Lecture #4 – Impedance Transformations & Matching; L-matches

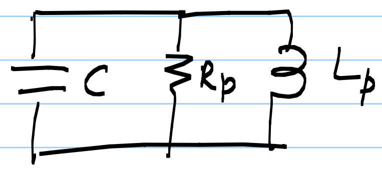
Why?

* RF input and output impedances are standardised to 50Ω (75Ω for TV components)
* 50Ω is approx. tradeoff between max. power handling capability and min. loss
* on-chip – try to minimise 50Ω impedances (large power needed to drive 50Ω)
* match at LNA input and DA/PA output

Series – parallel transformations:

neither series nor parallel RLC network

make series-parallel conversion at resonance ⇒



$$R_s + j\omega_0 L_s = \frac{R_p \cdot j\omega_0 L_p}{R_p + j\omega_0 L_p}$$

$$= \frac{(\omega_0 L_p)^2 R_p + j\omega_0 L_p R_p^2}{R_p^2 + \omega_0^2 L_p^2}$$

Equate real and imaginary components:

Real part:

$$R_s = \frac{(\omega_0 L_p)^2 R_p}{R_p^2 + \omega_0^2 L_p^2}\ ;$$

we know $Q_p = \dfrac{R_p}{\omega_0 L_p}$

$Q_s = \dfrac{\omega_0 L_s}{R_s}$

and $Q_p = Q_s = Q$

$$\therefore R_s = \frac{R_p}{1 + \dfrac{R_p^2}{(\omega_0 L_p)^2}}$$

$$\boxed{R_p = R_s(1+Q^2)}$$

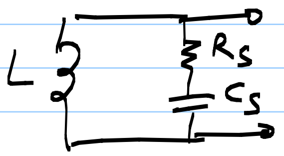
Imaginary part:

$$\omega_0 L_s = \frac{\omega_0 L_p R_p^2}{R_p^2 + (\omega_0 L_p)^2}$$

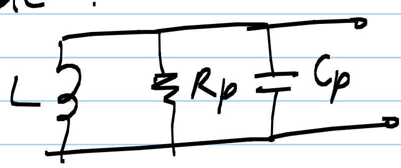
$$L_s = L_p \cdot \frac{R_p^2/(\omega_0 L_p)^2}{1 + \left(\dfrac{R_p}{\omega_0 L_p}\right)^2} = L_p \cdot \frac{Q^2}{1+Q^2}$$

$$\boxed{L_p = \frac{L_s(1+Q^2)}{Q^2}}$$

Note: HW 1 will include:



$\Longrightarrow$



$R_p = R_s(1+Q^2)$

$C_p = C_s \cdot \dfrac{Q^2}{1+Q^2}$

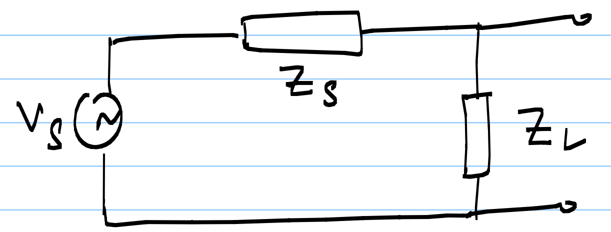
In general:

$$R_p = R_s(1+Q^2)$$

$$X_p = X_s \cdot \frac{1+Q^2}{Q^2}$$

Maximum Power Transfer Theorem:

"conjugate match"



$Z_s = R_s + jX_s$

$Z_L = R_L + jX_L$

Max power in Z_L is achieved when

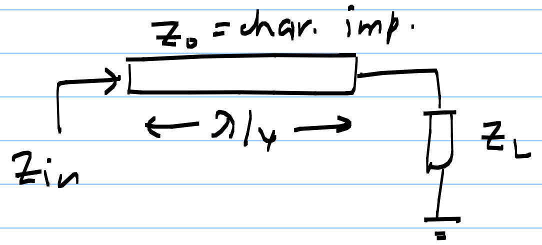
$R_L = R_s$ & $X_L = -X_s$

Note: in LNAs, optimum noise match $\neq$ optimum power match

Impedance matching:

