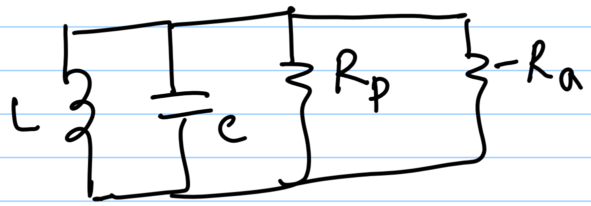
Lecture 31 : Phase Noise (cont.)

Last class : LTI analysis



Remember $\Rightarrow |-R_a| = R_p$ only @ resonance

If $-R_a$ were to cancel R_p @ all freq.,

$\Rightarrow$ ideal LC osc. !

at other freq. $|-R_a| > R_p$, but $|-R_a| \sim R_p$

$$R_{eq.} = \frac{-R_a \cdot R_p}{R_p - R_a} = \frac{R_p R_a}{\underbrace{R_a - R_p}_{\text{very small}}} \gg R_p$$

$$Q_{eq.} = \omega_0 C \cdot R_{eq.} \gg Q_0$$

$$Y(\omega_0 + \Delta\omega) = G_{eq.} + j \frac{2\omega_0 \Delta\omega}{(\omega_0 + \Delta\omega)} \cdot C$$

$$\Rightarrow Z(\omega_0 + \Delta\omega) = \frac{1}{G_{eq.} + j \frac{2\omega_0 \Delta\omega}{(\omega_0 + \Delta\omega)} \cdot C}$$

$$= \frac{1}{G_{eq.}} \frac{1}{1 + j \frac{2\omega_0 \Delta\omega}{(\omega_0 + \Delta\omega)} R_{eq.} \cdot C}$$

$$= R_{eq.} \frac{1}{1 + 2Q_{eq.} \cdot j \cdot \frac{\Delta\omega}{\omega_0}}$$

$$\simeq \frac{\omega_0 R_{eq.}}{j(2Q_{eq.} \Delta\omega)} \quad \{Q_{eq.} >> 1\}$$

$$|Z(\omega_0 + \Delta\omega)| = \frac{\omega_0 R_{eq.}}{2Q_{eq.}\Delta\omega}$$

$$Q = \omega_0 R_p C$$

$$Q_{eq.} = \omega_0 R_{eq.} C$$

$$\Rightarrow \frac{\omega_0 R_{eq.}}{Q_{eq.}} = \frac{\omega_0 R_p}{Q} \quad (= C)$$

$$\Rightarrow |Z(\omega_0 + \Delta\omega)| = \frac{\omega_0 R_p}{2Q\Delta\omega}$$

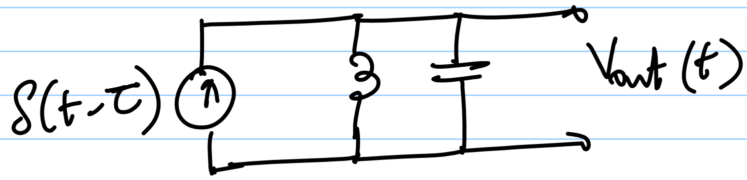
$$\Rightarrow \frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \cdot |Z|^2$$

$$= \frac{4kT}{R_p} \cdot \frac{(\omega_0 R_p)^2}{(2Q\Delta\omega)^2}$$

$$= 4kTR_p \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

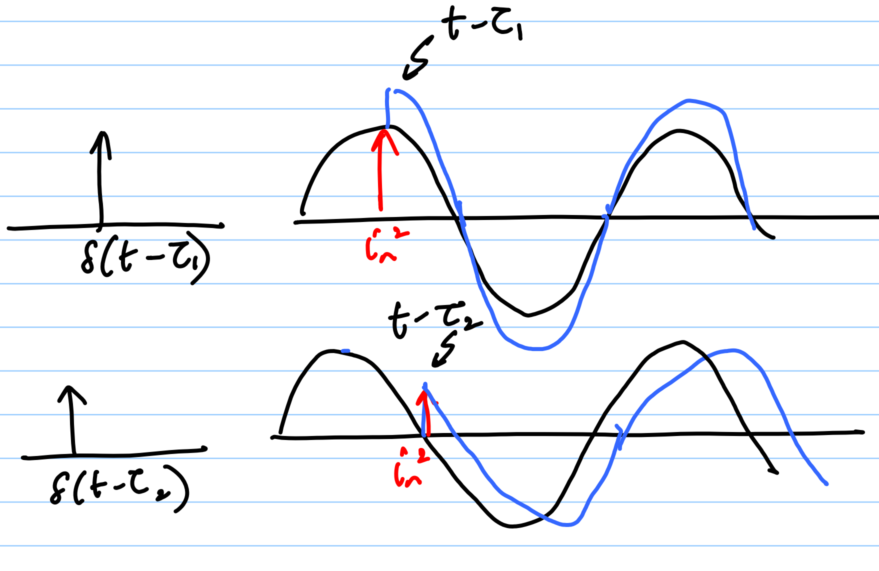
rest of the analysis proceeds as before.

