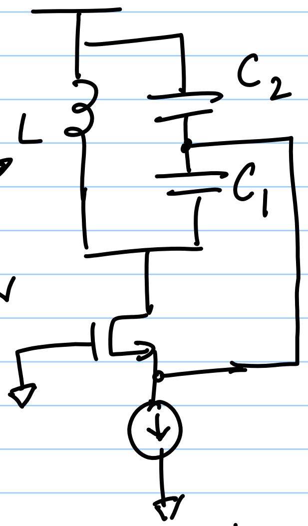
Lecture 29 : Colpitts Oscillator ; Quadrature Signal Generation

Colpitts oscillator

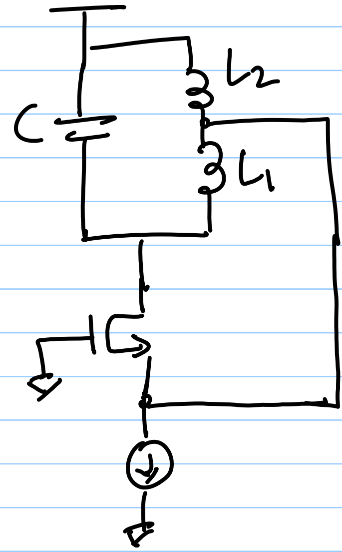


Only 1 inductor

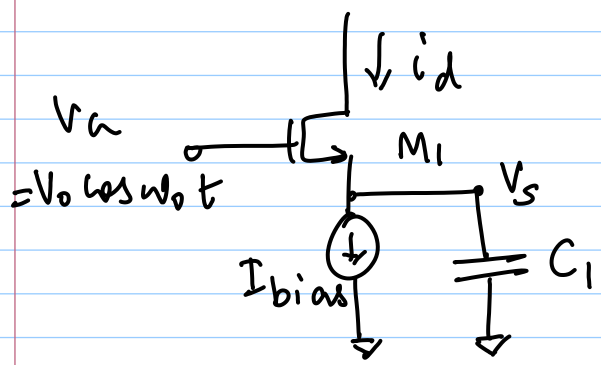
we will analyze this

$$\omega_0 = \frac{1}{\sqrt{L \cdot \left(\frac{C_1 C_2}{C_1 + C_2}\right)}}$$

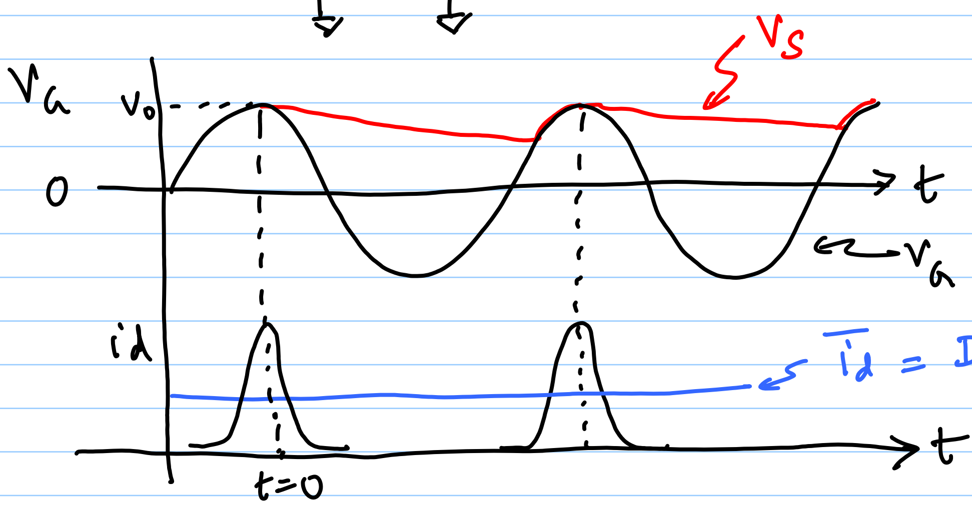
Hartley oscillator



Colpitts oscillator



* Assume large amplitude
* Assume ω_0 = high freq.
* Assume C_1 is short @ ω_0



$$\bar{I}_d = I_{bias} \left\{ = \frac{1}{T} \int_0^T i_d(t)\, dt \right\}$$

* M_1 conducts only when V_a is large ($\sim V_0$)
  → when $V_a \ll V_0$, M_1 cuts off, I_{bias} discharges C_1
* i_d = periodic pulses of current @ ω_0

Fourier series:

$$i_d(t) = I_0 + \sum_{n=1}^{\infty} \hat{I}_n \cos n\omega_0 t$$

→ $t=0$ reference = peak of $V_0 \cos\omega_0 t$

$I_{DC}(C_1) = 0 \Rightarrow I_0 = I_{bias}$

→ fundamental component of i_d is

$$\hat{I}_1 = \frac{2}{T}\int_0^T i_d(t)\cos\omega_0 t\, dt$$

$$\approx \frac{2}{T}\int_0^T i_d(t)\, dt \quad \left\{\begin{matrix}\text{current exists} \\ \text{only @ peak } V_a\end{matrix}\right\}$$

$$\boxed{\hat{I}_1 = 2 I_{bias}}$$

* If i_d flows through a tank tuned to ω_0, only $\hat{I}_1$ creates a voltage

$$\boxed{G_m \approx \frac{\hat{I}_1}{V_0} = \frac{2 I_{bias}}{V_0}}$$

← applicable to any device

← assume V_s is constant (due to C_1)

