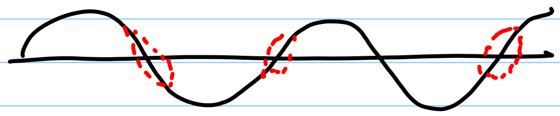
Lecture 30 : Phase Noise

* VCO is an autonomous ckt (i.e. its phase is not tied to any reference)
* ideal VCO : $v(t) = V_0 \cos(\omega_0 t)$
* Real VCO : random amplitude / phase changes due to noise

$$V(t) = V_0(t) \cos(\omega_0 t + \varphi(t))$$

Amplitude noise

Phase Noise
≡ Jitter in time domain



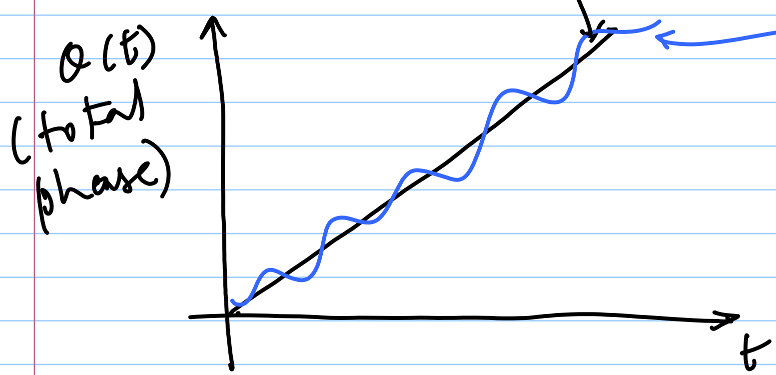
← zero crossings are modulated by noise

Instantaneous freq.

$$\omega = \omega_0 + \frac{d\varphi(t)}{dt}$$

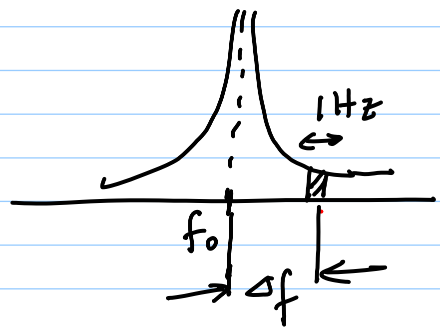
Ideal Osc.

Real osc.



$\theta(t)$ slope $= \omega_0$

SSB phase noise



$$L(\Delta f) = 10 \cdot \log_{10}\left[\frac{P_{1Hz}(f_0+\Delta f)}{P_s}\right]$$

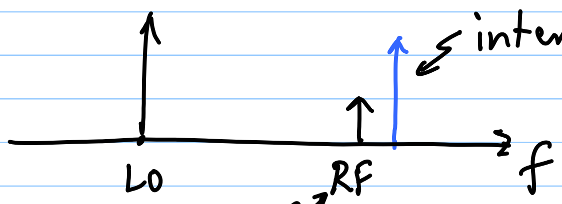
Units : dBc/Hz

* $P_{1Hz}(f_0+\Delta f) \equiv$ power in a 1 Hz BW at offset Δf from f_0
* P_s = mean-square carrier power
* $L(\Delta f)$ is expressed in units of dB's below carrier per Hz {at offset Δf}

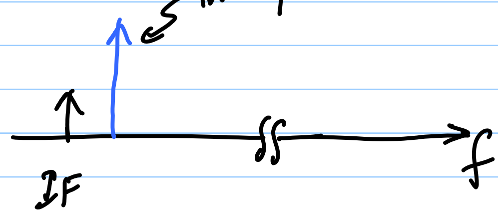
Effects of PN

1) Reciprocal Mixing

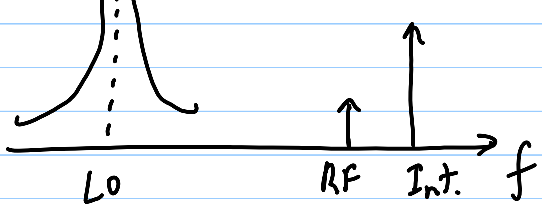
Ideal mixer :



