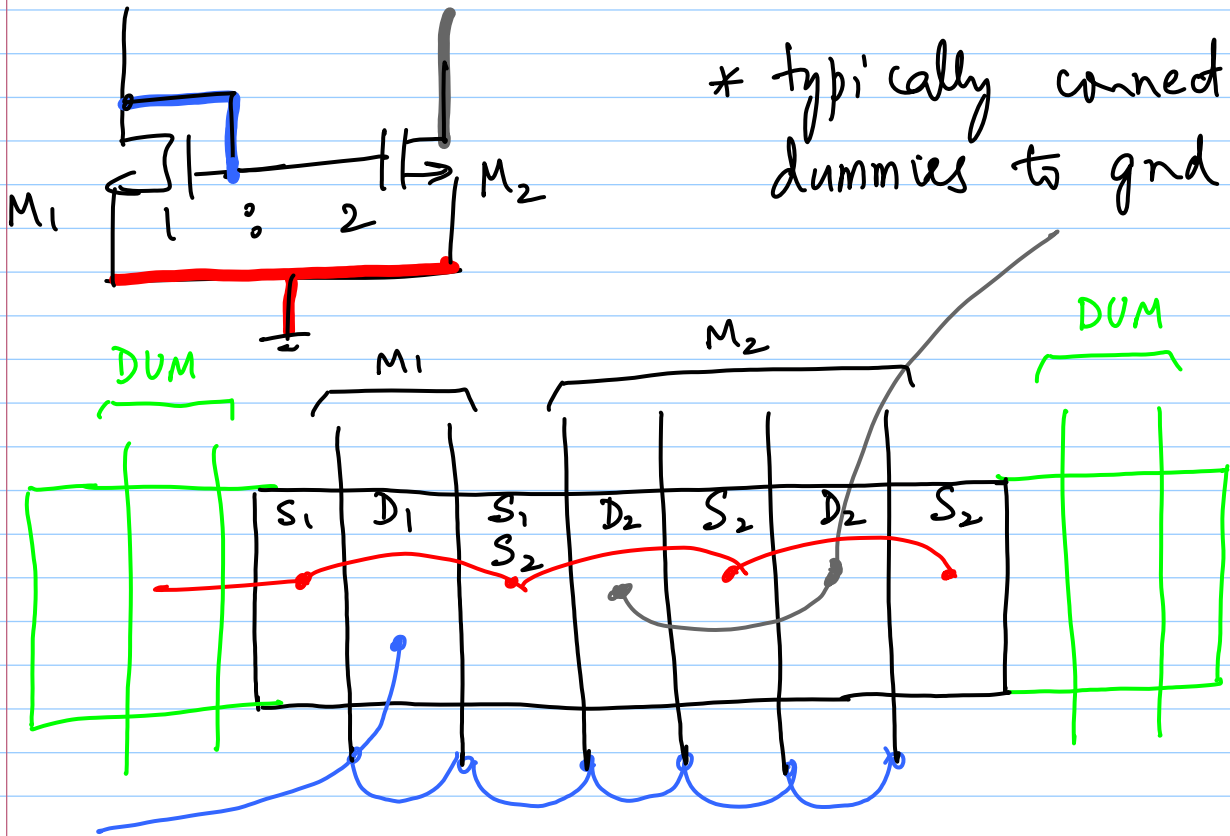


27-1-12

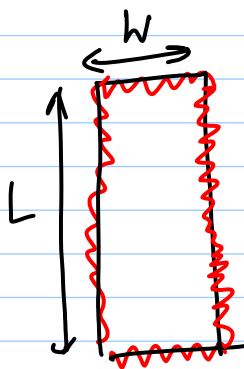
Lec 11



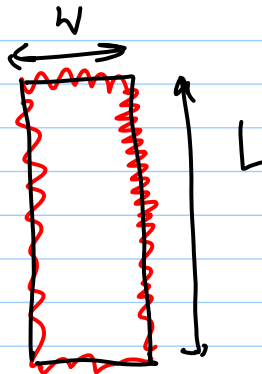
Random Mismatch

$$R_{sh}: 100 \Omega / \square \pm 30\%$$

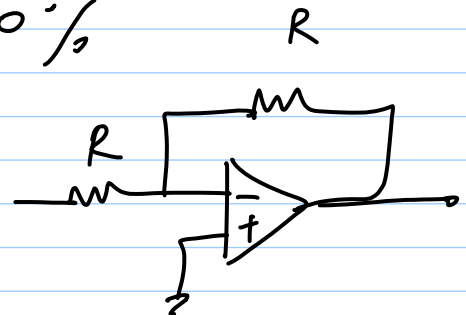
$$C: 1 \text{ fF} / \mu\text{m}^2 \pm 20\%$$