Recall that

$$g_m(\text{long channel}) = \frac{2 I_{bias}}{V_{DSAT}}$$

short-channel:

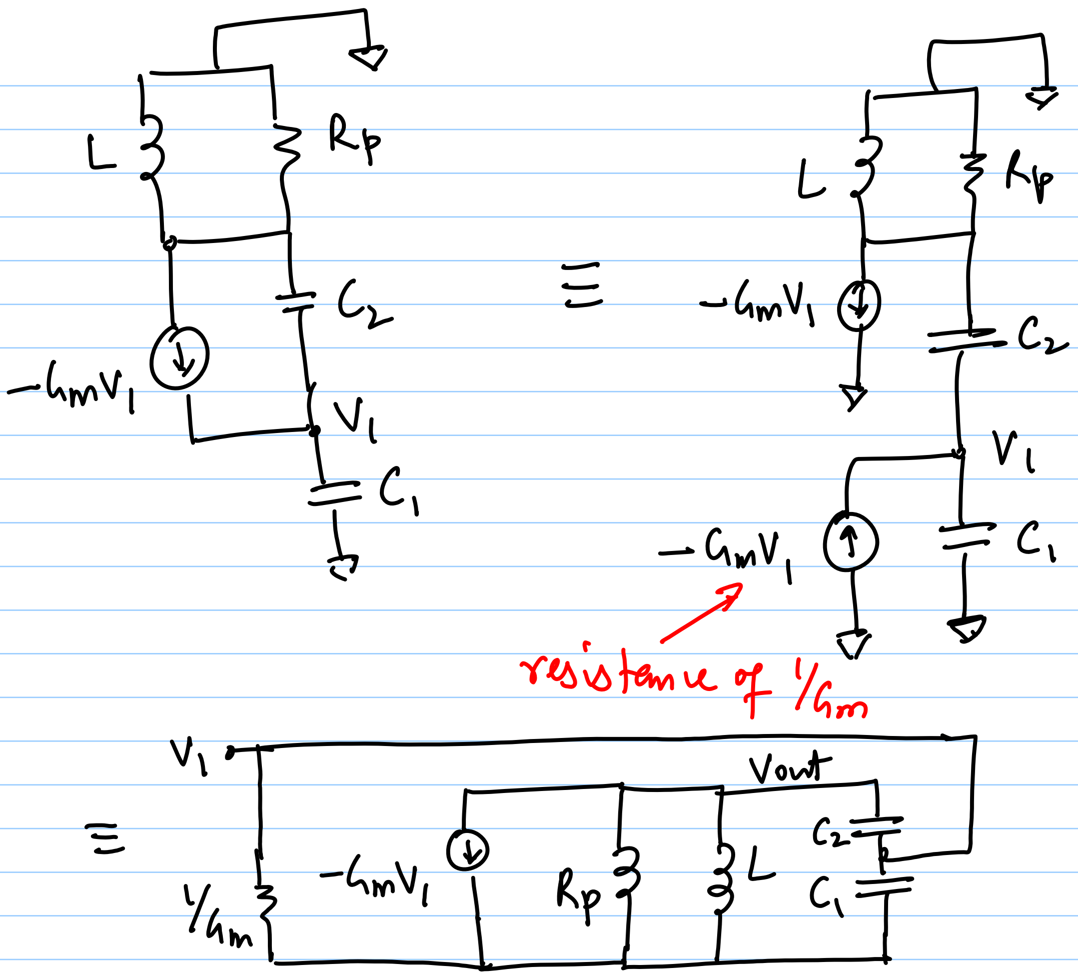
$$I_D = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right) (V_{GS} - V_T) \cdot L \, \mathcal{E}_c \qquad \text{(vel. sat.)}$$

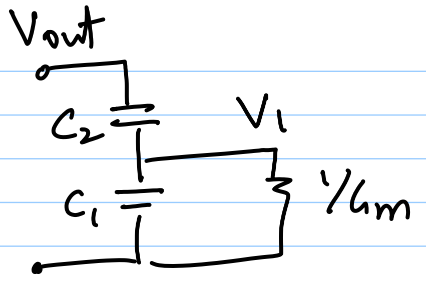
$$\Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{I_{bias}}{V_{DSAT}}$$

$\Rightarrow$ in general,

$$\frac{I_{bias}}{V_{DSAT}} \leq g_m \leq \frac{2 I_{bias}}{V_{DSAT}}$$

$$\Rightarrow \boxed{\frac{V_{DSAT}}{V_0} \leq \frac{G_m}{g_m} \leq \frac{2 V_{DSAT}}{V_0}}$$



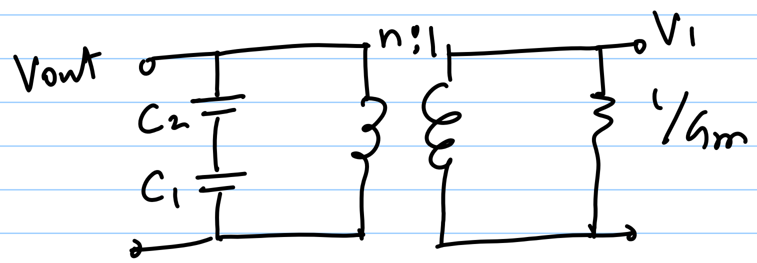


assume 1/Gm does not load the cap divider (i.e. Q >> 1)

