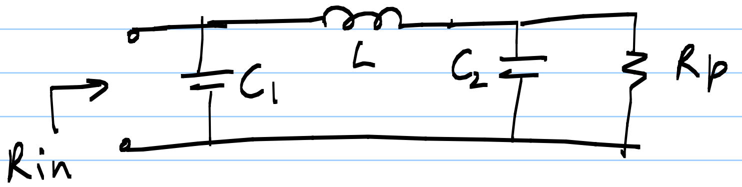
Lecture # 5 : π and T-matches ; Other matches

L-match – 2 degrees of freedom (L & C)

But, we want to fix 3 parameters } ω_0 , $\frac{R_p}{R_s}$ and Q

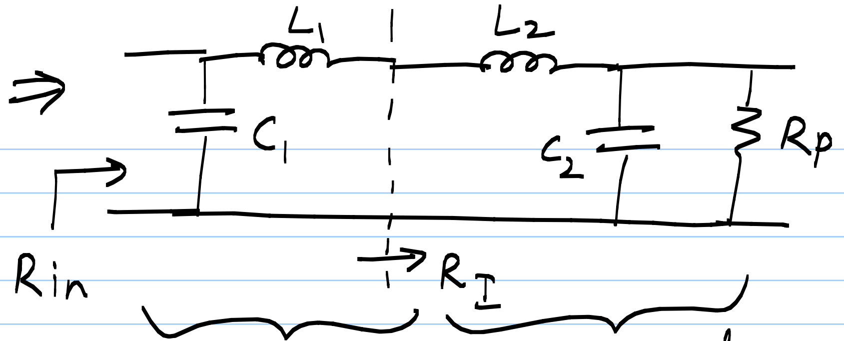
Remember that $Q \longleftrightarrow BW$ $\left\{ Q = \frac{\omega_0}{BW} \right\}$

Solution : add a third element to the matching network – one more degree of freedom



π – match network

Decompose into 2 L-matches $\Rightarrow$

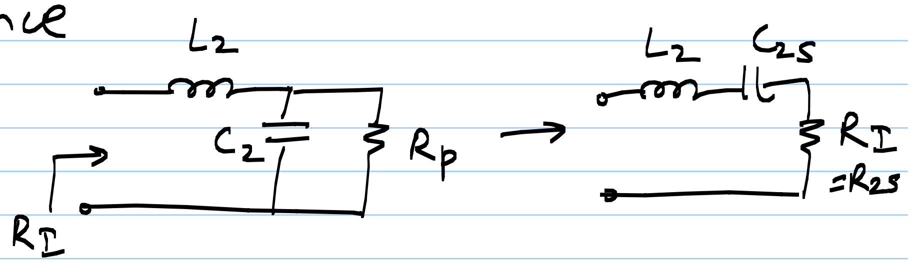


upward transformer

downward transformer

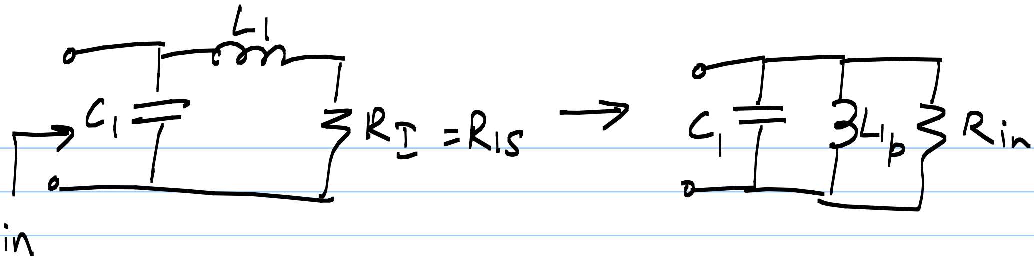
$R_I \equiv$ intermediate resistance

R.H. Section $\Rightarrow$



$$Q_{right} = \sqrt{\frac{R_p}{R_I} - 1} = \frac{\omega_0 L_2}{R_I}$$

L.H. Section:



$$Q_{left} = \sqrt{\frac{R_{in}}{R_I} - 1} = \frac{\omega_0 L_1}{R_I}$$

It can be shown that

See posted document for this derivation

$$\text{overall } Q = Q_{left} + Q_{right}$$

$$= \frac{\omega_0 (L_1 + L_2)}{R_I}$$

$$\boxed{Q = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_p}{R_I} - 1}}$$

e.g. $R_p = 200\,\Omega$, $R_{in} = 50\,\Omega$, $f_0 = 2.4\,\text{GHz}$, $Q = 10$