Hajimiri – Lee PN model



t = observation time

τ = excitation time



← noise injected @ peak does not affect PN

← very sensitive to noise injected @ zero crossings

Impulse response (linearity still holds) for phase:

$$h_\varphi(t,\tau) = \frac{\Gamma(\omega_0\tau)}{q_{max}}\, u(t-\tau)$$

q_{max} = max charge displacement across cap.
(to make $\Gamma(\omega_0 t)$ ampl. independent)

$$\varphi(t) = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(\omega_0\tau)\cdot i(\tau)\, d\tau$$

$i(\tau)$ = noise current

$\Gamma(\omega_0\tau)$ = ISF (normalised)

$\varphi(t)$ ← total phase @ t = sum of all phase disturbances due to $i_n(t)$ from $-\infty$ to t

ISF

$\Gamma \rightarrow$ dimensionless,

$\rightarrow$ freq. & amplitude independent

$\rightarrow$ periodic w/ period 2π

$$\Gamma(\omega_0 t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

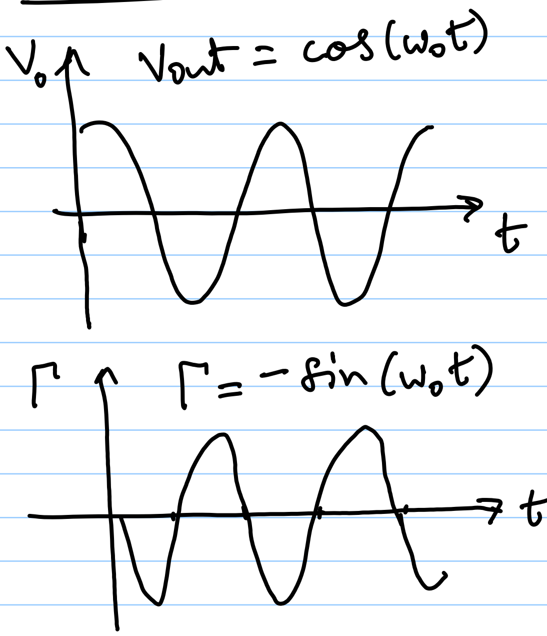
c_n = Fourier Series coeffts.

θ_n = phase of n^{th} harmonic of ISF

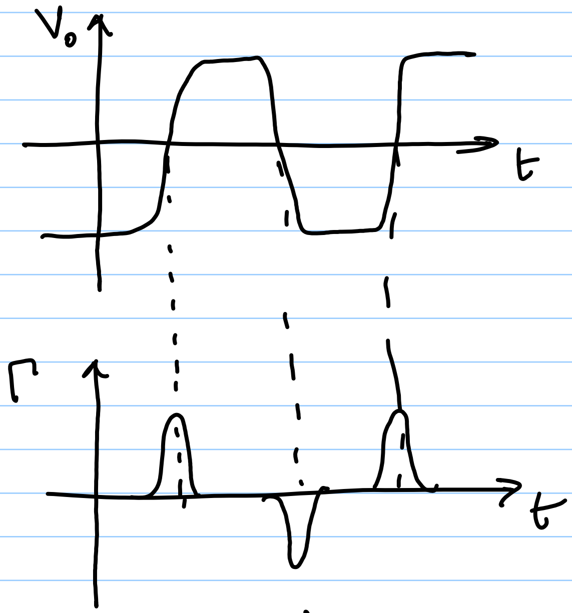
Note that from Parseval's Theorem,

$$\sum_{n=0}^{\infty} c_m^2 = \frac{1}{\pi}\int_0^{2\pi} |\Gamma(\varphi)|^2 \, d\varphi = 2\Gamma_{rms}^2$$

LC osc.



Ring Osc.