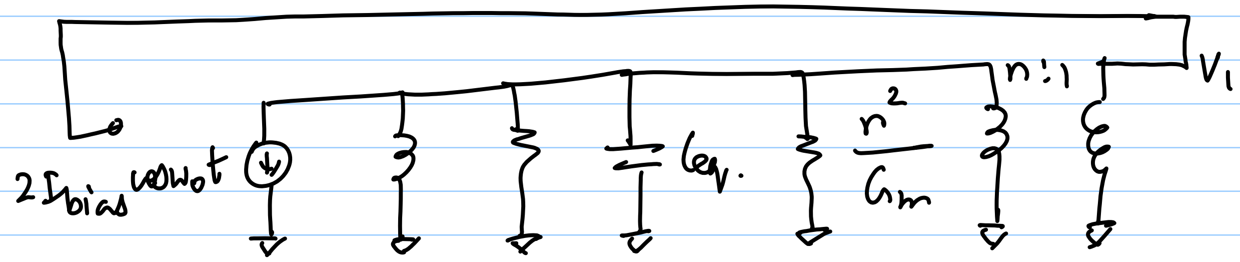
$$V_1 = \frac{C_2}{C_1 + C_2} \cdot V_{out} = \frac{1}{n} \cdot V_{out}$$

$$n = \frac{C_1 + C_2}{C_2} = 1 + \frac{C_1}{C_2}$$ ← equivalent turns ratio

equivalent ckt:



overall oscillator ckt becomes:



$$C_{eq.} = \frac{C_1 C_2}{C_1 + C_2}$$

resonance: set $\omega_0 = \dfrac{1}{\sqrt{L \cdot C_{eq.}}}$

$$V_{out} = 2 I_{bias} \cdot R_p \,||\, \frac{n^2}{G_m}$$

$$= 2 I_{bias} \cdot \frac{R_p \cdot n^2/G_m}{R_p + n^2/G_m}$$

$$= \frac{2 I_{bias} R_p}{1 + \frac{R_p G_m}{n^2}}$$

$$\Rightarrow \quad V_{out}\left[1 + \frac{R_p G_m}{n^2}\right] = 2 I_{bias} R_p$$

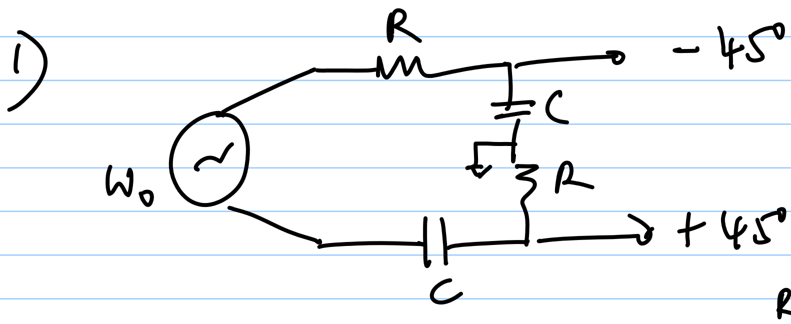
$$V_{out}\left[1 + \frac{R_p}{n^2} \cdot \frac{2 I_{bias}}{V_{out}/n}\right] = 2 I_{bias} R_p$$

$$\Rightarrow \quad V_{out} + \frac{2 I_{bias} R_p}{n} = 2 I_{bias} R_p$$

$$\boxed{V_{out} = 2 I_{bias} R_p (1 - 1/n)}$$

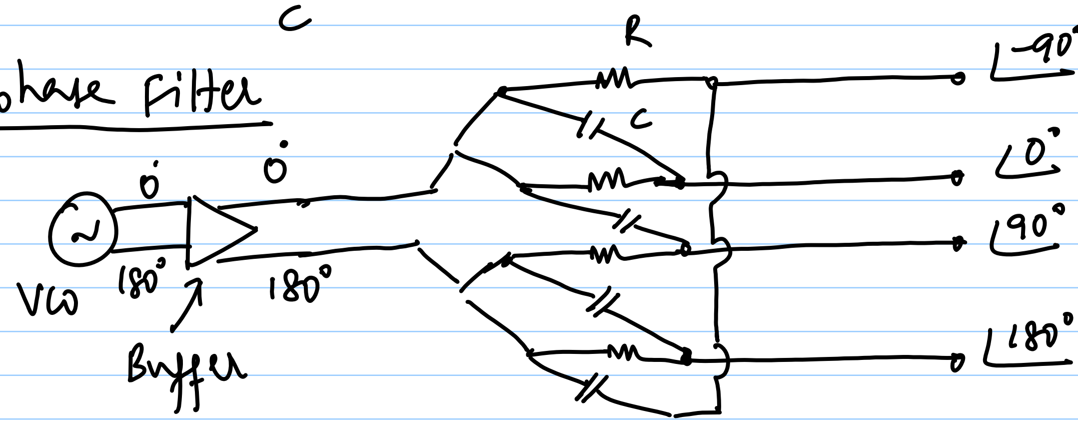
* For startup, use small-signal g_m

Quadrature Signal Generation

1) 

