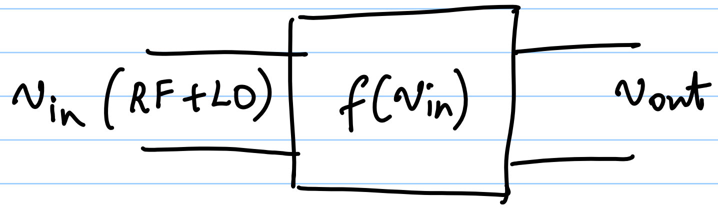
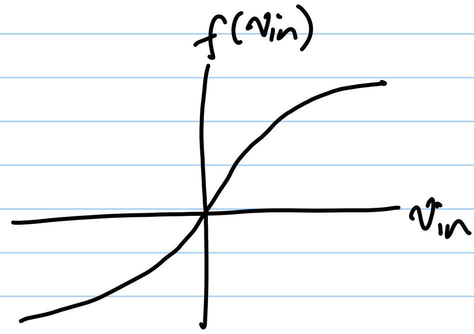
Lecture 20: Two- & Three-port Mixers





"soft nonlinearity"

$$v_{out} = \sum_{n=0}^{N} c_n (v_{in})^n$$

output contains:

* DC terms – from even order terms
  – use AC coupling to filter these
* harmonics of inputs – $m\omega_{LO}$ & $m\omega_{RF}$ ($m = 1$ to N)
  – can be filtered as these are $\gg \omega_{IF}$ (Rx Mixer)
* IM terms – $p\omega_{RF} \pm q\omega_{LO}$

  $p, q > 0$ & $p+q = 2, 3 \ldots N$

IM_2: $\omega_{RF} \pm \omega_{LO}$ (i.e. $p=q=1$) = desired output

$\Rightarrow$ square-law (quadratic) behavior

IM_3: $\left.\begin{array}{l} 2\omega_{RF} \pm \omega_{LO} \\ \omega_{RF} \pm 2\omega_{LO} \end{array}\right\}$ $p+q=3$ undesired terms

Square-law mixer

let $c_i = 0$ for $i \neq 1, 2$

$$v_{out} = c_1 v_{in} + c_2 v_{in}^2$$

$$v_{in} = v_{RF} \cos\omega_{RF}t + v_{LO}\cos\omega_{LO}t$$

(A) fund. terms: $c_1 [v_{RF}\cos\omega_{RF}t + v_{LO}\cos\omega_{LO}t]$

(B) square terms: $C_2\{[v_{RF}\cos\omega_{RF}t]^2 + [v_{LO}\cos\omega_{LO}t]^2\}$

(C) cross terms: $2C_2 v_{RF} v_{LO}[\cos\omega_{RF}t]\cdot[\cos\omega_{LO}t]$

(A) – scaled versions of input

(B) – rewrite $(\cos\omega t)^2$ as $\frac{1}{2}(1+\cos 2\omega t)$

$\Rightarrow$ DC & 2^{nd} harmonic terms

not useful, remove by filtering

rewrite (C): $C_2 v_{RF} v_{LO}[\cos(\omega_{RF}-\omega_{LO})t + \cos(\omega_{RF}+\omega_{LO})t]$