* Need to find out R_I first.

$$Q = Q_{left} + Q_{right}$$

$$10 = \sqrt{\frac{50}{R_I} - 1} + \sqrt{\frac{200}{R_I} - 1}$$

Solve using 1) quadratic equations

or 2) iteration

try $R_I = 10\,\Omega \Rightarrow \sqrt{4} + \sqrt{19} = 6.36$

try $R_I = 4\,\Omega \Rightarrow \sqrt{11.5} + \sqrt{49} = 10.39$

⋮

after several iterations, $R_I = 4.3\,\Omega$

$$Q_{left} = \sqrt{\frac{50}{4.3} - 1} = 3.26$$

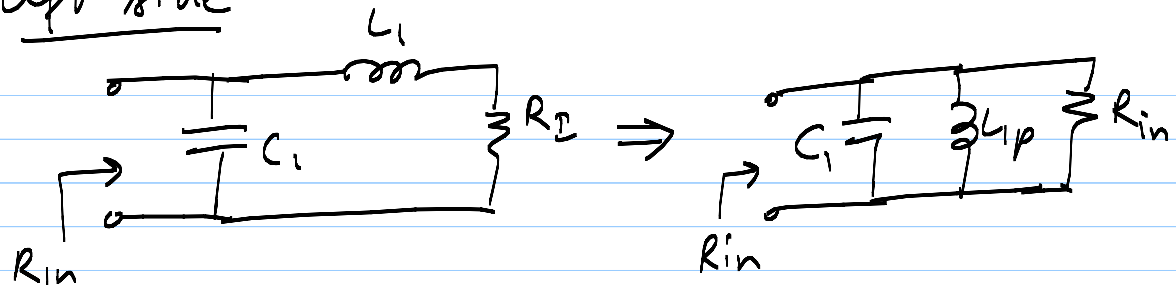
$$Q_{right} = \sqrt{\frac{200}{4.3} - 1} = 6.74$$

$$L_1 = \frac{Q_{left} \cdot R_I}{\omega_0} = \frac{(3.26)(4.3)}{2\pi \cdot 2.4GHz} = 0.93\,nH$$

$$L_2 = \frac{Q_{right} \cdot R_I}{\omega_0} = \frac{(6.74)(4.3)}{2\pi \cdot 2.4GHz} = 1.92\,nH$$

$$L = L_1 + L_2 = \underline{\underline{2.85\,nH}}$$

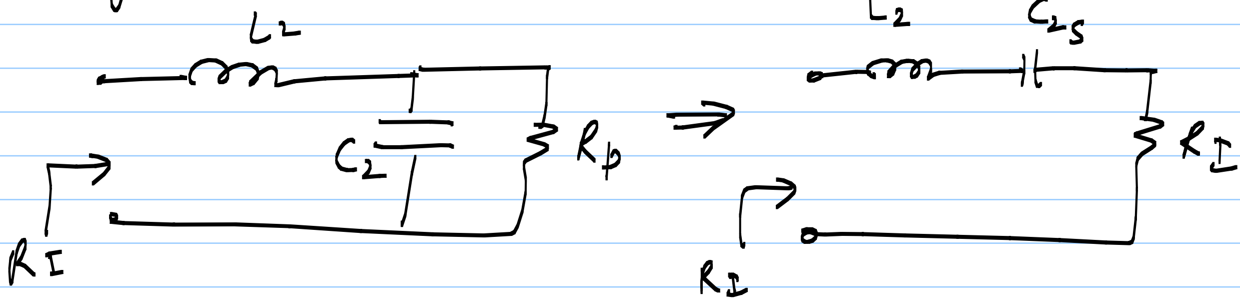
<u>Left side</u>



$$Q_{left} = \omega_0 R_{in} C_1$$

$$C_1 = \frac{Q_{left}}{\omega_0 R_{in}} = \frac{3.26}{(2\pi \cdot 2.4GHz) \cdot (50)} = 4.32\,pF$$