we discussed this before

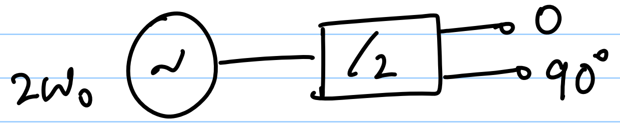
Polyphase Filter

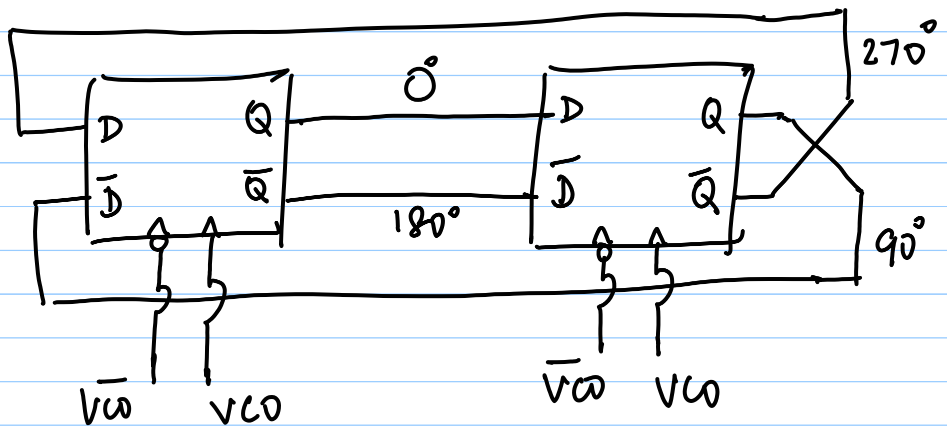


these drive more buffers

* Buffers consume extra power
* R, C mismatches affect phase error

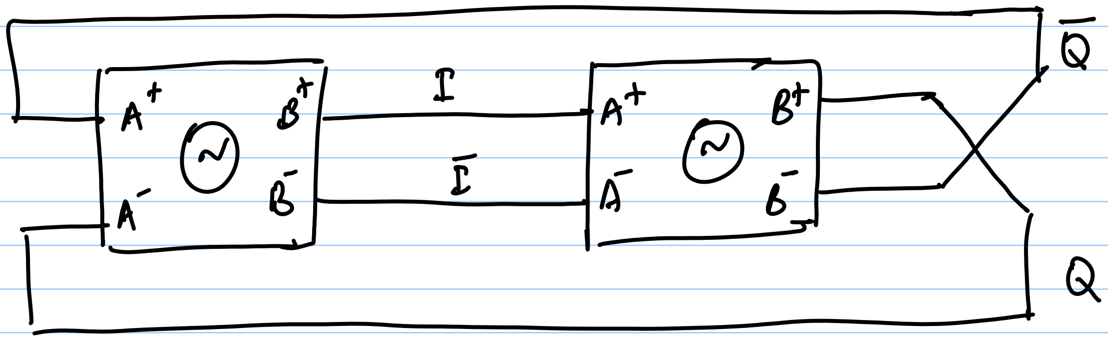
2)





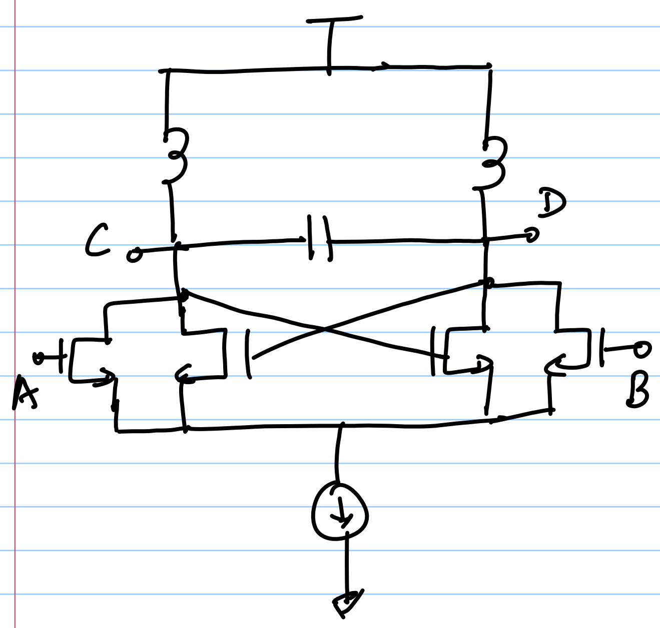
* Basically a synchronous counter (that counts to 2)
* power consumption @ high frequencies ↑

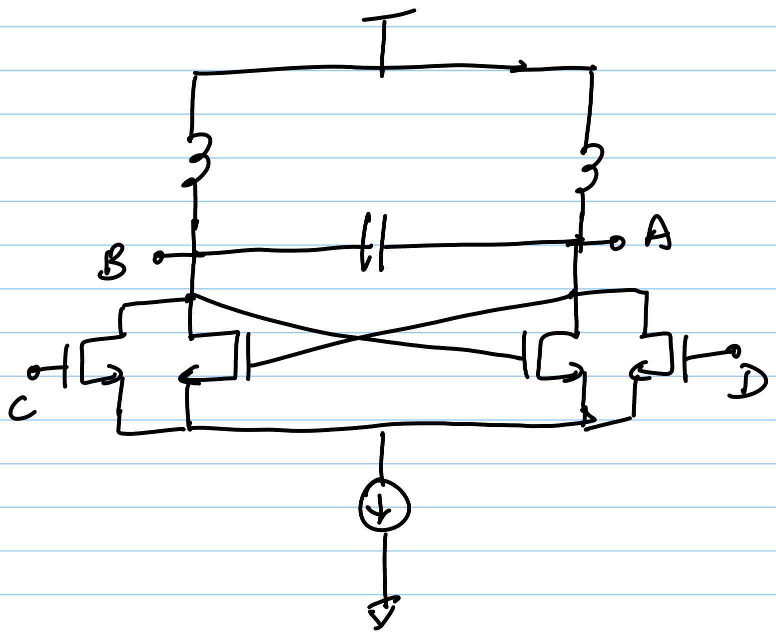
3) Quadrature VCOs
* Couple 2 identical oscillators in quadrature