Traditional μWave techniques:

1) λ/4 transformer:



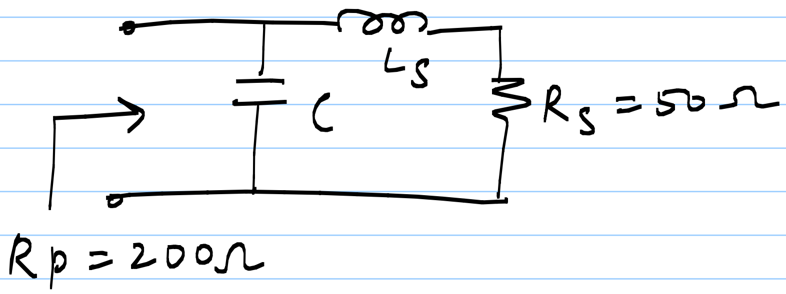
$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

2) Stub matching:

* Use open and short T-Lines to rotate impedance in Smith Chart, to obtain desired Z_{in}
* Use series or shunt stub depending on whether Z or Y is being manipulated

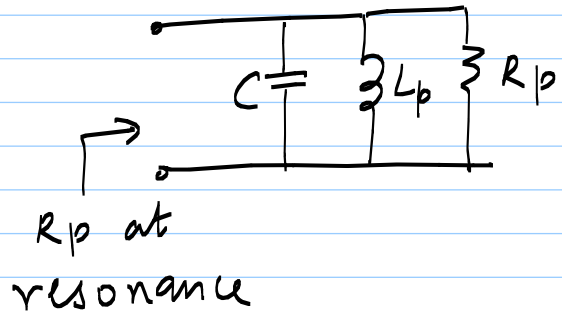
L-match networks: use lumped components

(A) Upward impedance transformers:



"Low-pass" L-match

$\Downarrow$



$$L_p = L_s \frac{(1+Q^2)}{Q^2}$$

$$R_p = R_s (1+Q^2)$$

i.e. $$Q = \sqrt{\frac{R_p}{R_s} - 1} \quad \text{—— (1)}$$

$$Q = \frac{R_p}{\omega_o L_p} \Rightarrow L_p = \frac{R_p}{\omega_o Q} \quad \text{—— (2)}$$

$$L_s = \frac{Q^2 L_p}{1+Q^2} \quad \text{—— (3)}$$

$$C = \frac{1}{L_p \omega_o^2} \quad \text{—— (4)}$$

Use these 4 equations to determine C, L_s

e.g. Match 50Ω to 200Ω at 2.4GHz

We know: R_s, R_p, ω_o

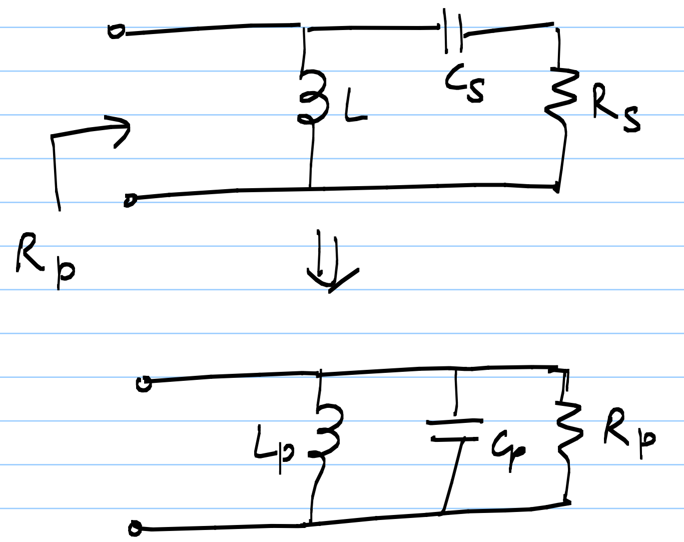
$R_s = 50\Omega$, $R_p = 200\Omega$, $\omega_o = 2\pi \cdot 2.4\text{GHz}$

$$Q = \sqrt{\frac{200}{50} - 1} = \sqrt{3} = 1.73$$

$\Rightarrow L_p = 7.67\text{nH}$ (2) $\Rightarrow L_s = 5.75\text{nH}$ (3) $\Rightarrow C = 0.573\text{pF}$ (4)

reasonable values for IC implementation

Alternative upward transformer:



"High-pass" L-match

$$C_p = C_s \cdot \frac{Q^2}{1+Q^2}$$

$$R_p = R_s(1+Q^2)$$

i.e. $Q = \sqrt{\frac{R_p}{R_s} - 1}$

e.g. $50\,\Omega \longrightarrow 200\,\Omega$ at 2.4GHz

$$Q = \sqrt{3} = 1.73$$

$$Q = \omega_0 C_p R_p \Rightarrow C_p = \frac{Q}{\omega_0 R_p} = \frac{1.73}{2\pi \times 2.4\text{GHz} \times 200}$$

$$= 0.574\,\text{pF}$$

$$C_s = C_p \frac{1+Q^2}{Q^2} = 0.765\,\text{pF}$$

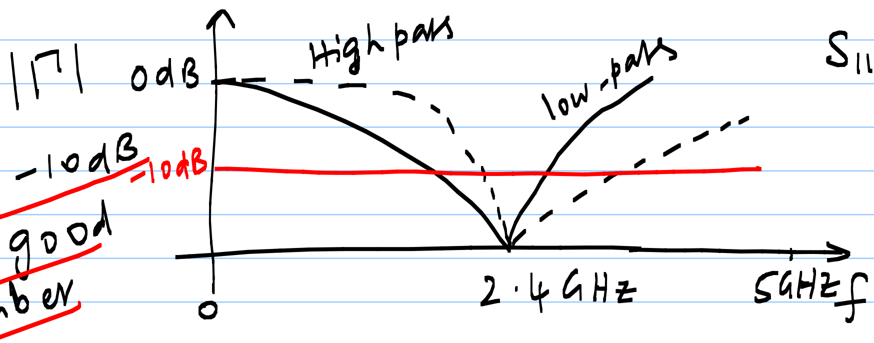
$$L = \frac{1}{\omega_0^2 C_p} = 7.66\,\text{nH}$$

High-pass or Low-pass?

Considerations for Choice:

1) Die area: smaller inductor, also lower loss (low-pass in this case)

2) Quality of match vs. frequency



$S_{11} < -10$dB is a good number

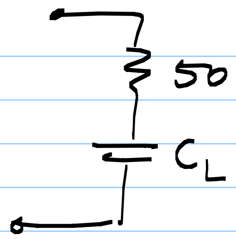
$S_{11} = \Gamma$ (for a 1-port) reflection coefficient

$$= \frac{Z_L - Z_0}{Z_L + Z_0}$$ where Z_0 = source imp.

3) Relationship to parasitics (e.g. output bondwire etc.)

Note: Matching to Complex load

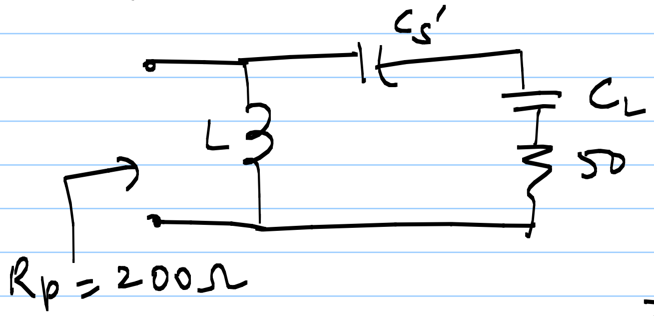
e.g. $Z_L = 50 - j25 \Rightarrow$



at 2.4 GHz,

$$\frac{-j}{\omega_0 C_L} = -j25$$

$$\Rightarrow C_L = 2.65\,pF$$



Total $C_s = C_L$ series C_s'