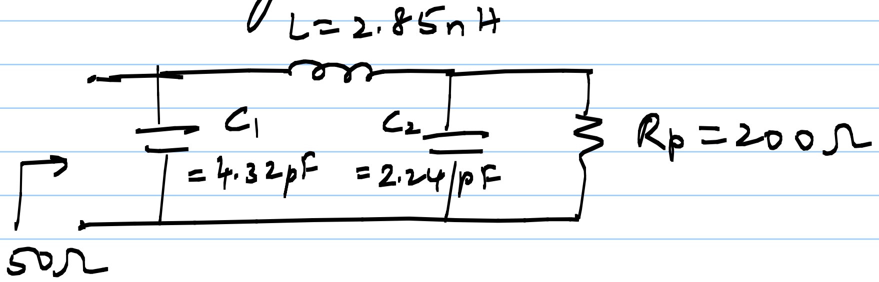
<u>Right side</u>



$$Q_{right} = \omega_0 R_p C_2$$

$$C_2 = \frac{Q_{right}}{\omega_0 R_p} = \frac{6.74}{(2\pi \cdot 2.4GHz)\cdot(200)} = 2.24pF$$

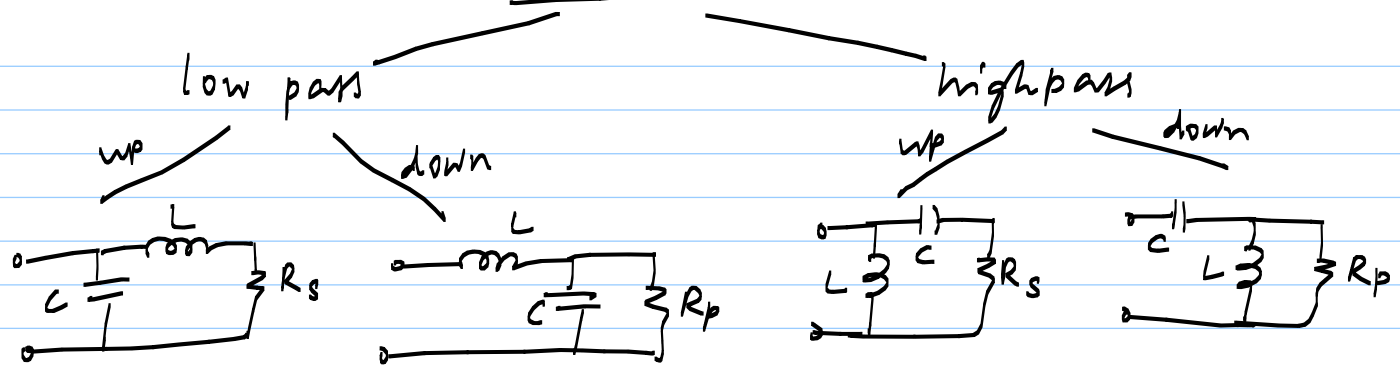
Final matching network:



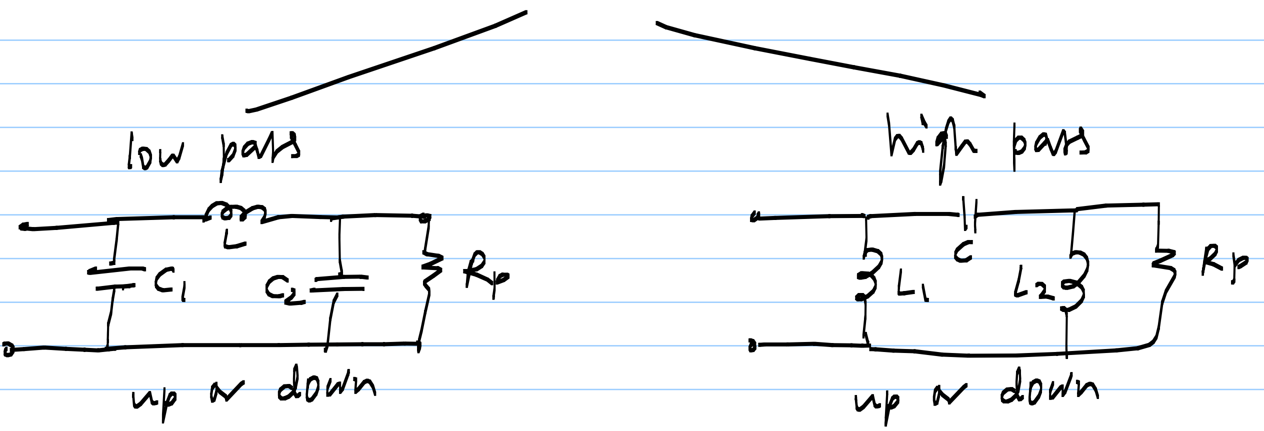
* Capacitive parasitics (including from L) are absorbed into C_1 & C_2!

* $R_I < R_{in}, R_p$ for a π-match

L-match



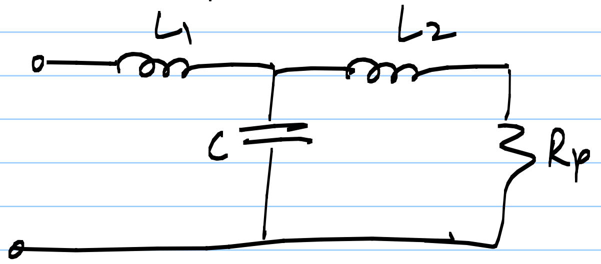
π-match



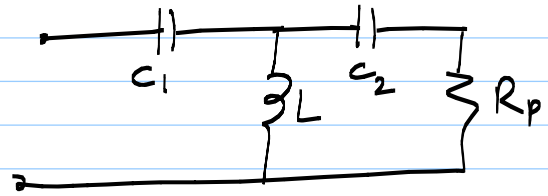
T- match

(will have a problem on T- match in HW1)

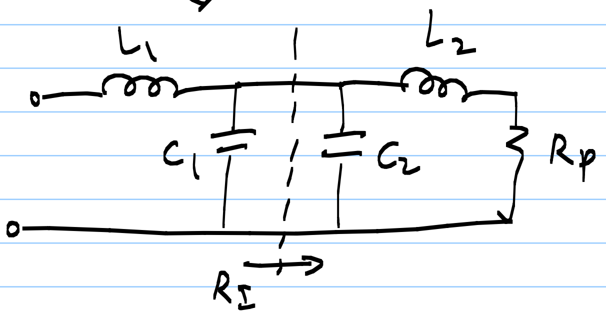
low pass



high pass



⇓



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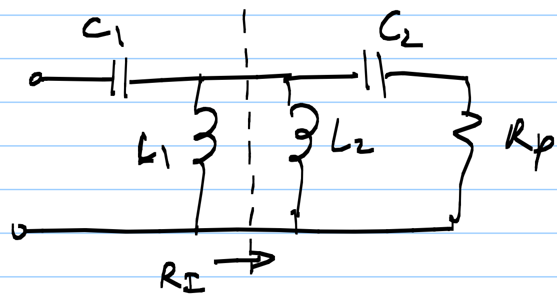
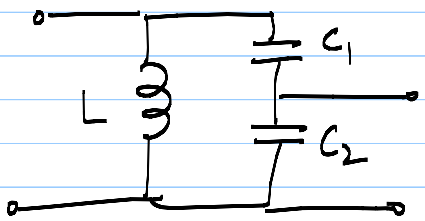