* In general, determine ISF accurately from simulation

$$\phi(t) = \frac{1}{q_{max.}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)\,d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)\,d\tau\right]$$

Consider sinusoidal current at $(m\omega_0 + \Delta\omega)$

$$i(t) = I_m \cos[(m\omega_0 + \Delta\omega)t]$$

$$\Rightarrow \varphi(t) \simeq \frac{I_m c_m \sin(\Delta\omega t)}{2 q_{max.}\,\Delta\omega}$$

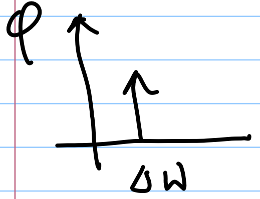
{ Even though noise is @ $m\omega_0 + \Delta\omega$ }

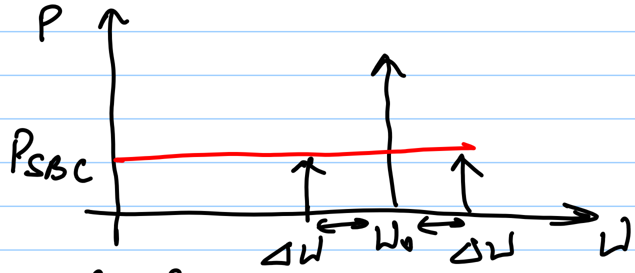
$v_o(t) = \cos(\omega_0 t + \varphi(t))$ ⇒ phase to voltage converter

(square $V_o^2(t)$ to get power)

→ fundamentally nonlinear (PM)

$\varphi(t)$ @ $\Delta\omega$ → (~) → $v_o(t)$ (side bands @ $\Delta\omega$)





$$P_{SBC}(\Delta\omega) \simeq 10\log\left[\frac{I_m^2 c_m^2}{4 q_{max}^2 \Delta\omega^2}\right]$$

narrowband FM approx.

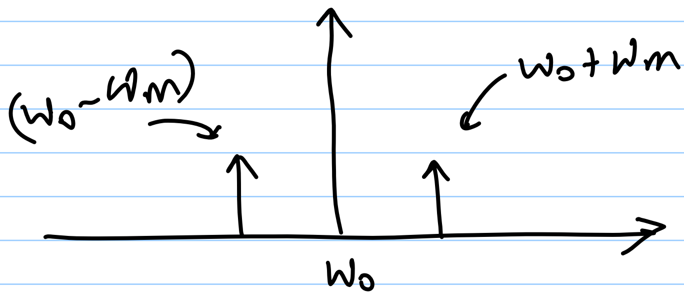
* see Razavi 3.2.2 for narrowband FM approx.

$V_C + V_m \cos\omega_m t$ → (~) → $v_o(t) = A_0\cos\left(\omega_0 t + \int_{-\infty}^{t} V_c(t)\,dt\right)$

