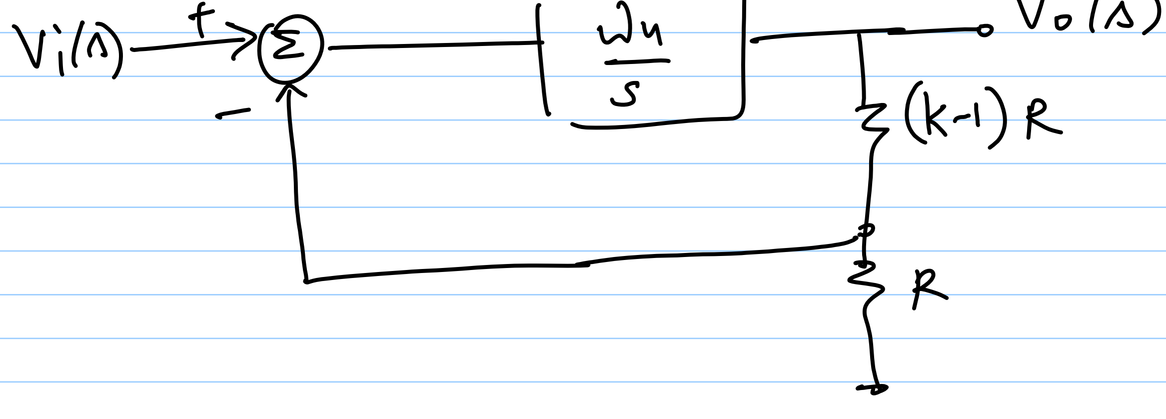
* V_i = constant

→ $V_o = k V_i$

→ initial condition not a problem

→ $\tau = k/\omega_n$

Laplace domain



* Laplace domain

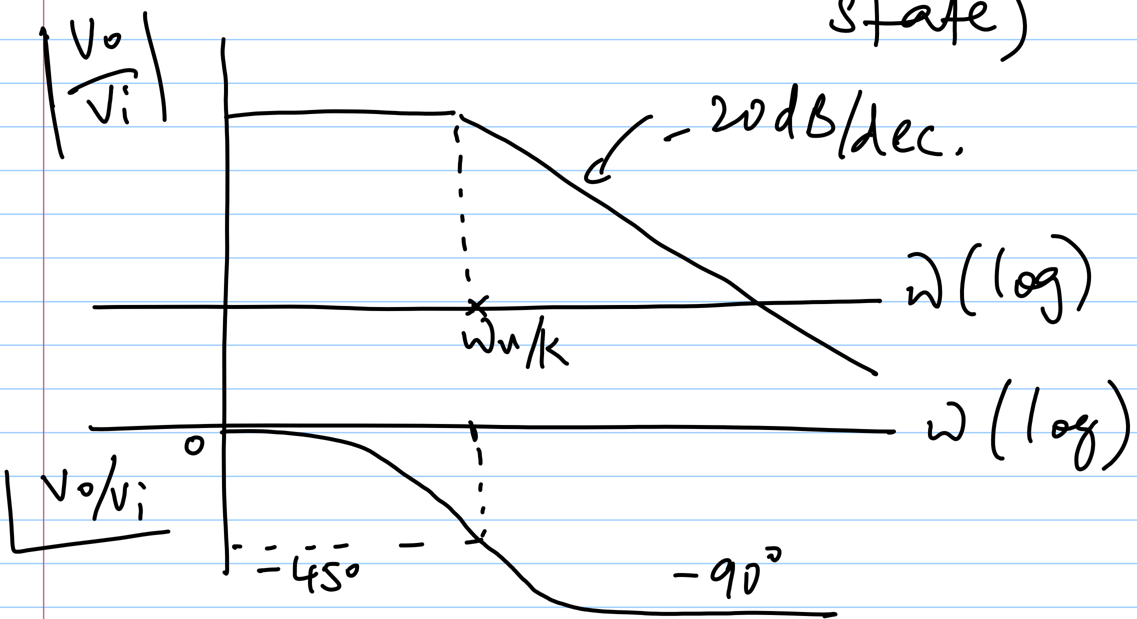
differential eqn $\Rightarrow$ algebraic eqn!

$$V_o(s) = V_i(s) \frac{\omega_u}{s + \frac{\omega_u}{k}}$$

$$\frac{V_o(s)}{V_i(s)} = k \frac{1}{1 + s/(\omega_u/k)}$$

constant V_i $\Rightarrow$ $s = 0$ (DC) $\Rightarrow$ $\frac{V_o}{V_i} = k$ ✓

Freq. response (Sinusoidal steady state)

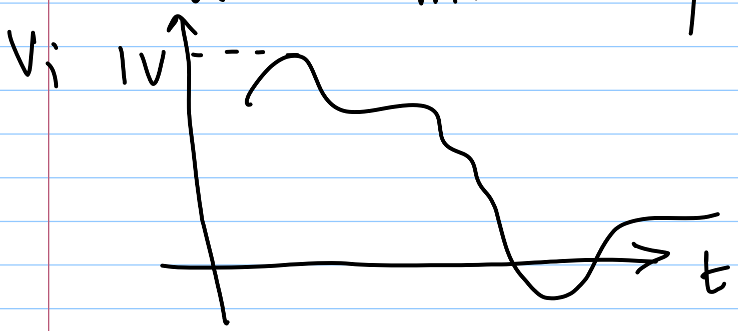


$V_i = A \cos \omega t$

$$\left|\frac{V_o}{V_i}\right| = \frac{k}{\sqrt{1+\frac{k^2\omega^2}{\omega_n^2}}}$$

$$\angle\frac{V_o}{V_i} = -\tan^{-1}\left(\frac{k\omega}{\omega_n}\right)$$

We want amp with gain 'k'



till what freq. do you get exact same shape?

* each freq component has a different gain & phase based on $\left|\frac{V_o}{V_i}\right|$ & $\angle V_o/V_i$

"Linear phase" system

phase $\propto$ freq.

$\Rightarrow$ every sine component has same delay

* phase is linear only for small ω (note that plot has log axis)

$$v_i = A\cos\omega t$$

$$v_o = \frac{kA}{\left(1 + \frac{k^2\omega^2}{\omega_n^2}\right)^{1/2}} \cdot \cos\left[\omega t - \tan^{-1}\left(\frac{\omega k}{\omega_n}\right)\right]$$

$$\omega \ll \frac{\omega_n}{k} \Rightarrow \quad v_o \approx kA \cdot \cos\left[\omega\left(t - \frac{k}{\omega_n}\right)\right]$$

If $\omega \gg \frac{\omega_n}{k}$

$$v_o \simeq \frac{A\,\omega_n}{\omega} \cdot \cos\left(\omega t - \pi/2\right)$$

looks like an integrator

$$\longrightarrow \boxed{\omega_n/s} \longrightarrow$$

f.b. is not working!

$\omega << \dfrac{\omega_u}{k} \Rightarrow$ amplification $= k$

delay $= \dfrac{k}{\omega_u}$

$\omega >> \dfrac{\omega_u}{k} \Rightarrow$ gain $\propto 1/\omega$

90° phase wrt input

approx. BW $\boxed{BW = \dfrac{\omega_u}{k}}$

how to increase BW? (given ampl. value)

$\Rightarrow \omega_u$ should increase (faster integrator)