Note : $R_I > R_{in}, R_p$ for T-match

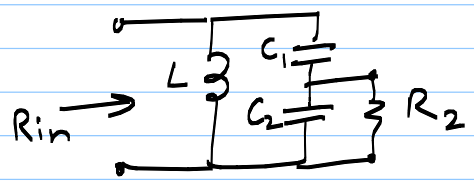
Tapped Capacitor Match



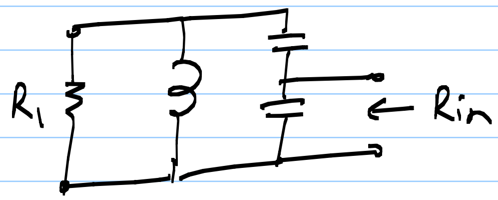
3 degrees of freedom : L, C_1, C_2

⇒ can set ω_0, Q and $\frac{R_{out}}{R_{in}}$

up   down

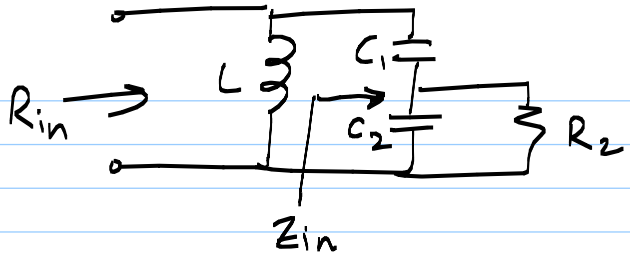


$R_{in} > R_2$



$R_{in} < R_1$

* You will see this type of circuit used in certain types of oscillators.



$$Z_{in} = \frac{1}{sC_1} + \frac{R_2}{1 + sC_2R_2}$$

$$= \frac{1 + sC_2R_2 + sC_1R_2}{sC_1 + s^2C_1C_2R_2}$$

$$Y_{in}(j\omega) = \frac{j\omega C_1 - \omega^2 C_1C_2R_2}{1 + j\omega R_2(C_1+C_2)}$$

$$= \frac{j\omega C_1 - \omega^2 \cancel{C_1} C_2 R_2 + \omega^2 C_1 R_2 (C_1 + \cancel{C_2}) + j\omega^3 C_1 C_2 R_2^2 (C_1+C_2)}{1 + [\omega R_2 (C_1+C_2)]^2}$$

Real part:

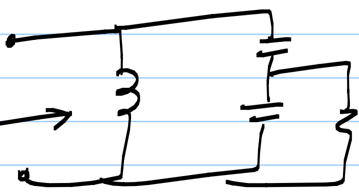
$$G_{in} = \frac{\omega^2 R_2 C_1^2}{1 + [\omega R_2 (C_1+C_2)]^2} \approx \frac{\cancel{\omega^2} \cancel{R_2} C_1^2}{\cancel{\omega^2} R_2^{\cancel{2}} (C_1+C_2)^2}$$

$$\approx G_2 \cdot \left(\frac{C_1}{C_1+C_2}\right)^2 = \frac{G_2}{n^2}$$

where $G_2 = \frac{1}{R_2}$

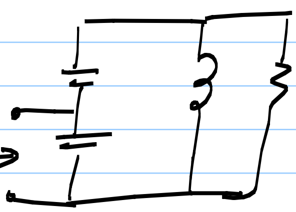
and $n \equiv$ turns ratio $= \frac{C_1+C_2}{C_1}$

"higher side"



upwards transform

"lower side"



downwards transform

Imaginary Part:

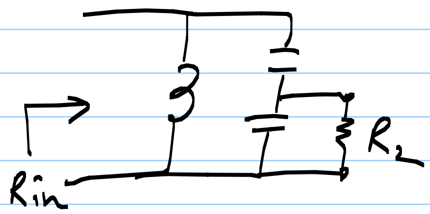
$$B_{in} = \frac{\omega C_1 + \omega^3 C_1 C_2 R_2^2 (C_1 + C_2)}{1 + \omega^2 R_2^2 (C_1 + C_2)^2}$$

$$\approx \frac{\omega^{\cancel{3}} C_1 C_2 \cancel{R_2^2} \cancel{(C_1 + C_2)}}{\cancel{\omega^2} \cancel{R_2^2} (C_1 + C_2)^{\cancel{2}}} \quad \text{at high freq.}$$