* A - inputs (coupling)
B - oscillator output

a) parallel coupling

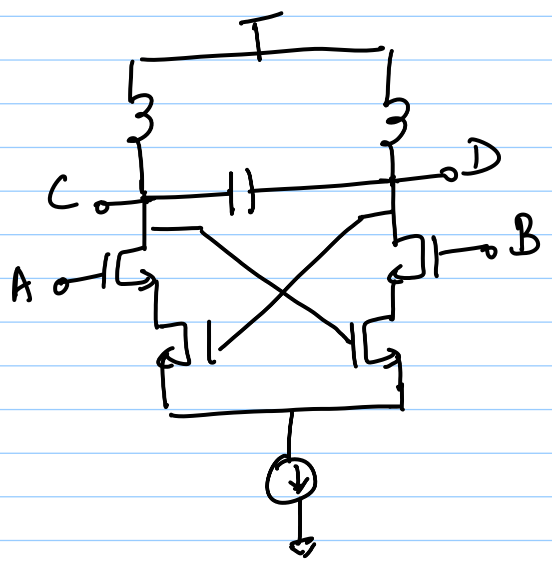


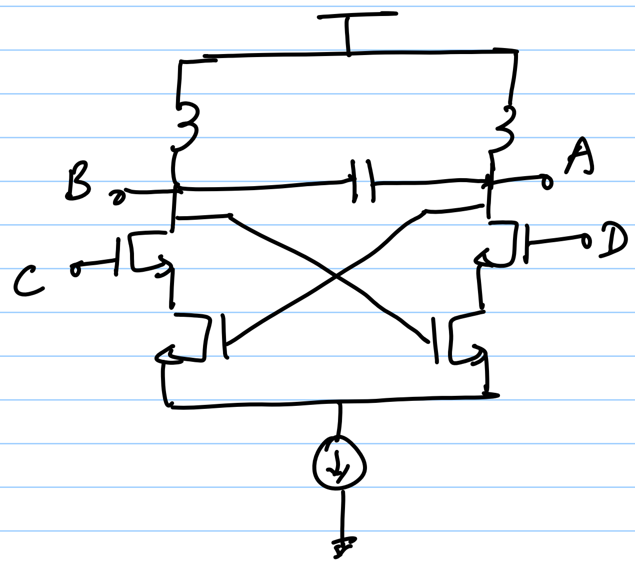


* Exercise: Analyse the ckt with waveforms
→ assume outputs @ 0° & 180° phases and

see what happens (both cases will result in no oscillations – i.e. any 0°/180° components will die out)

b) Series coupling



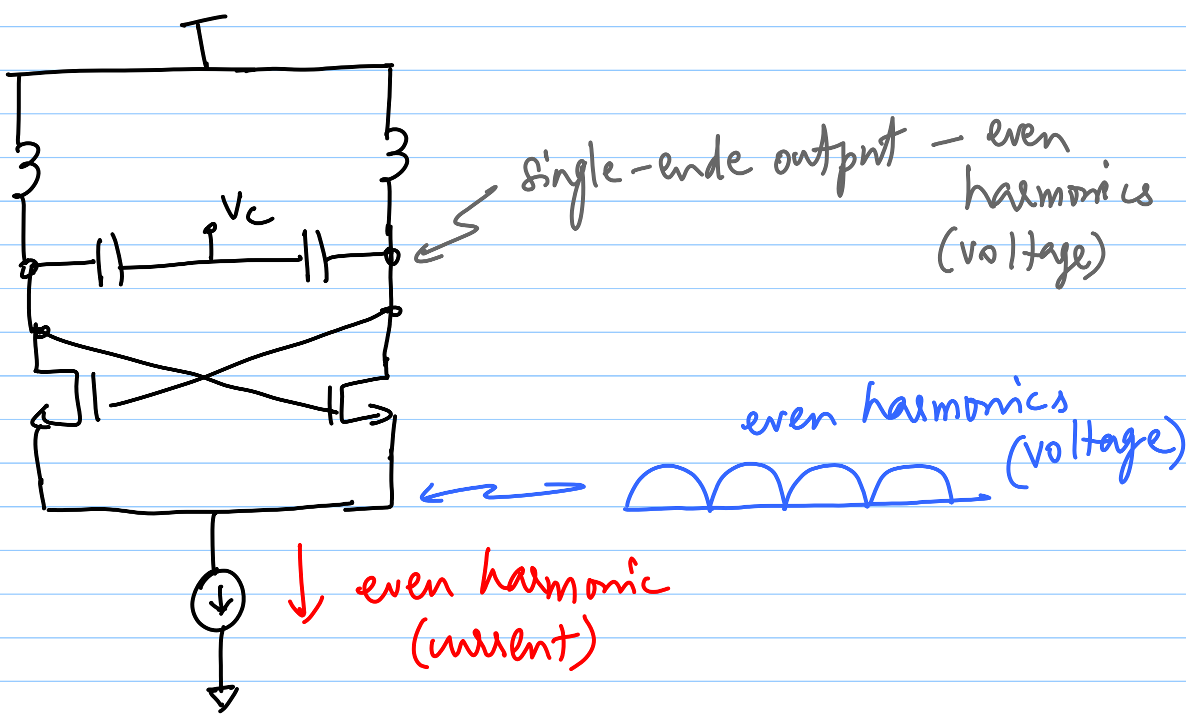


* (a) & (b) : extra parasitic cap on tank (↓ tuning range)
* (b) : quadrature coupling devices have to be much larger than cross-coupled devices (to increase headroom for cc devices)