⇒



Real mixer



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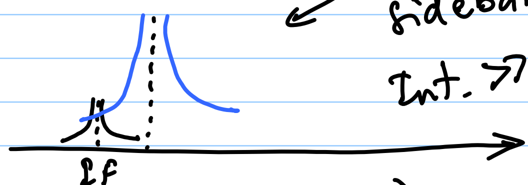
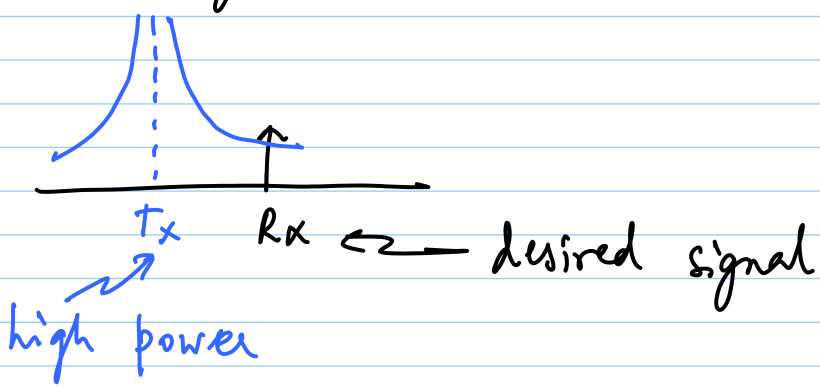


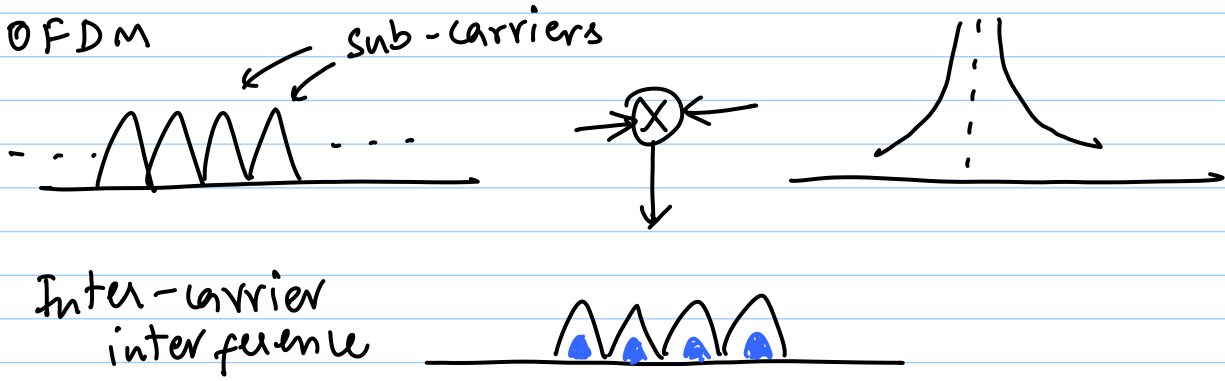
noise sidebands of Int. ⇒ desired IF

(R_x desensitised)