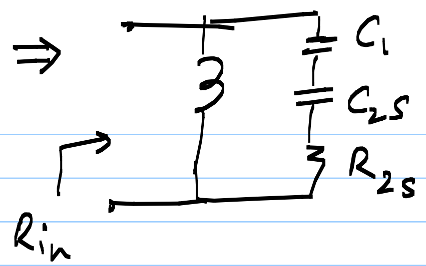
$$\approx \frac{\omega C_1 C_2}{C_1 + C_2} = \omega C_{eq} \quad \text{as expected}$$

Matching Equations:

$$Q = \frac{R_{in}}{\omega_0 L} \Rightarrow \boxed{L = \frac{R_{in}}{\omega_0 Q}}$$



$\Rightarrow$



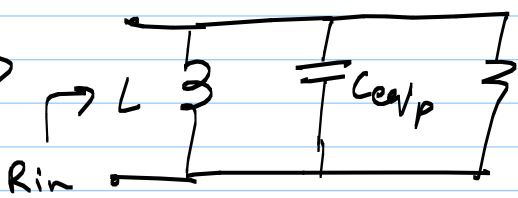
$$C_{2s} = C_2 \left(\frac{Q_2^2 + 1}{Q_2^2} \right)$$

$$Q_2 = \omega_0 C_2 R_2$$

$$R_{2s} = \frac{R_2}{Q_2^2 + 1}$$

$$C_{eq_s} = \frac{C_1 C_{2s}}{C_1 + C_{2s}}$$

$\Rightarrow$



$$R_p = R_{in} = R_{2s} (Q^2 + 1)$$

$\neq Q_2$, $\because C_1$ is also included

$$\frac{R_2}{Q_2^2 + 1} = \frac{R_{in}}{Q^2 + 1} \Rightarrow Q_2 = \sqrt{\frac{R_2}{R_{in}} (Q^2 + 1) - 1}$$

$$Q_2 = \omega_0 R_2 C_2 \Rightarrow C_2 = \frac{Q_2}{\omega_0 R_2}$$

$$\boxed{C_2 = \frac{\sqrt{\frac{R_2}{R_{in}}(Q^2+1) - 1}}{\omega_0 R_2}}$$

$$C_{eq_s} = \frac{C_1 C_{2s}}{C_1 + C_{2s}} \Rightarrow Q = \frac{1}{\omega_0 C_{eq} R_{2s}} = \frac{(C_1 + C_{2s})}{\omega_0 C_1 C_{2s} R_{2s}}$$

$$\Rightarrow \boxed{C_1 = \frac{C_2 (Q_2^2 + 1)}{Q Q_2 - Q_2^2}}$$

e.g. $R_2 = 50\Omega$, $R_{in} = 200\Omega$, $f_0 = 2.4GHz$, $Q = 10$

$$L = \frac{R_{in}}{\omega_0 Q} = \frac{200}{2\pi \cdot 2.4GHz \cdot 10} = \underline{\underline{0.75nH}}$$

$$Q_2 = \sqrt{\frac{R_2}{R_{in}}(Q^2+1) - 1} = \sqrt{\frac{50}{200}(100+1) - 1} = 4.92$$

$$C_2 = \frac{Q_2}{\omega_0 R_2} = \frac{4.92}{2\pi \cdot 2.4GHz \times 50}$$

$$= \underline{\underline{6.53pF}}$$

$$C_1 = \frac{C_2 (Q_2^2 + 1)}{Q Q_2 - Q_2^2} = \frac{6.53 (4.92^2 + 1)}{10 \cdot 4.92 - 4.92^2} = \underline{\underline{6.61}}$$

Recall : $R_{in} \simeq R_2 \left(\frac{C_1 + C_2}{C_1} \right)^2 = n^2 R_2$

in this case, $R_{in} = 200$, $R_2 = 50$

$\Rightarrow n = 2$ ("turns ratio" of 2)

$\Rightarrow C_1 \approx C_2$ (as computed)