7-2-12

Lec - 14

Optimum gain per stage

* Given total gain $A_{tot.}$, we want to find optimal n & maximize BW

$$A_0^n = A_{tot.} \Rightarrow A_0 = A_{tot.}^{1/n}$$

$$\omega_{on} = \frac{\omega_u}{A_{tot.}^{1/n}} \sqrt{2^{1/n} - 1}$$

apply $\frac{d\omega_{on}}{dn} = 0$

after some algebra:

$$n_{opt.} = \frac{\ln 2}{\ln\left\{1 + \frac{\ln 2}{2 \ln A_{tot.}}\right\}}$$

for large $A_{tot.}$,

$$\ln\left\{1 + \frac{\ln 2}{2 \ln A_{tot.}}\right\} \approx \frac{\ln 2}{2 \ln A_{tot.}}$$

$$\ln(1+x) \approx x \quad \text{for} \quad x << 1$$

$$\Rightarrow \boxed{n_{opt.} \approx 2 \ln A_{tot.}}$$

optimum gain/stage:

$$A_{o_{opt.}} = (A_{tot.})^{1/n_{opt}} = \exp\left\{\frac{1}{n_{opt.}}\ln A_{tot.}\right\}$$

$$\approx e^{1/2}$$

$$\boxed{A_{o,opt.} = \sqrt{e}}$$

optimum BW:

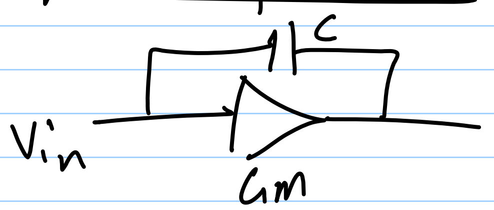
$$\omega_{on,opt.} = \frac{\omega_u}{A_{tot.}^{1/n_{opt.}}}\sqrt{2^{1/n_{opt.}} - 1}$$

$$\approx \frac{\omega_u}{\sqrt{e}}\left[\exp\left\{\frac{1}{n_{opt.}}\ln 2\right\} - 1\right]^{1/2}$$

$$\boxed{\omega_{on,opt.} \approx \omega_u\sqrt{\frac{\ln 2}{2e\ln A_{tot.}}}}$$

Summary of freq. response

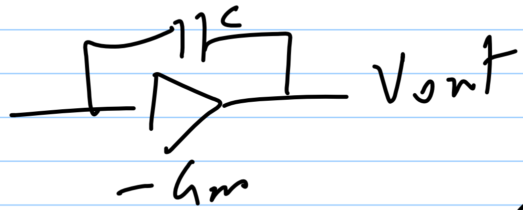
LHP zero:



$-\frac{G_m}{C}$

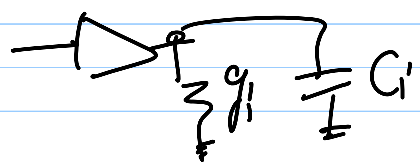
not the only source of zeroes

RHP zero:



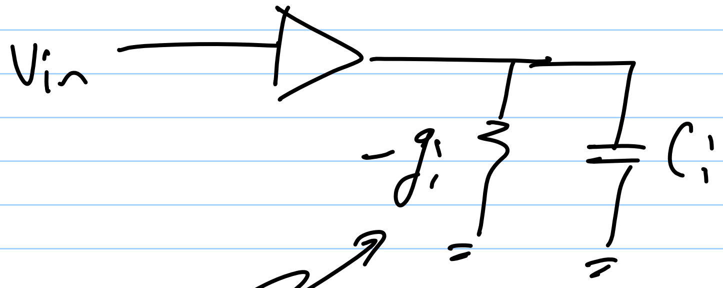
$\frac{G_m}{C}$

LHP pole:



$\frac{-g_i}{C_i}$

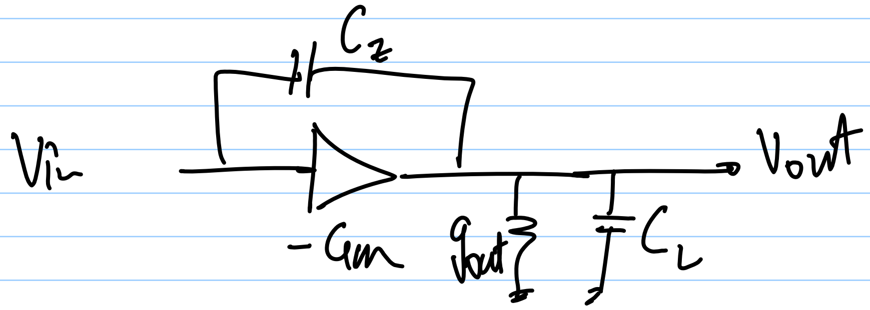
RHP pole



$$\frac{+g_i}{C_i}$$

-ve g/R (typically from +ve f.b. or instability)

