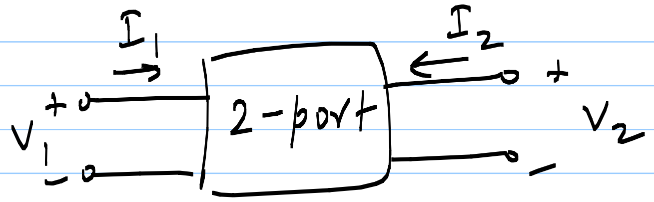
Lecture #3 – S-parameters, ABCD parameters, Resonance

Recall :



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

In general, $[V] = [Z] \cdot [I]$

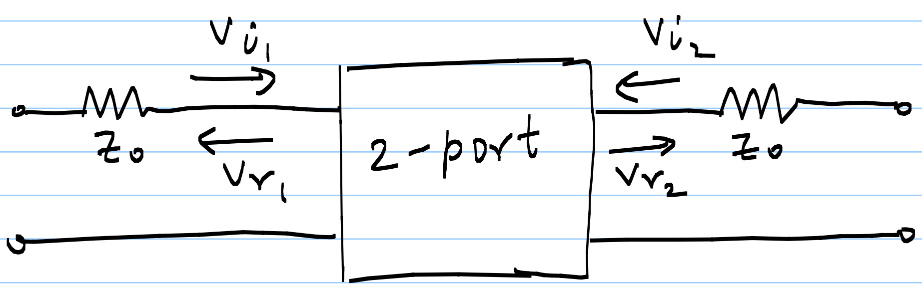
lly $[Y]$, $[H]$, $[G]$ can also be defined

All of these relate voltages and currents of the network

For a distributed network, we talk of incident & reflected waves.

⇒ Scattering (S-) parameters

$$[V_r] = [S] \cdot [V_i]$$



$$\begin{bmatrix} V_{r_1} \\ V_{r_2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_{i_1} \\ V_{i_2} \end{bmatrix}$$

Usually normalised w.r.t. Z_0 :

$$a_1 = \frac{V_{i_1}}{\sqrt{Z_0}} \quad ; \quad a_2 = \frac{V_{i_2}}{\sqrt{Z_0}}$$

$$b_1 = \frac{V_{r_1}}{\sqrt{Z_0}} \quad ; \quad b_2 = \frac{V_{r_2}}{\sqrt{Z_0}}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$a_1^2, a_2^2, b_1^2, b_2^2 \Rightarrow$ powers of incident & reflected waves

S_{ij} are usually represented in dB
i.e. $20\log(\text{value})$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{V_{r_1}}{V_{i_1}} = \Gamma_1$$ (input reflection coefficient)

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{V_{r_2}}{V_{i_1}}$$ (gain)

$$S_{21}^2 = \frac{V_{r_2}^2}{V_{i_1}^2} = \frac{V_{r_2}^2/Z_0}{V_{i_1}^2/Z_0} = \text{power gain}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{V_{r_2}}{V_{i_2}} = \Gamma_2$$ (output port reflection coefficient)

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{V_{r_1}}{V_{i_2}}$$ (reverse transmission gain)

Short note on units etc:

let x be any linear electrical value (e.g. V, I etc.)

$$x \text{ (in dB)} = 20 \log(x)$$

power $P \propto V^2$ or I^2

$$\therefore \quad P \text{ (in dB)} = 10 \log(P)$$

$$\boxed{P \text{ (in dBm)} = 10 \log\left(\frac{P}{1\,mW}\right)}$$

i.e. 0 dB reference value is 1 mW

$$1\,mW \text{ in dBm} = 10 \log\left(\frac{1\,mW}{1\,mW}\right) = 0\,dBm$$

$$10\,mW \text{ in dBm} = 10 \log(10) = 10\,dBm$$

If $Z_0 = 50\,\Omega$, 0 dBm corresponds to $\approx 223\,mV$ rms

We can also talk about dBV etc.