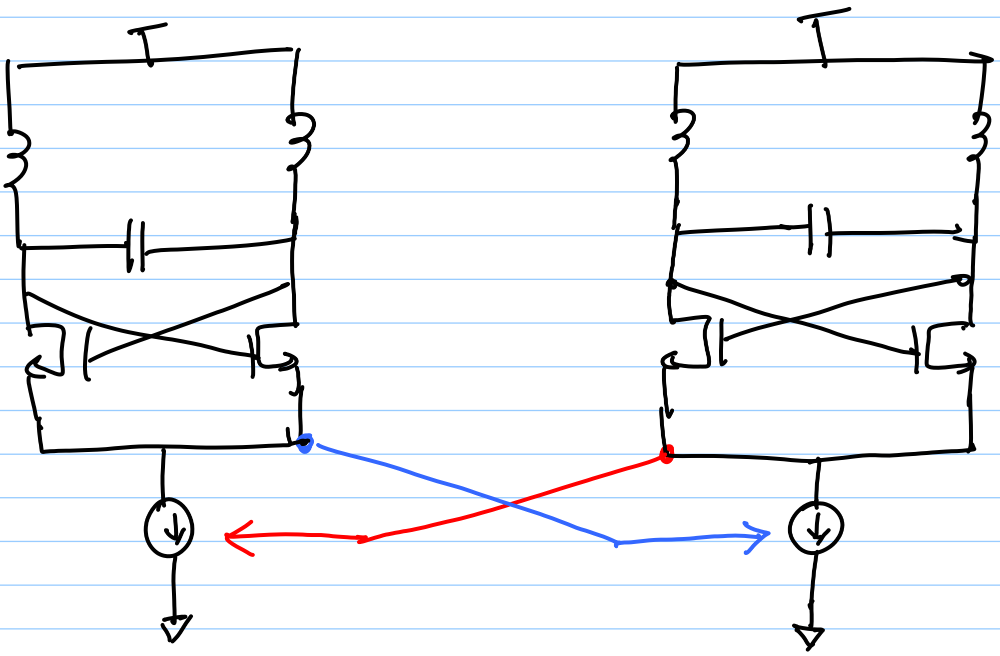
(c) <u>Harmonic injection</u>

* Single-ended outputs, Common-mode nodes, common-mode paths } even harmonics exist in voltages and/or currents

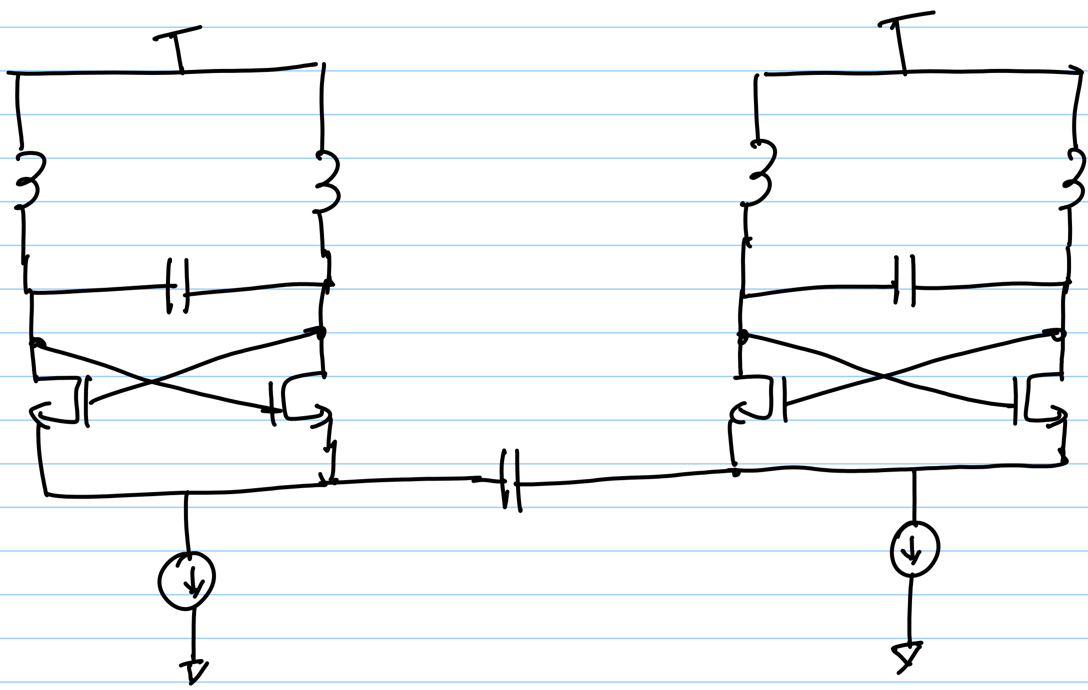
* Harmonics have specific phase relationship with fundamental freq.