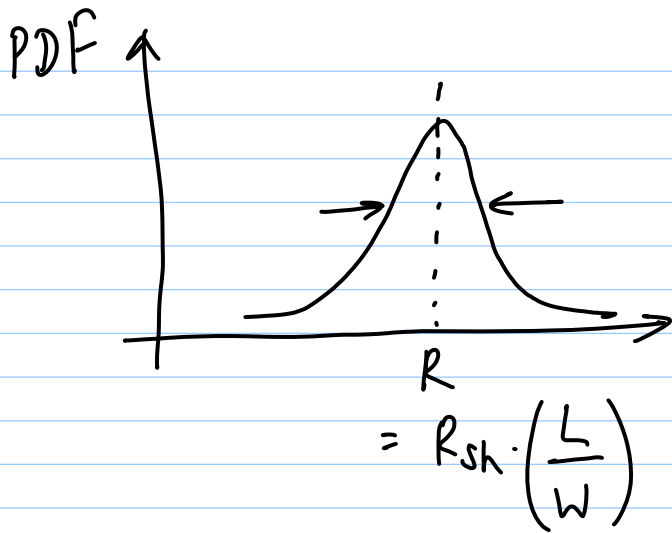
R



$R + \Delta R$



identical structures have different values



$$(\mu \pm \sigma) \Rightarrow 68\%$$

$$(\mu \pm 2\sigma) \Rightarrow 95\%$$

$$(\mu \pm 3\sigma) \Rightarrow 99.7\%$$

$$I_D = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L} \right) (V_{hs} - V_T)^2$$

components: μ, C_{ox}, W, L, V_T

Observations:

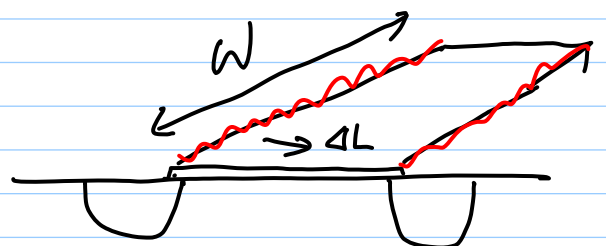
* as $W, L \uparrow \Rightarrow \frac{\Delta W}{W} \& \frac{\Delta L}{L} \downarrow$

$\Rightarrow$ larger device exhibit smaller mismatches

* as $(W \cdot L) \uparrow, \frac{\Delta W}{W} \& \frac{\Delta L}{L} \downarrow$

e.g. as $W \uparrow, \frac{\Delta L}{L} \downarrow$ due to

"averaging" effect



* Averaging: view devices as parallel combination of many smaller ones

$$W = W_0 \cdot n \quad \left\{ n \text{ parallel devices} \right\}$$

$$L_{eq.} = \frac{1}{n} \sum_{i=1}^n L_i$$

Overall variation:

$$\Delta L_{eq.} = \frac{1}{n} \sqrt{\sum_{i=1}^n \Delta L_i^2}$$

$$\approx \frac{1}{n} \sqrt{n \Delta L_0^2} = \frac{\Delta L_0}{\sqrt{n}}$$

$\Rightarrow$ i.e. if L_0 is statistical variation of length for width W_0 , increasing n decreases overall

$$\Delta L_{eq.}$$

* Similarly, μ , C_{ox} & V_T also show decreased mismatches when $(WL) \uparrow$.