zeros



$$V_{out} = i_{out} Z_{out}$$

$$= \left\{ -G_m \cdot V_{in} + (V_{in} - V_{out}) \cdot s C_z \right\} R_{out}$$

$$Y_{out} V_{out} = (-G_m + s C_z) V_{in} - s C_z V_{out}$$

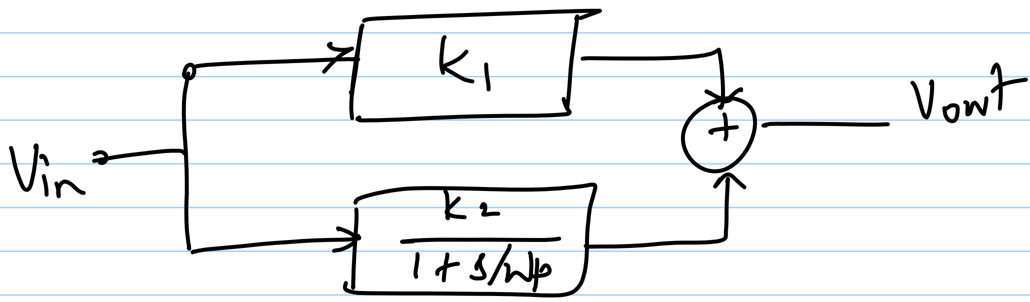
$$V_{out} \left[g_{out} + s(C_z + C_L) \right] = (-G_m + s C_z) V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{-(G_m - s C_z)}{g_{out} + s(C_z + C_L)}$$

$$= \frac{-G_m}{g_{out}} \left[\frac{1 - sC_z/G_m}{1 + \frac{s(C_z+C_L)}{g_{out}}} \right]$$

* presence of zero is not always obvious

1)



$$V_{out}(s) = k_1 V_{in}(s) + \frac{k_2}{1+s/\omega_p} V_{in}(s)$$

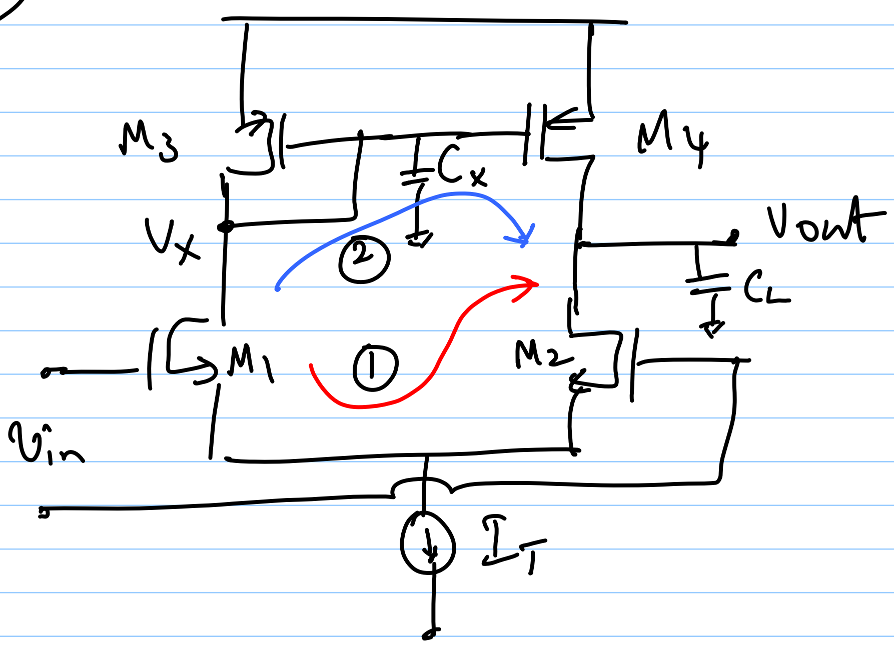
$$\frac{V_{out}}{V_{in}}(s) = k_1 + \frac{k_2}{1+s/\omega_p}$$

$$= \frac{(k_1+k_2) + k_1 s/\omega_p}{1 + s/\omega_p}$$

$$= (k_1+k_2) \cdot \frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

$$\omega_z = \omega_p \cdot \frac{(k_1+k_2)}{k_1}$$

2)