$\omega_m \ll \omega_0$

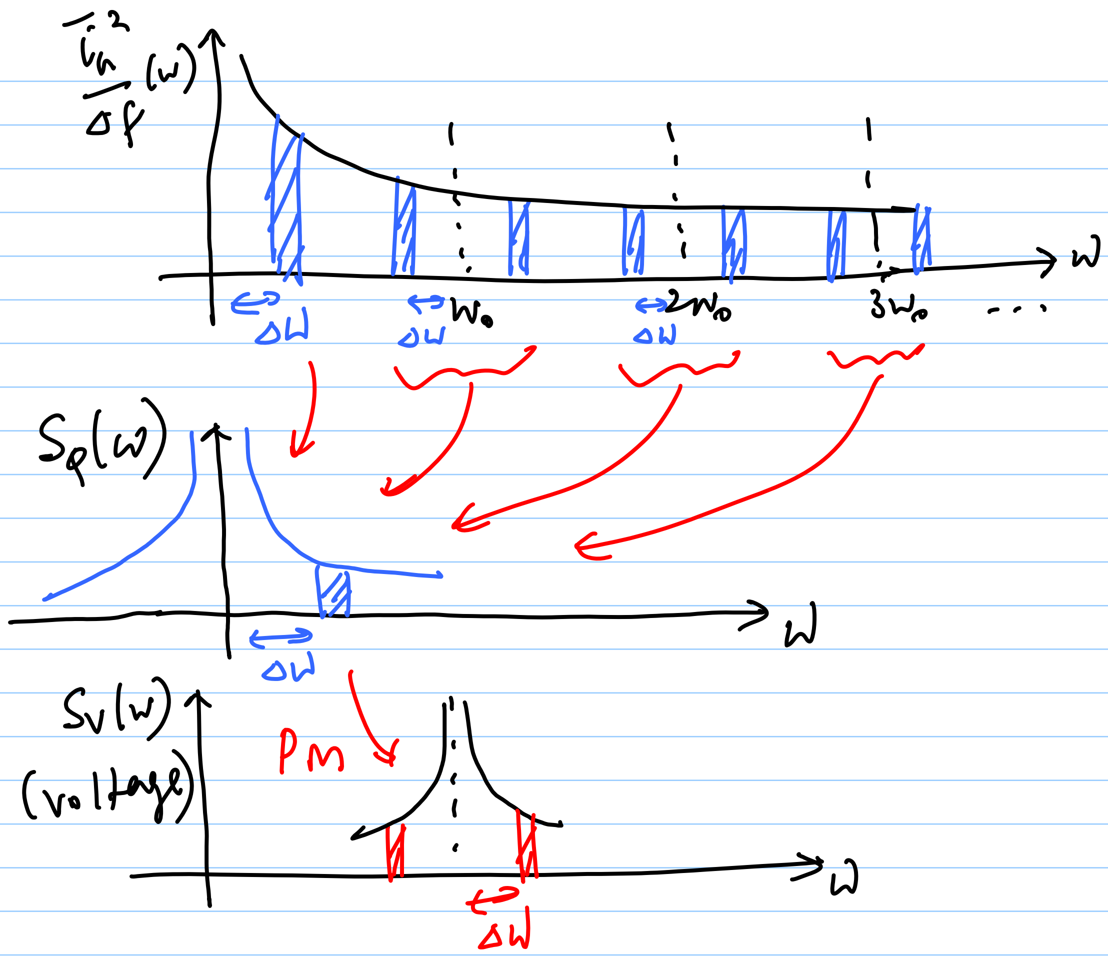
Use Bessel function approx.

$$v_o(t) \simeq A_o \cos w_o t + \frac{A_o V_m K_{VCO}}{2 w_m} \cdot \left[\cos(w_o + w_m)t - \cos(w_o - w_m)t \right]$$



for a white noise source

$$P_{SBC}(\Delta w) \simeq 10 \log \left[\frac{\left(\overline{i_n^2}/\Delta f\right) \cdot \sum_{m=0}^{\infty} c_m^2}{4 q^2 \, \Delta w^2} \right]$$



Thankfully $|C_m|$ reduces as $m \uparrow$, so only the first few terms are significant

spectrum in $1/f^2$ region:

$$L(\Delta\omega) = 10 \log \left[\frac{(\overline{i_n^2}/\Delta f) \cdot \Gamma_{rms}^2}{2 q_{max}^2 \, \Delta\omega^2} \right]$$

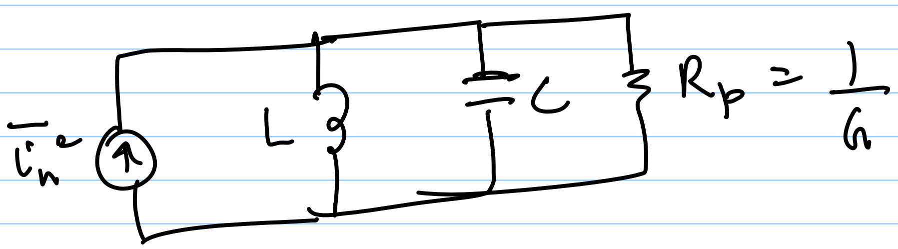
$\Rightarrow$ reduce Γ_{rms} to reduce PN

C = tank cap.
V_{pk} = peak amplitude across tank $\Big\}$ $q_{max} = C \cdot V_{pk}$

$$L(\Delta\omega) = 10 \log \left[\frac{\overline{i_n^2} \, \Gamma_{rms}^2}{2 (C V_{pk}^2) (\Delta\omega)^2} \right]$$

consider an LC oscillator

$$\Gamma(\omega_0 t) = -\sin(\omega_0 t) \Rightarrow \Gamma_{rms}^2 = \frac{1}{2}$$



$$L(\Delta\omega) = 10\log\left[\frac{4kTG \cdot 1/2}{2(CV_{pk})^2 (\Delta\omega)^2}\right]$$

$$= 10\log\left[\frac{kTG}{C^2 \cdot (2V_0^2)} \cdot \frac{1}{(\Delta\omega)^2}\right]$$

$$= 10\log\left[\frac{kTG}{2\omega_0^2 C^2 V_0^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2\right]$$

$$= 10\log\left[\frac{kT \cdot 1/R}{2V_0^2 \cdot (Q^2/R^2)} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2\right]$$

$$= 10\log\left[\frac{2kTR}{V_0^2} \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]$$ ← Same as Leeson's linear result