$$\sigma_{V_T} = \frac{A_{V_T}}{\sqrt{WL}}$$

e.g. $A_{V_T} = 5 \text{ mV} \cdot \mu\text{m}$

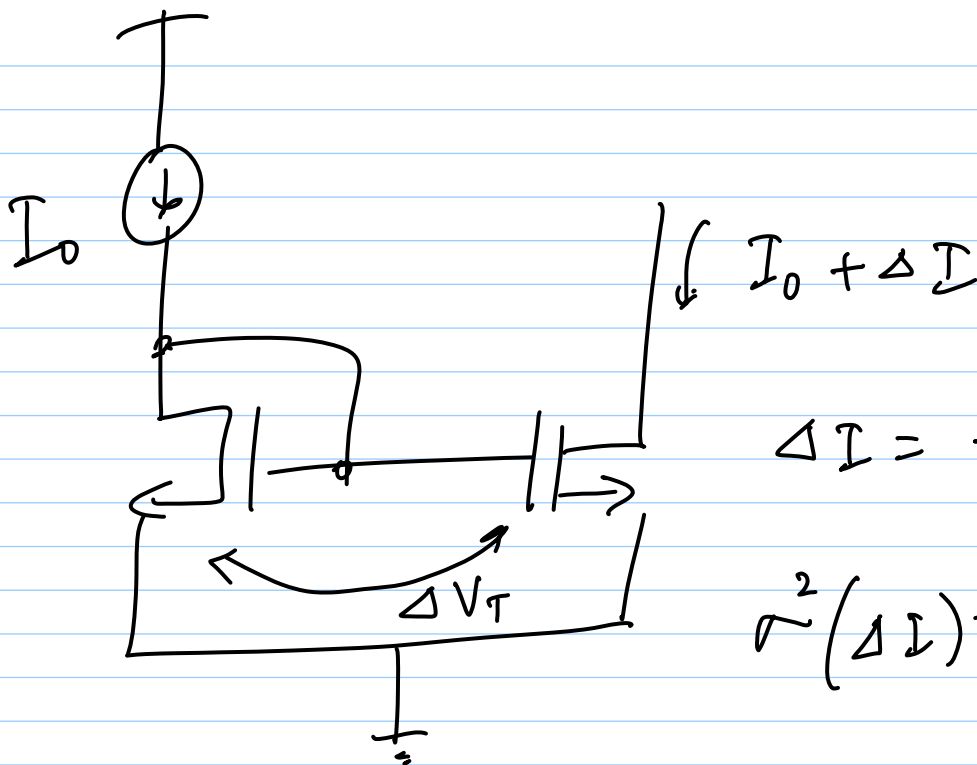
$$\sigma_{\beta} = \frac{A_{\beta}}{\sqrt{WL}}$$

$$\beta = \mu C_{ox}$$

111ly, $\sigma\left(\frac{\Delta R}{R}\right) = \frac{A_R}{\sqrt{WL}}$ averaging effects in R, C also

$\sigma\left(\frac{\Delta C}{C}\right) = \frac{A_C}{\sqrt{WL}}$

$A_R, A_C \rightarrow \% \cdot \mu\text{m} \text{ (units)}$



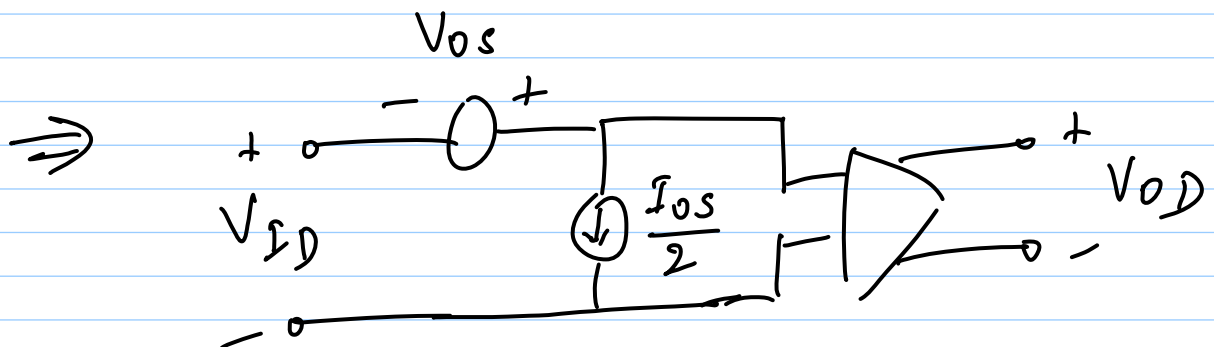
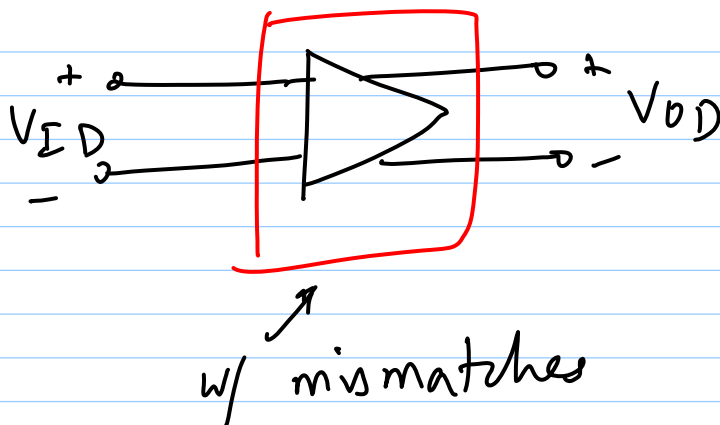
$$\sigma^2\left(\frac{\Delta I}{I_0}\right) = \frac{g_m^2}{I_0^2} \sigma_{V_T}^2$$