Read Razavi Section 6.6

$$\omega_{p_1} \approx -\frac{g_{ds_1} + g_{ds_3}}{C_L}$$

$$\omega_{p_2} \approx -\frac{g_{m_3}}{C_X}$$

1 : $\dfrac{A_0}{1 + s/\omega_{p_1}}$

2 : $\dfrac{A_0}{(1 + s/\omega_{p_1})(1 + s/\omega_{p_2})}$

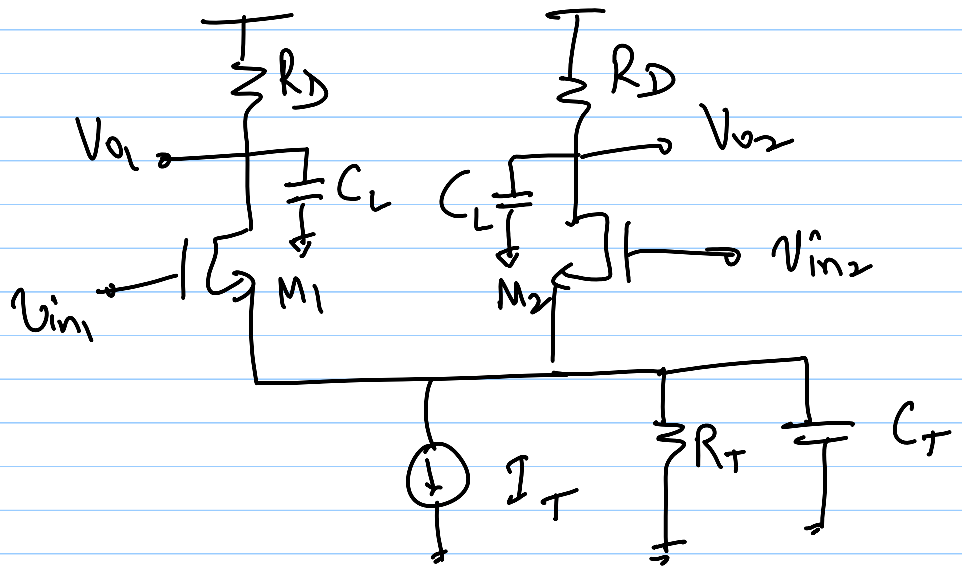
$\Rightarrow \quad A_{tot.}(s) = A_{1} + A_{2}$

$$= \frac{A_0}{(1+s/\omega_{p1})}\left[1+\frac{1}{1+\frac{s}{\omega_{p2}}}\right]$$

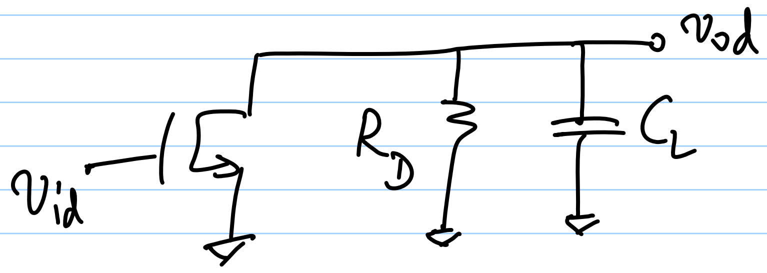
$$= \frac{A_0\,(2+s/\omega_{p2})}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$\omega_z = 2\omega_{p2}$$

Diff pairs



DM half ckt



(neglect device caps for simplicity)