Flicker noise

$$\overline{i_n^2}_{1/f} = \overline{i_n^2} \cdot \frac{\omega_{1/f}}{\Delta\omega}$$ in the 1/f noise region

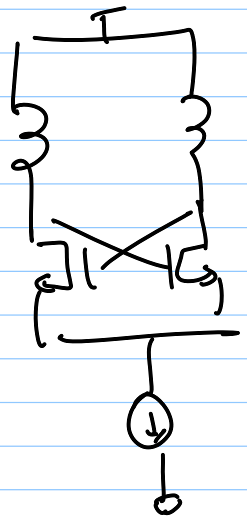
flicker noise only around DC!

$$\Rightarrow L(\Delta\omega) = 10\log\left[\frac{(\overline{i_n^2}/\Delta f) \cdot C_0^2}{8q_{max}^2 \cdot \Delta\omega^2} \cdot \frac{\omega_{1/f}}{\Delta\omega}\right]$$

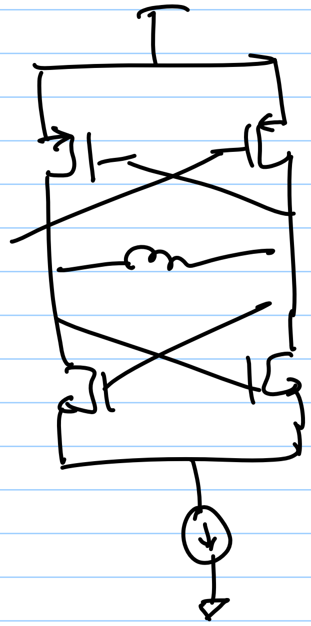
$$\Rightarrow \Delta\omega_{1/f^3} = \omega_{1/f} \cdot \frac{C_0^2}{4\Gamma_{rms}^2} = \omega_{1/f} \cdot \left(\frac{\Gamma_{dc}}{\Gamma_{rms}}\right)^2$$

$\Rightarrow$ $1/f^3$ noise corner can be reduced by decreasing Γ_{dc} (also $\neq \omega_{1/f}$)

$\Gamma_{dc} = 0 \Rightarrow$ symmetrical rise & fall times!

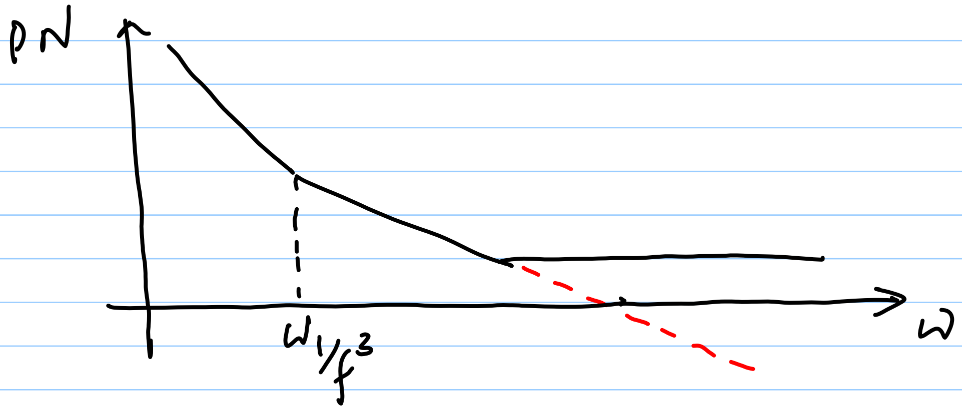


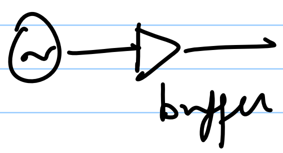
$t_r \neq t_f$



$t_r = t_f$ is possible by careful Design!

* One last point: why do you have a flat noise region?





* noise of buffer
* noise floor of meas. equipment (if it is high)