$$1\,V \longleftrightarrow 0\,dBV$$

$$\boxed{dBc \equiv \text{power w.r.t. carrier}}$$

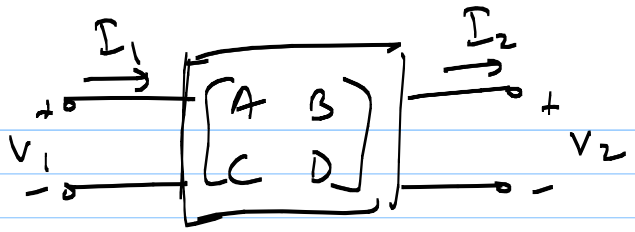
"c" $\longrightarrow$ 0 dB reference is power of carrier

$\Rightarrow$ used for noise, distortion etc.

ABCD parameters (Transmission matrix)

very useful for cascaded 2-ports

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



Note: I_2 flows <u>out</u> of port 2

Consider a cascade of two 2-port networks

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

can easily relate input & output of cascade.

HW 1 will include the following:

1) Derive [S] in terms of [Z]

2) Derive [ABCD] in terms of [Z]

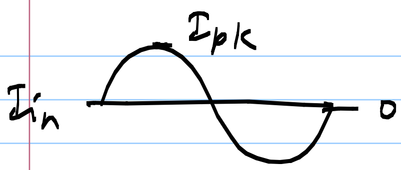
<u>Resonance</u>:

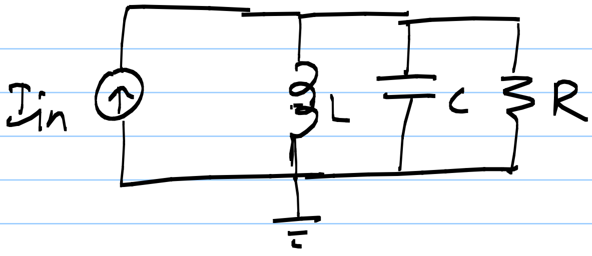
All (narrowband) RF systems employ resonance

* tuned bandpass amplifiers
* impedance transformations and matching
* RF oscillators

Series and Parallel RLC categories

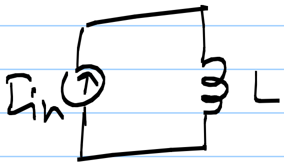
Parallel RLC





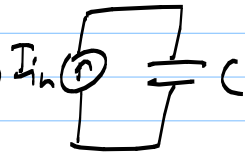
Intuitively:

low freq. $\Rightarrow$

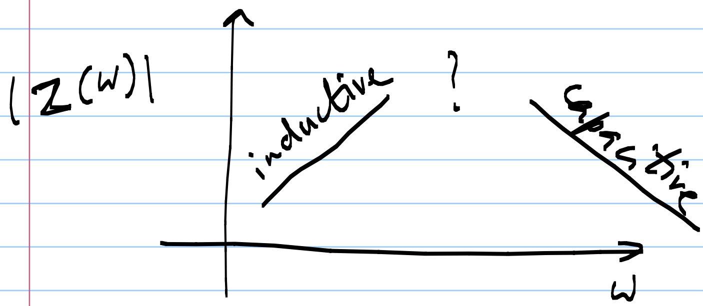


inductive

high freq. $\Rightarrow$



capacitive



what happens in-between?