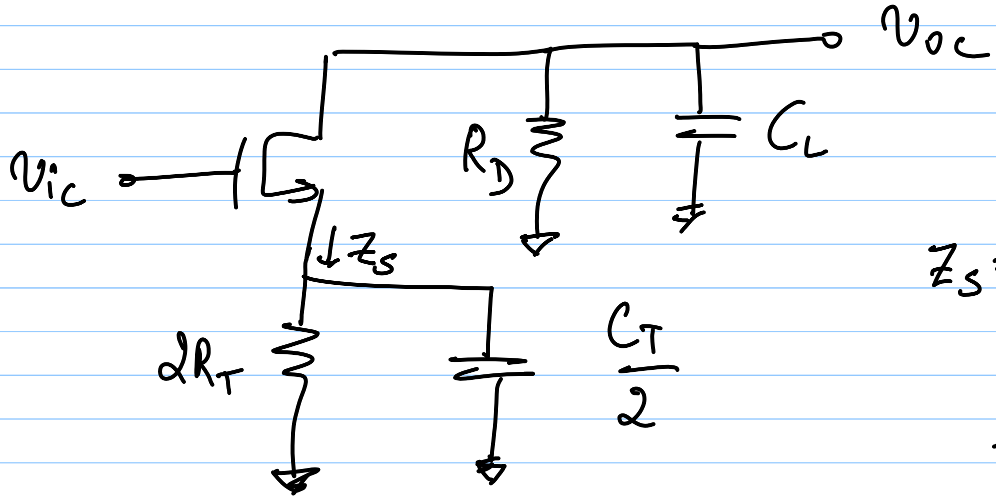
$$g_{out} = g_D + g_{ds}$$

$$a_{dm}(s) = \frac{-g_m}{g_{out}} \cdot \frac{1}{1 + \frac{sC_L}{g_{out}}}$$

in reality C_{gd} causes a zero

($|\omega_z| \gg |\omega_p|$ here)

CM half ckt



$$Z_s = \frac{2R_T/(sC_T/2)}{2R_T + \frac{2}{sC_T}} = \frac{2R_T}{1 + sC_T R_T}$$

$$a_{CM}(s) = -\underbrace{\left(\frac{g_m}{1 + g_m Z_s}\right)}_{G_{mc}} \cdot \frac{1}{g_{out}} \cdot \frac{1}{1 + \frac{sC_L}{g_{out}}}$$

ω_{p1}

$$G_{m_c} = \frac{g_m}{1 + g_m Z_s}$$

$$= \frac{g_m}{1 + g_m \cdot \dfrac{(2R_T)}{(1 + sC_T R_T)}}$$

w_z

$$= \frac{g_m \left(1 + sC_T R_T\right)}{\left(1 + 2g_m R_T\right) + sC_T R_T}$$

w_{p_2}

$$a_{cm}(s) = \left(\frac{g_m}{1 + 2g_m R_T}\right) \cdot \left(\frac{1}{g_D + g_{ds}}\right)$$

$$\cdot \frac{\left(1 + s/w_z\right)}{\left(+ s/w_{p_1}\right)\left(1 + \dfrac{s}{w_{p_2}}\right)}$$

$$w_z = -\frac{1}{R_T C_T} \qquad \text{LHP zero}$$

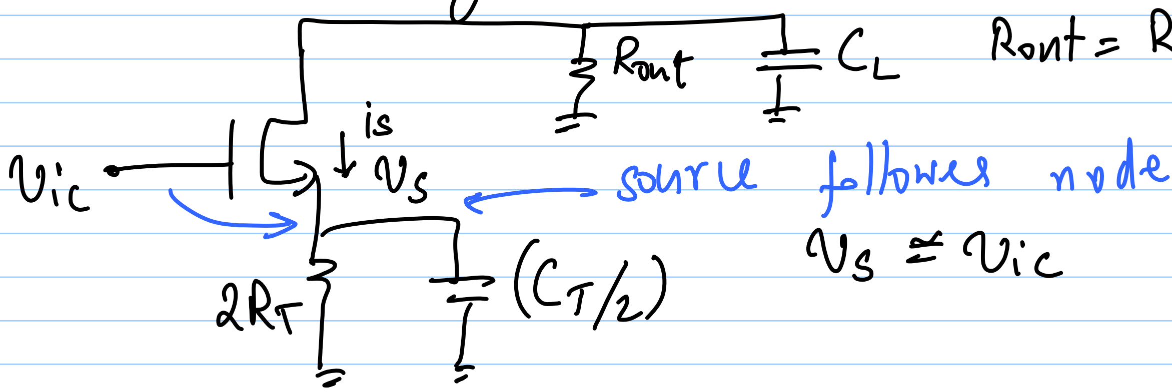
$$w_{p_1} = -\frac{\left(g_{ds} + g_D\right)}{C_L} \qquad \text{LHP pole}$$

$$\omega_{p_2} = -\frac{1+2g_mR_T}{R_TC_T} \qquad \text{LHP pole}$$

$$\simeq -\frac{g_m}{(C_T/2)} \qquad \text{if} \quad 2g_mR_T \gg 1$$

(cap in signal path)

Another way to look at it: (approx.)