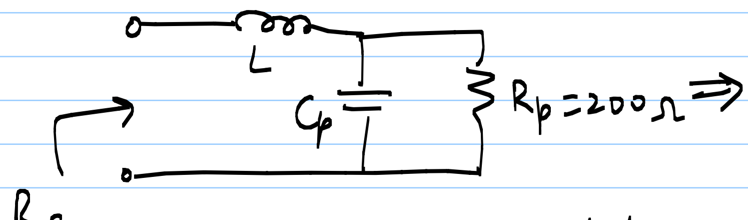
we know $C_s = 0.765\,pF$ from before

$$\Rightarrow C_s = \frac{C_s' C_L}{C_s' + C_L}$$

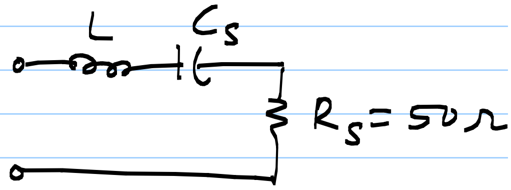
$$\Rightarrow C_s' = \frac{C_L C_s}{C_L - C_s} = \frac{(2.65\,pF)(0.765\,pF)}{2.65\,pF - 0.765\,pF} = \underline{\underline{1.08\,pF}}$$

(B) Downward Impedance Transformers:

Swith ports!



$\Rightarrow$



low-pass L-match

$$R_p = R_s(1 + Q^2)$$

$$C_p = C_s\left(\frac{Q^2}{Q^2+1}\right)$$

e.g. $R_p = 200\Omega$, $R_s = 50\Omega$ at $f_0 = 5.6\,GHz$

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{3} = 1.73$$

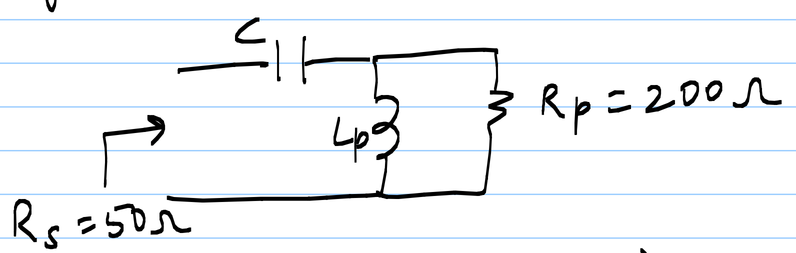
$$Q = \frac{1}{\omega_0 R_s C_s} \Rightarrow C_s = \frac{1}{\omega_0 Q R_s} = \frac{1}{2\pi \cdot 5.6\,\text{GHz} \cdot 1.73 \cdot 50}$$

$$= 0.329\,\text{pF}$$

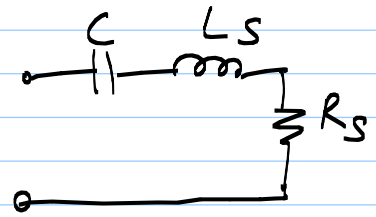
$$\Rightarrow C_p = C_s \frac{Q^2}{1+Q^2} = 0.246\,\text{pF}$$

$$L = \frac{1}{\omega_0^2 C_s} = 2.46\,\text{nH}$$

High-pass L-match:



$\Rightarrow$



$$R_p = R_s(1+Q^2)$$
$$L_p = L_s \left(\frac{Q^2+1}{Q^2}\right)$$

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{3} = 1.73$$

$$Q = \frac{\omega_0 L_s}{R_s} \Rightarrow L_s = \frac{Q R_s}{\omega_0} = \frac{1.73 \times 50}{2\pi \times 5.6\,\text{GHz}} = 2.46\,\text{nH}$$

$$\Rightarrow L_p = (2.46\,\text{nH}) \cdot \left(\frac{4}{3}\right) = \underline{\underline{3.28\,\text{nH}}}$$

$$C = \frac{1}{\omega_0^2 L_s} = \underline{\underline{0.328\,\text{pF}}}$$

What was common to all L-matches?

$$Q = \sqrt{\frac{R_p}{R_s} - 1}$$

Quality of match is fixed once R_{in}, R_L are known!