$$= \frac{4 \sigma_{V_T}^2}{(V_{GS} - V_T)^2}$$

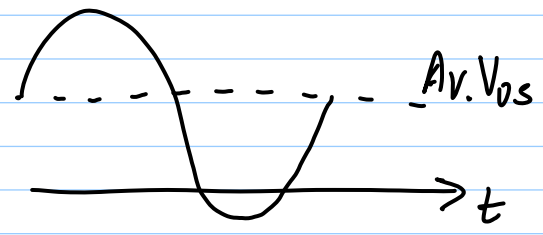
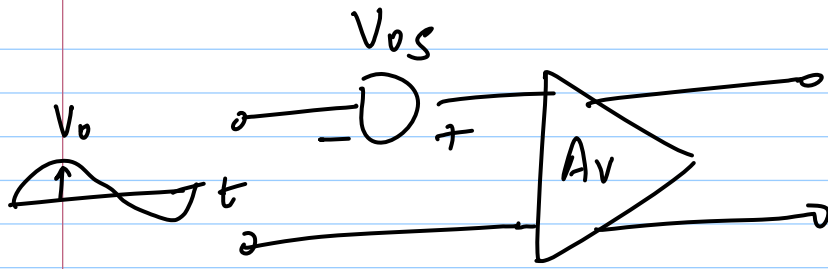
$$= \frac{2 \mu C_{ox}}{I_0} \left(\frac{A_{V_T}}{L^2} \right)$$

Effects of mismatch:

- * DC offsets
- * Even-order distortion { ideally gets cancelled, but due to mismatches ... }



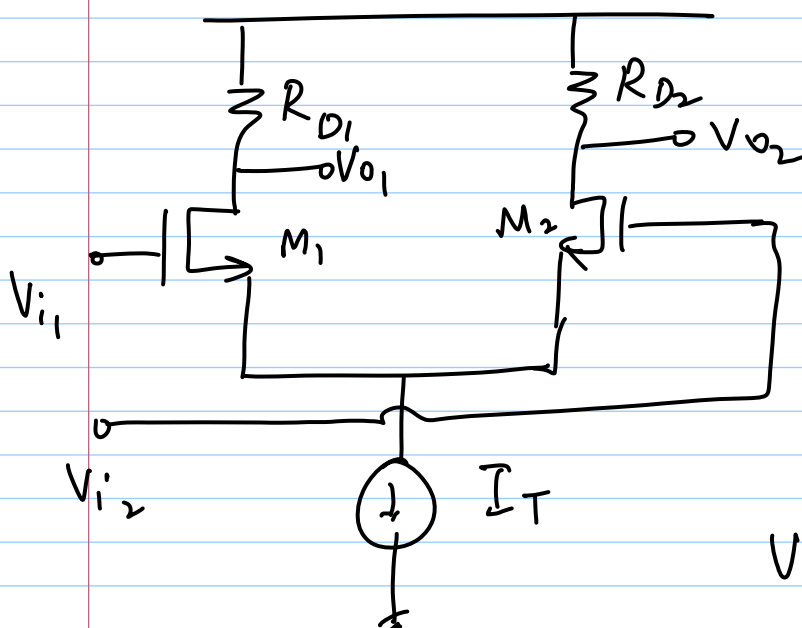
(valid for any source Res.)