$R_{out} = R_D \| r_{ds}$

$v_s \simeq v_{ic}$

$$\Rightarrow i_s = v_s \cdot Y_s$$

$$= v_{ic} \cdot \left[\frac{1}{2R_T} + s\frac{C_T}{2}\right]$$

$$v_{oc} = -i_s \cdot Z_{out} = -Z_{out} \cdot \left(\frac{1}{2R_T} + \frac{sC_T}{2}\right) v_{ic}$$

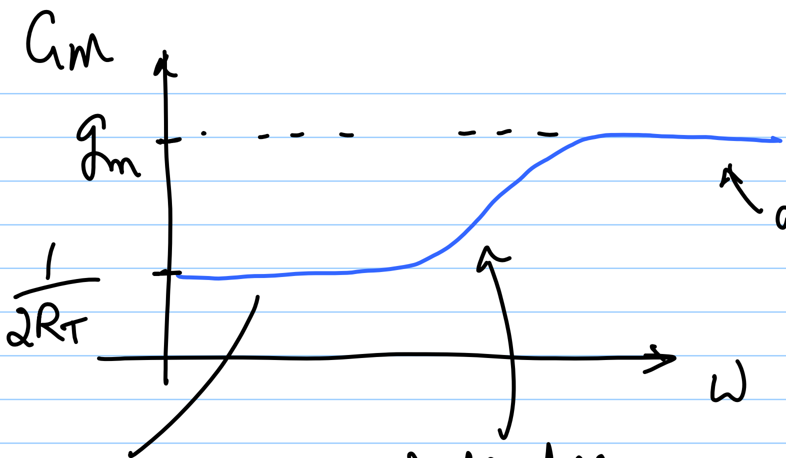
$$a_{cm}(s) = \frac{v_{o1}}{v_{ic}}$$

$$= -\left(\frac{R_{out}}{1 + sC_L R_{out}}\right)\left(\frac{1}{2R_T} + \frac{sC_T}{2}\right)$$

$$= \left(\frac{-R_{out}}{2R_T}\right)\left(\frac{1}{1 + sC_L R_{out}}\right) \cdot \left(1 + sC_T R_T\right)$$

ω_{p1} ω_z

$v_s = v_{ic}$ → caused us to neglect ω_{p2}



at high freq.,
C_T is a short
$\Rightarrow G_m = g_m$

indicates presence of zero

assuming
$2g_m R_T >> 1$
$\Rightarrow g_m >> \frac{1}{2R_T}$

1) increase in gain ⇒ zero

2) @ ω_z, $V_{out} = 0$ ⇒ use this to find out ω_z

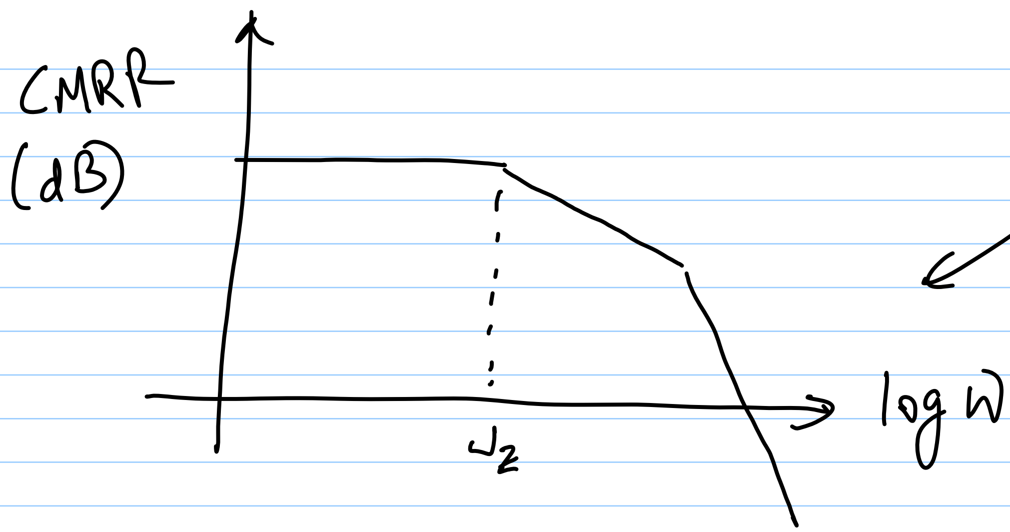
CMRR = Common-mode Rejection ratio

$$\equiv 20 \log \left(\frac{|a_{dm}|}{|a_{cm}|} \right) \text{ in dB}$$

@ low freq.:

$$\text{CMRR} = 20 \log \frac{(g_m / g_{out})}{(R_{out} / 2R_T)}$$

$$= 20 \log (2 g_m R_T)$$



rapid loss of CMRR @ high freq. due to a_{CM} zero