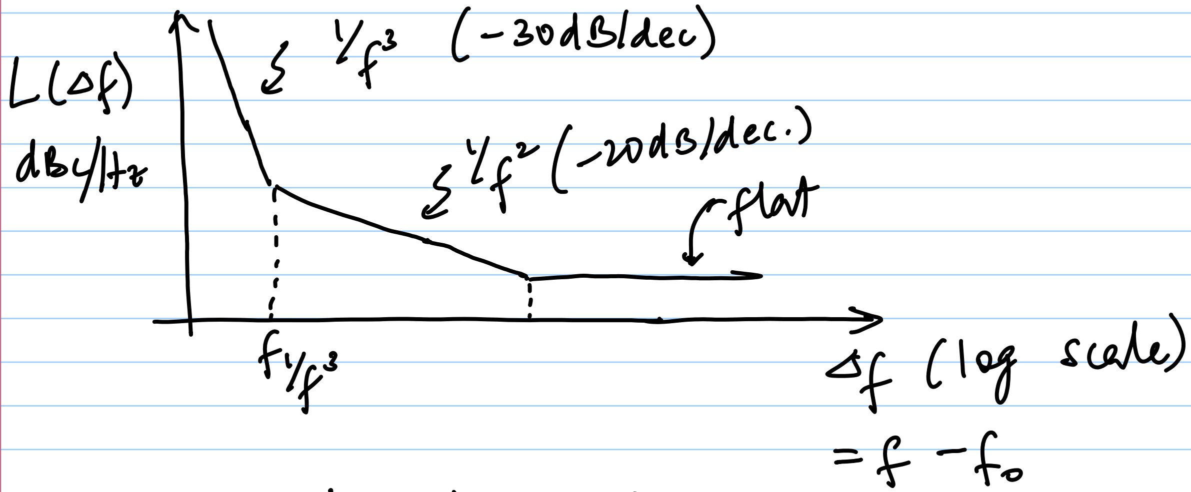
2) Powerful Tx with noisy LO can desensitise nearby Rx with weak signal



3) OFDM



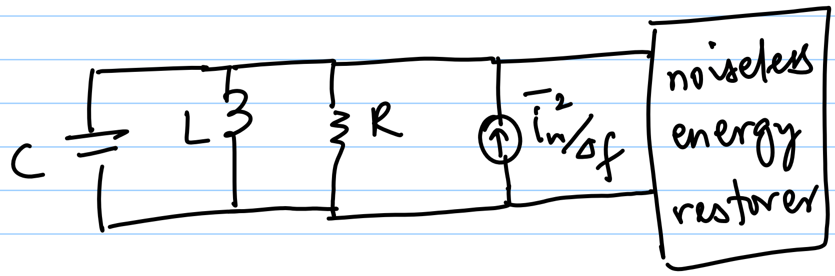
Phase noise (zoomed in)



Leeson's phase-noise model

Ref: D. B. Leeson, "A simple model of feedback oscillator noise spectrum", Proceedings of the IEEE, pp. 320-330, Feb. 1966.

* LTI analysis of LC oscillator in steady-state using feedback theory
* Assumes noise from resistor only
  → excess noise compensated using empirical factor 'F'



idealised osc. model

$$\frac{\overline{i_n^2}}{\Delta f} = 4kTG \quad A^2/Hz$$

Energy $E = \frac{1}{2} C V_c^2 \Rightarrow V_c^2 = \frac{E}{C}$

$$Q = \omega \cdot \frac{E}{P_D}$$

$\overline{v_n^2}$ = total mean squared noise over entire BW

$$= 4kTG \int_0^{\infty} |Z(f)|^2 \, df$$

where $Z(s) = \dfrac{sL}{s^2LC + \frac{sL}{R} + 1}$

$$= \frac{kT}{C}$$

$$N/S = \frac{\overline{v_n^2}}{\overline{V_c^2}} = \frac{kT}{E} = \frac{\omega kT}{Q \cdot P_D}$$

At a small freq. offset $\Delta\omega$,

$$Y(\omega_0+\Delta\omega) = G + j(\omega_0+\Delta\omega)\cdot C + \frac{1}{j(\omega_0+\Delta\omega)L}$$

$$= \frac{jG(\omega_0+\Delta\omega)\cdot L - (\omega_0+\Delta\omega)^2 LC + 1}{j(\omega_0+\Delta\omega)\cdot L}$$

$$\approx \frac{jG(\omega_0+\Delta\omega) - 2\omega_0\Delta\omega C}{j(\omega_0+\Delta\omega)}$$

$$= G + j\frac{2\omega_0\Delta\omega}{(\omega_0+\Delta\omega)}\cdot C$$

$$Z(\omega_0+\Delta\omega) = \frac{1}{G + j\cdot\frac{2\omega_0\Delta\omega}{(\omega_0+\Delta\omega)}\cdot C}$$

$$= \frac{1}{G}\cdot\frac{1}{1+2j\frac{\omega_0\cdot\Delta\omega}{(\omega_0+\Delta\omega)}\cdot RC}$$