non-linearities

- * Can saturate cascade of gain stages
- * DC offsets can be time-varying
e.g. with temperature
- * DC offsets can limit minimum V_{in} that can be detected

MOS Diff-pair



DC quantities

$$V_{ID} = V_{I1} - V_{I2}$$

$M_1 - M_2$ } not
 $R_{D1} - R_{D2}$ } identical

$$V_{ID} = V_{as1} + V_{as2}$$

$I_{OS} = 0$ (All gate currents = 0)

$$V_{ID} = V_{T1} + \sqrt{\frac{2I_{D1}}{\beta(\frac{W}{L})_1}} - V_{T2} - \sqrt{\frac{2I_{D2}}{\beta(\frac{W}{L})_2}}$$

Input offset voltage

$V_{OS} \equiv$ voltage to be applied to input to drive V_{OD} to 0

$$V_{OD} = 0 \Rightarrow I_{D1}R_{D1} = I_{D2}R_{D2} \text{ (constraint)}$$

* for small mismatches, an approx analysis can be done

$$\Delta I_D = I_{D1} - I_{D2}$$

$$I_D = \frac{I_{D1} + I_{D2}}{2}$$

$$\Delta(W/L) = (W/L)_1 - (W/L)_2$$

$$(W/L) = \frac{(W/L)_1 + (W/L)_2}{2}$$

$$\Delta V_T = V_{T1} - V_{T2}$$

$$V_T = (V_{T1} + V_{T2})/2$$

$$\Delta R_D = R_{D1} - R_{D2}$$

$$R_D = (R_{D1} + R_{D2}) / 2$$

You can write

$$I_{D1} = I_D + \frac{\Delta I_D}{2} ; I_{D2} = I_D - \frac{\Delta I_D}{2}$$

$$(W/L)_1 = (W/L) + \frac{\Delta(W/L)}{2}$$

$$(W/L)_2 = (W/L) - \frac{\Delta(W/L)}{2}$$

$\therefore$

$$V_{os} = \Delta V_T + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\beta \left[(W/L) + \frac{1}{2} \Delta(W/L) \right]}}$$

$$- \sqrt{\frac{2(I_D - \Delta I_D/2)}{\beta \left[(W/L) - \frac{1}{2} \Delta(W/L) \right]}}$$

$$= \Delta V_T + (V_{as} - V_T) \left[\sqrt{\frac{1 + \Delta I_D / (2 I_D)}{1 + \frac{\Delta(W/L)}{2(W/L)}}} \right]$$

$$- \sqrt{\frac{1 - \frac{\Delta I_D}{2 I_D}}{1 - \frac{\Delta (W/L)}{2 (W/L)}}}$$

Simplifies to:

$$V_{os} = \Delta V_T + \frac{(V_{as} - V_T)}{2} \left(\frac{\Delta I_D}{I_D} - \frac{\Delta (W/L)}{W/L} \right)$$

$$\text{also } I_{D1} R_{D1} = I_{D2} R_{D2}$$

$$\Rightarrow I_D R_D = (I_D + \Delta I_D) (R_D + \Delta R_D)$$

$$\Rightarrow I_D R_D \approx I_D R_D + R_D \Delta I_D + I_D \Delta R_D$$

$$\Rightarrow \frac{\Delta I_D}{I_D} = - \frac{\Delta R_D}{R_D}$$

$$\therefore V_{os} = \Delta V_T + \frac{(V_{as} - V_T)}{2} \left(- \frac{\Delta R_D}{R_D} - \frac{\Delta (W/L)}{W/L} \right)$$

* minimise $(V_{as} - V_T)$ to reduce V_{os}

* ΔV_T is directly referred to output

* MOS $V_{os} >$ BJT V_{os}

Since mismatches are independent:

$$\sigma_{V_{OS}}^2 = \left(\frac{V_{AS} - V_T}{2} \right)^2 \cdot \left[\sigma_{\frac{\Delta R_D}{R_D}}^2 + \sigma_{\left(\frac{\Delta W/L}{W/L} \right)}^2 \right] + \sigma_{V_T}^2$$