Admittance $Y(\omega) = \dfrac{1}{R} + j\omega C + \dfrac{1}{j\omega L}$

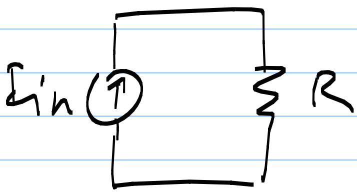
$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

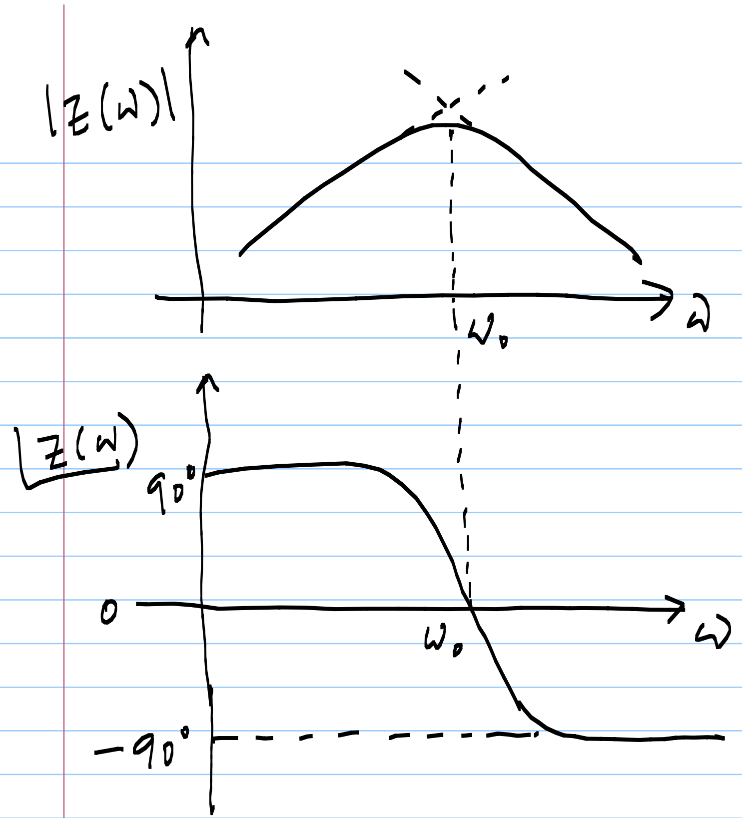
At resonance, $\omega_0 C = \dfrac{1}{\omega_0 L} \Rightarrow \omega_0^2 = \dfrac{1}{LC}$, $\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Do <u>not</u> confuse ω (rad/s) & f (Hz)

$Y(\omega_0) = \dfrac{1}{R}$ $\Rightarrow$ purely resistive





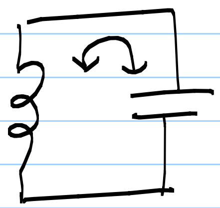
convenient on-chip values:

$1\,pF \parallel 1\,nH \simeq 5\,GHz$

<u>Quality Factor</u> (physical definition)

$$Q \equiv \omega \cdot \frac{\text{Energy stored}}{\text{Average Power dissipated}}$$

at $\omega = \omega_0$, $V_{out} = I_{in} \cdot R$



stored energy moves back and forth between L & C

$$V_{out_{pk}} = I_{pk} \cdot R \Rightarrow E_{tot.} = \frac{1}{2} C \left(I_{pk} \cdot R\right)^2$$

$$P_{ave} = \left(I_{RMS}\right)^2 \cdot R = \left(\frac{I_{pk}}{\sqrt{2}}\right)^2 \cdot R = \frac{1}{2} I_{pk}^2 R$$

$$\Rightarrow Q = \omega_0 \cdot \frac{E_{tot.}}{P_{ave}} = \frac{1}{\sqrt{LC}} \cdot \frac{\frac{1}{2} C I_{pk}^2 R^2}{\frac{1}{2} I_{pk}^2 R}$$

$$\boxed{Q = \frac{R}{\sqrt{L/C}}}$$

Intuitive check: as $R \to \infty$, $Q \to \infty$

seems correct ⇒ no average power dissipated

$\sqrt{\frac{L}{C}}$ = <u>characteristic impedance</u> of network

At resonance,

$$|Z_L| = \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

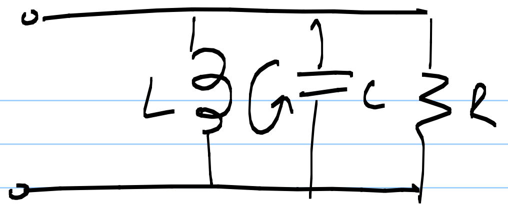
$$|Z_C| = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}}$$