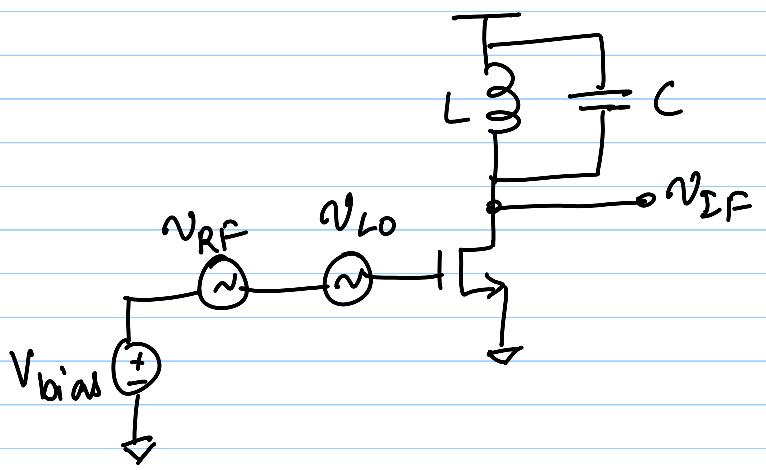
* If v_{LO} is fixed,

$A_{IF} \propto A_{RF} \Rightarrow$ linear mixing

$$G_c = \frac{C_2 v_{RF} v_{LO}}{v_{RF}} = C_2 \cdot v_{LO}$$

* Long-channel MOSFETs can serve as good square-law mixers

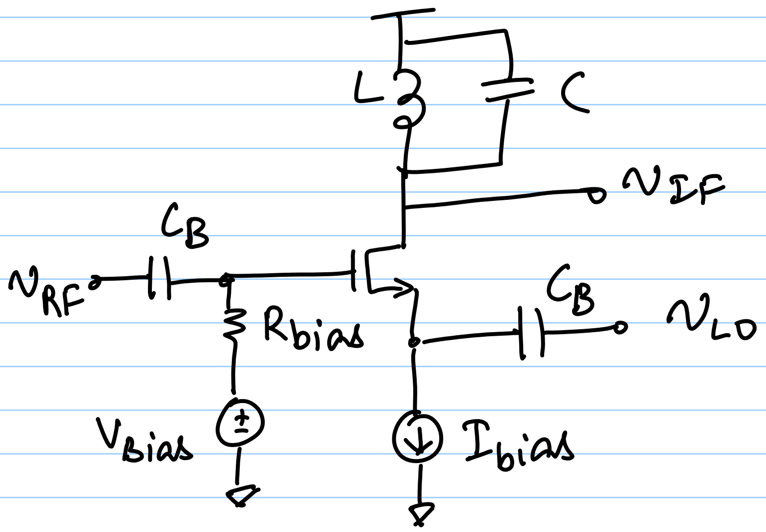
<u>example 1</u>



* summation thru' resistive/reactive networks

* poor isolation between LO & RF

example 2



* $v_{gs} = v_{RF} - v_{LO}$
* R_{Bias} = large enough so that a) min. loading b) min. noise
* Assume V_{Bias}, I_{bias} & L are chosen such that:

$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS}-V_T)^2$$ is valid

* short-channel devices are inferior
* V_{Bias}, I_{bias} chosen to avoid sub-threshold operation (exponential)

$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[(V_{Bias}-V_T)+(v_{RF}\cos\omega_{RF}t - v_{LO}\cos\omega_{LO}t)\right]^2$$

Expand into 3 terms:

(i): $\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[V_{Bias}-V_T\right]^2 \Rightarrow$ DC bias term

(ii): $\underbrace{\frac{1}{\cancel{2}}\mu C_{ox}\frac{W}{L}\cdot\cancel{2}(V_{Bias}-V_T)}_{g_m}\cdot(v_{RF}\cos\omega_{RF}t - v_{LO}\cos\omega_{LO}t)$

$= g_m(v_{RF}\cos\omega_{RF}t - v_{LO}\cos\omega_{LO}t)$

$\Rightarrow$ fundamental gain terms

(iii): $\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[v_{RF}\cos\omega_{RF}t - v_{LO}\cos\omega_{LO}t\right]^2$

$= \frac{1}{2}\mu C_{ox}\frac{W}{L}\Big[\underset{A}{v_{RF}^2\cos^2\omega_{RF}t} + \underset{B}{v_{LO}^2\cos^2\omega_{LO}t}$

$\underset{C}{-2v_{RF}v_{LO}\cos\omega_{RF}t\cdot\cos\omega_{LO}t}\Big]$

Ⓐ : $\frac{v_{RF}^2}{2}(1+\cos 2\omega_{RF}t)$

DC term   2nd harmonic term

Ⓑ : $\frac{v_{LO}^2}{2}(1+\cos 2\omega_{LO}t)$

Ⓒ : $\frac{1}{2}\mu C_{ox}\frac{W}{L}\cdot v_{RF}v_{LO}\cdot\left[\cos(\omega_{RF}-\omega_{LO})t + \cos(\omega_{RF}+\omega_{LO})t\right]$

mixing terms

conversion gain

$$G_c = \frac{\frac{1}{2}\mu C_{ox}\frac{W}{L}\cdot v_{RF}v_{LO}}{v_{RF}}$$

$$= \frac{1}{2}\mu C_{ox}\frac{W}{L}\cdot v_{LO} = C_2\cdot v_{LO}$$

G_c is

* independent of bias
* temp. dependent (through μ)
* dependent on v_{LO}

Bipolar mixer (perfect square-law is not necessary)