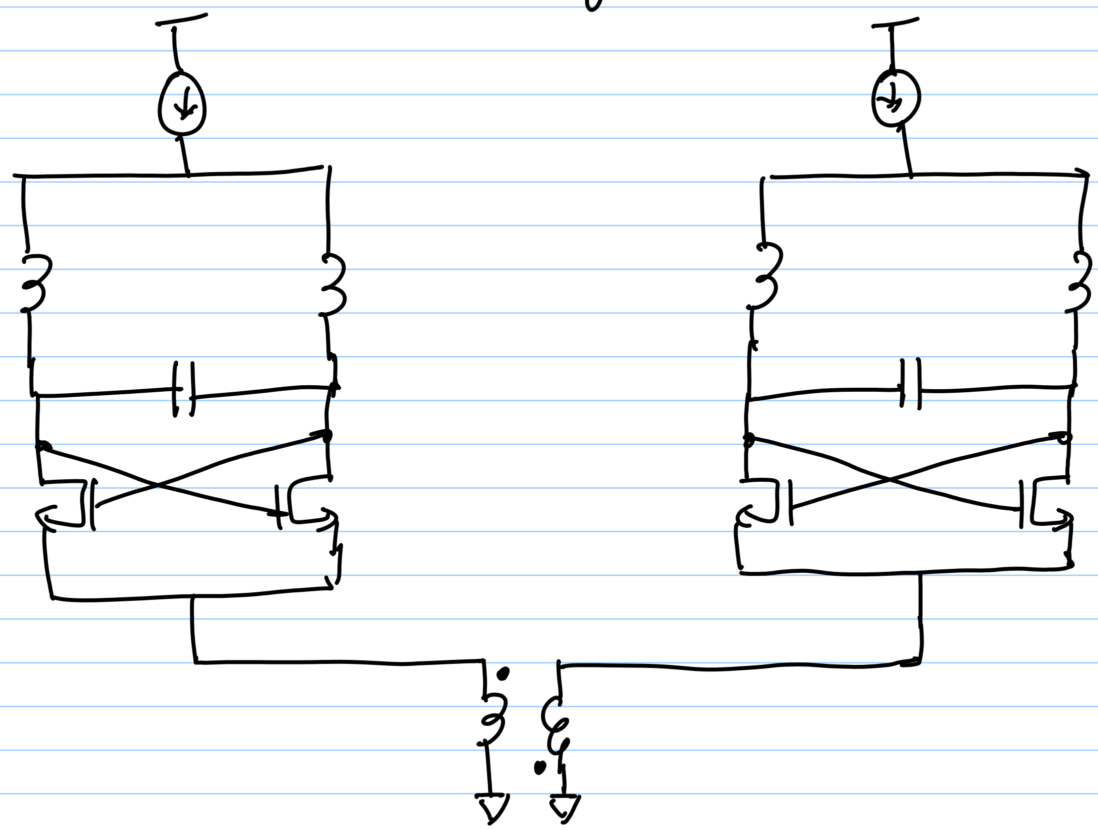
(i) C-S. coupling



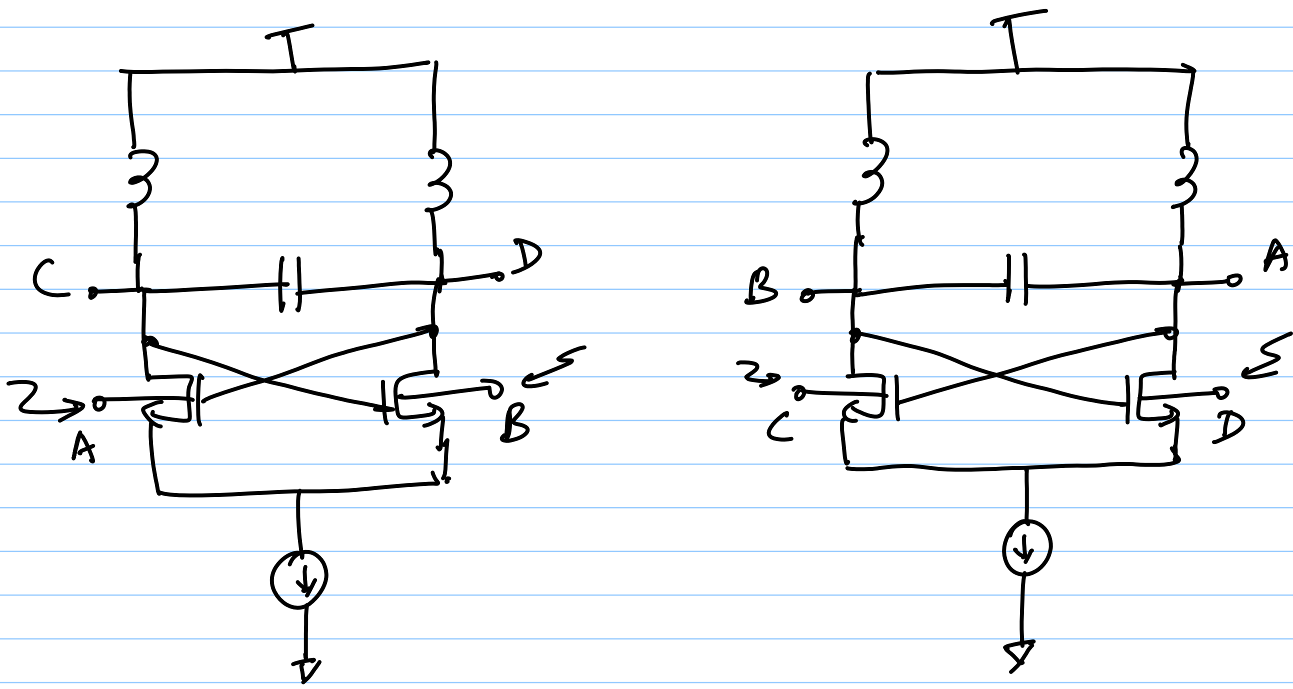
(ii) Capacitive coupling



(iii) Transformer coupling

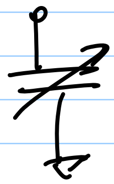


(iv) Backgate coupling

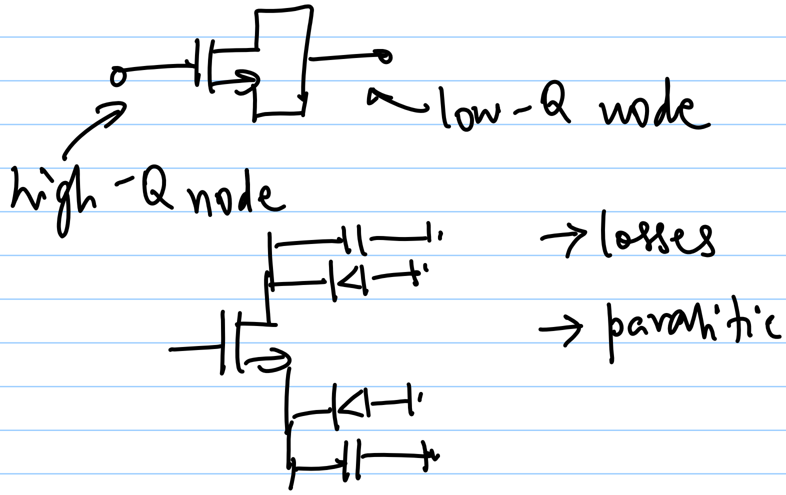


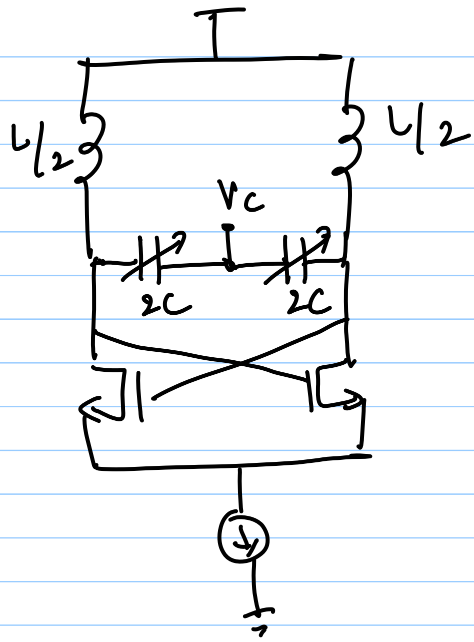
Other features of the VCO

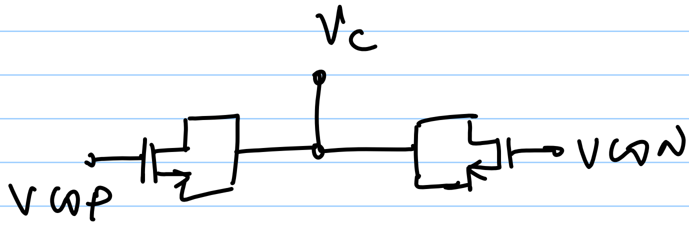
1) tuning & tuning range

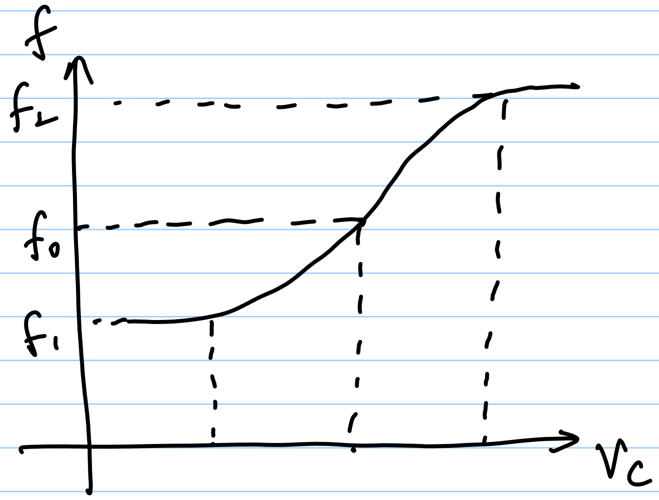


varactor is a 2-terminal device









$$\left.\begin{array}{l}\text{tuning}\\ \text{range}\end{array}\right\} = \frac{f_2 - f_1}{f_0}$$

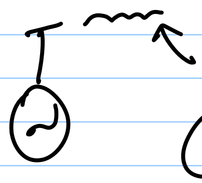
2) centre frequency $f_0 = \dfrac{f_1+f_2}{2}$

3) VCO gain

$$K_{VCO} = \frac{f_2-f_1}{\Delta V_c} \quad \left\{ \text{typically MHz/V} \right\}$$

4) Power consumption

5) Supply pushing / rejection



noise on supply can show up @ VCO output (around VCO freq.) as phase noise or spurs

6) Phase noise