Basic forms of Q for parallel RLC:

$$Q = \frac{R}{\sqrt{L/C}}$$

$$Q = \frac{R}{|Z_L|} = \frac{R}{\omega_0 L}$$

$$Q = \frac{R}{|Z_C|} = \omega_0 RC$$



Beware! Branch currents at resonance

$$|I_L| = |I_C| = \frac{|V_{out}|}{\omega_0 L} = \frac{|I_{in}|R}{\omega_0 L} = Q \cdot |I_{in}|$$

* very large currents can flow through L & C at resonance
* Careful layout is required to ensure current carrying capability (esp. for C)
* It is dangerous to think of resonance as L & C "cancelling" each other out!

BW & Q relationship:

Calculate impedance close to resonance at a frequency $\omega = \omega_0 + \Delta\omega$

$$Y(\omega) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= \frac{1}{R} + \frac{j}{\omega L}\left(\omega^2 LC - 1\right)$$

$$Y(\omega_0+\Delta\omega) = \frac{1}{R} + \frac{j}{(\omega_0+\Delta\omega)L}\left[(\omega_0+\Delta\omega)^2 LC - 1\right]$$

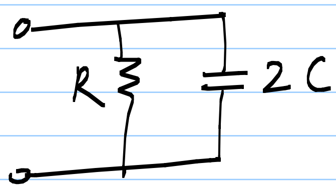
$$= \frac{1}{R} + \frac{j}{\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)} \cdot \left[\cancel{\omega_0^2 LC} + 2\omega_0\Delta\omega LC + \Delta\omega^2 LC \cancel{-1}\right]$$

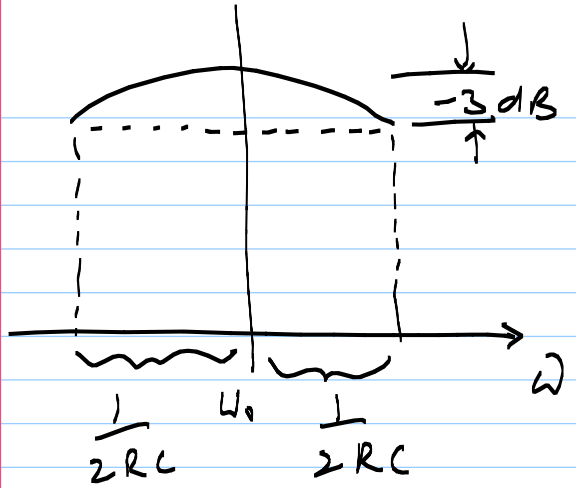
$$Y(\omega_0+\Delta\omega) = \frac{1}{R} + \frac{j}{\cancel{\omega_0}\cancel{L}\left(1+\frac{\Delta\omega}{\omega_0}\right)} \cdot \cancel{\omega_0}\Delta\omega \cancel{L} C\left[2 + \frac{\Delta\omega}{\omega_0}\right]$$

$$\simeq \frac{1}{R} + j\Delta\omega C\left(2+\frac{\Delta\omega}{\omega_0}\right)\left(1-\frac{\Delta\omega}{\omega_0}\right)$$

$$= \frac{1}{R} + j\Delta\omega C\left[2 - \frac{2\Delta\omega}{\omega_0} + \frac{\Delta\omega}{\omega_0} - \left(\frac{\Delta\omega}{\omega_0}\right)^2\right]$$

$$\simeq \frac{1}{R} + j2\Delta\omega C \qquad \{\text{neglecting } \Delta\omega^2, \Delta\omega^3\}$$

equivalent circuit $\Rightarrow$ 



by symmetry,

$$\text{total } BW = \frac{1}{RC}$$

$$\frac{\omega_0}{BW} = \frac{RC}{\sqrt{LC}} = \frac{R}{\sqrt{L/C}} = Q$$

Different definitions of Q:

1) Fundamental physical definition:

$$Q \equiv \omega_0 \frac{E_{tot.}}{P_{ave}}$$

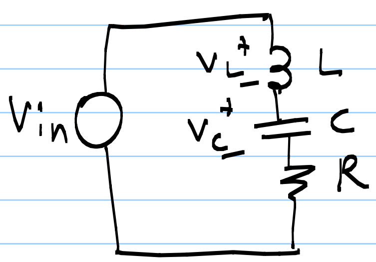
also applicable to distributed systems and non-resonant systems (e.g. Q of an RC network)

2) $$Q = \frac{\mathrm{Im}(Z(\omega))}{\mathrm{Re}(Z(\omega))}$$

3) $$Q = \frac{\omega_0}{BW}$$

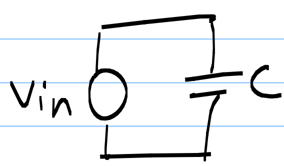
4) $$Q = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|$$ where $\phi(\omega)$ = phase of open loop TF.

Series RLC networks



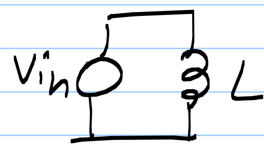
intuitively:

low-freq.



capacitive

high freq



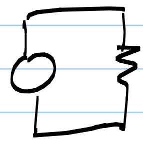
inductive

At resonance, reactive impedances are equal and opposite $\Rightarrow V_L + V_C = 0$

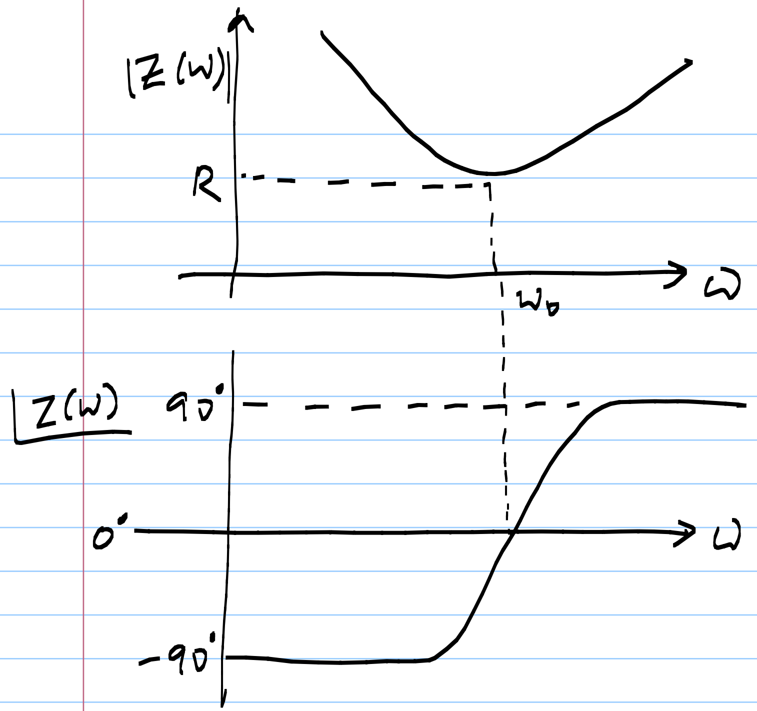
$$Z(\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \; ; \; f_0 = \frac{1}{2\pi\sqrt{LC}}$$

network behaves as if purely resistive



__Note:__ HW1 will have a problem on deriving $Z(\omega)$ & Q expressions for series RLC networks



Prove for yourself:

$|V_L| = |V_C| = |V_{in}| \cdot Q$

$\Rightarrow$ Useful for passive voltage amplification in LNAs

Series LC is not just a short!