Since $Q = \omega_0 RC$,

$$Z(\omega_0+\Delta\omega) \approx R\cdot\frac{1}{1+2Q\cdot j\cdot\frac{\Delta\omega}{\omega_0}}$$

$$|Z(\omega_0+\Delta\omega)| \approx \frac{R\omega_0}{2Q\,\Delta\omega}$$

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \cdot |Z|^2$$

$$= 4KTG \cdot R^2 \cdot \frac{w_0^2}{(2Q\Delta W)^2}$$

$$= 4KTR \cdot \left(\frac{W_0}{2Q\Delta W}\right)^2$$

* includes both amplitude & phase noise

→ Equipartition theorem of Thermodynamics states!

$$\text{Noise} \begin{cases} \to \text{Ampl. Noise} \\ \to \text{Phase Noise} \end{cases} \left\{\begin{matrix}\text{equally} \\ \text{split}\end{matrix}\right\}$$

→ assume amplitude limiter eliminates all amplitude noise

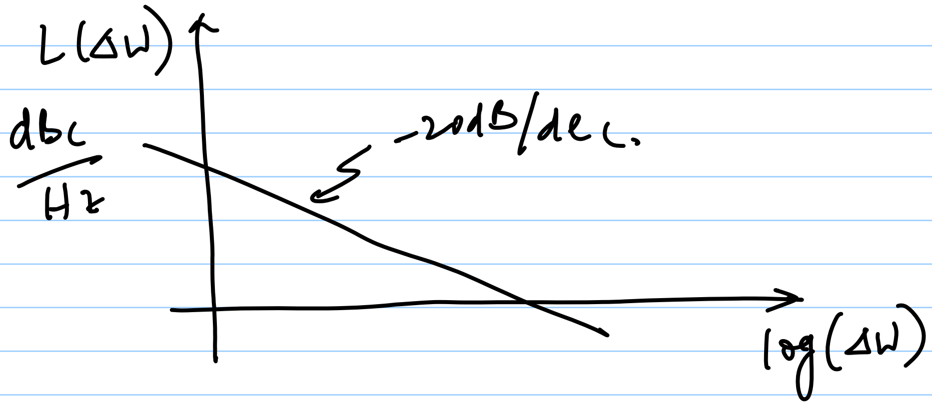
Phase component of Noise

$$= \frac{1}{2} \cdot \frac{\overline{v_n^2}}{\Delta f} = 2KTR \cdot \left(\frac{W_0}{2Q\Delta W}\right)^2$$

$$L(\Delta W) = 10\log_{10}\left[\frac{\frac{1}{2} \cdot \overline{v_n^2}/\Delta f}{v_{sig}^2}\right]$$

$$= 10\log_{10}\left[\frac{2KTR\left(\frac{W_0}{2Q\Delta W}\right)^2}{v_{sig}^2}\right]$$

$$= 10\log_{10}\left[\frac{2KT}{P_{sig.}} \cdot \left(\frac{W_0}{2Q\Delta W}\right)^2\right]$$



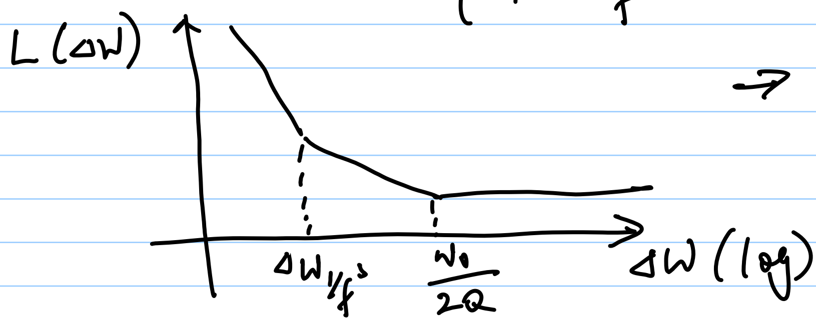
* we know actual VCO PN has different slopes not accounted for in this simplistic model
  - → $1/f^3$ region
  - → flat noise floor
  - → Device noise

Leeson's model:

$$L(\Delta W) = 10 \cdot \log_{10}\left[\frac{2KTF}{P_{sig.}}\left\{1+\left(\frac{W_0}{2Q\Delta W}\right)^2\right\}\left\{1+\frac{\Delta W_{1/f^3}}{|\Delta W|}\right\}\right]$$

→ F is an empirical factor {"device excess noise number"}

→ $\Delta W_{1/f^3}$ = $1/f^3$ noise corner
(≠ $1/f$ corner of device)



→ Q = Quality factor with loading

Leeson's model

→ accounts for trends in all regions of $L(\Delta\omega)$

→ provides some insight

e.g. $\omega_0 \uparrow \Rightarrow PN \uparrow$ (6dB for every 2x)

$Q \uparrow \Rightarrow PN \downarrow$ (?? for every 2x)

→ provides no info on F & $\Delta\omega_{1/f^3}$

→ We know that oscillators

have non-linearity

are time-variant

Rewrite:

$$L(\Delta\omega) = 10\log_{10}\left[\frac{2FkT}{P_{sig.}}\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]$$

$$= 10\log_{10}\left[\frac{2FkT}{\left(\frac{2}{\pi}I_T \cdot R_p\right)^2}\cdot\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]$$

$$= 10\log_{10}\left[\frac{(\pi^2/2)FkT}{I_T^2}\cdot\left(\frac{\omega_0}{QL}\right)\cdot\left(\frac{1}{2Q\Delta\omega}\right)^2\right]$$

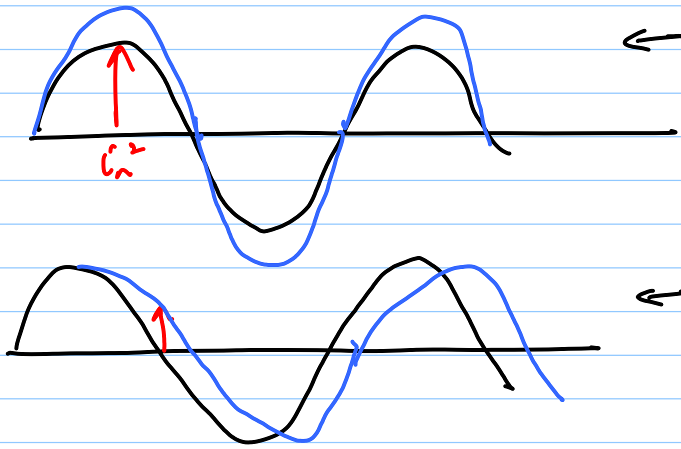
i.e. $PN \propto 10\log_{10}\left[\frac{1}{Q^3}\cdot(\ldots)\right]$

More sophisticated PN models

1) A. Hajimiri & T.H. Lee, "A General Theory of phase-noise in oscillators"

* Oscillators are time-variant (as far as noise is concerned)

e.g.



← noise injected @ peak does not affect PN

← very sensitive to noise injected @ zero crossings

* $\overline{v_n^2} << v_{sig}^2$ ⇒ Linearity assumption is valid for noise

⇒ LTV model

* "Impulse Sensitivity Function" or ISF can be determined for an oscillator

→ PN is determined based on this

2) J. Rael & A. Abidi, "Physical processes of phase noise in differential LC oscillators"

IEEE Custom Integrated Circuits Conf., 2000

* Non-linear, time-invariant analysis
* Similar to mixer noise analysis

* Expression for F in Leeson's Model :

$$\boxed{F = 2 + \frac{8 \gamma R I_T}{\pi V_0} + \gamma \cdot \frac{8}{9} g_m R}$$

where $V_0 = \frac{2}{\pi} I_T R$ = peak diff. ampl.

$$P_{sig.} = \frac{V_0^2}{R}$$

* If I_T is increased while keeping g_m constant $\Rightarrow$ F remains constant

$\Rightarrow$ $L(\Delta\omega) = 10 \log_{10} \left[\frac{4FkT}{V_0^2} (\ldots) \right] \downarrow$

PN decreases due to increase in V_0

* If I_T is increased beyond supply rail limitation, V_0 does not increase but $F \uparrow$ & PN $\uparrow$