$$i_c = I_S \exp(V_{BE}/V_T)$$

can be expanded as:

$$i_c = I_c\left[1+\frac{v_{in}}{V_T}+\frac{1}{2}\left(\frac{v_{in}}{V_T}\right)^2+\dots\right]$$

$$C_2 = \frac{1}{2}\frac{I_c}{(V_T)^2} = \frac{g_m}{2V_T}$$

$$\Rightarrow G_c = C_2 v_{LO} = g_m\cdot\frac{v_{LO}}{2V_T}$$

* Rewrite $G_{C_{MOS}}$ as :

$$G_{C_{MOS}} = \underbrace{\frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{BIAS}-V_T)}_{g_m} \cdot \frac{v_{LO}}{(V_{BIAS}-V_T)}$$

$$= g_m \frac{v_{LO}}{2V_{DSAT}}$$

$$\frac{G_{C_{BJT}}}{G_{C_{MOS}}} = g_m(BJT)\cdot\frac{v_{LO}}{2V_T}\cdot\frac{2V_{D,SAT}}{g_m(MOS)\cdot v_{LO}}$$

$$= \frac{g_m(BJT)}{g_m(MOS)}\cdot\frac{V_{D,SAT}}{V_T}$$

⇒ For same g_m, BJT has higher G_c

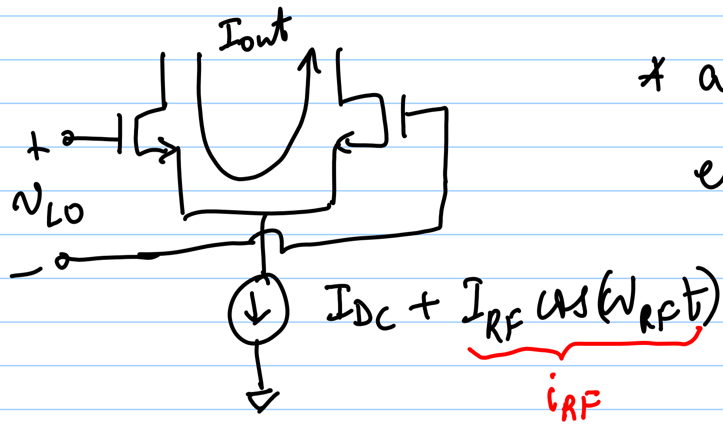
Issues with 2-port mixers :
→ No isolation between LO & RF (2-port)
→ generation of undesired spurs (multiplication is side-effect)

<u>Multiplier-based mixers</u>

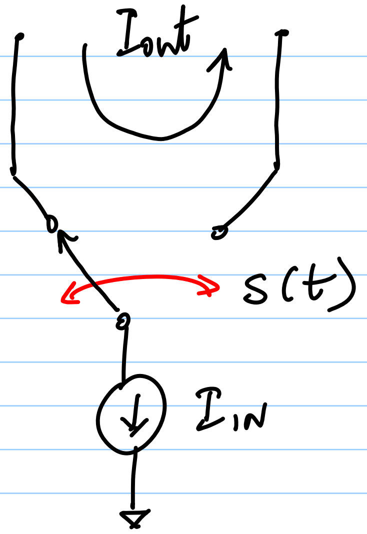
* ideally generate only desired IM product
* 3-port mixers ⇒ good isolation between RF, LO & IF
* CMOS – excellent switches

<u>Single-balanced mixer</u>

Based on "Gilbert multiplier" topology

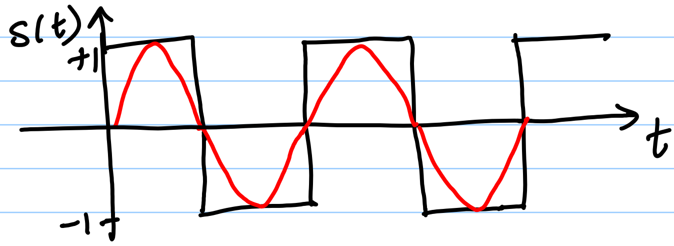


* assume v_{LO} is large enough to switch currents completely between two sides.



$$I_{out} = I_{IN} \times s(t)$$

$$s(t) = \text{sgn}\left(\sin \omega_{LO} t\right)$$

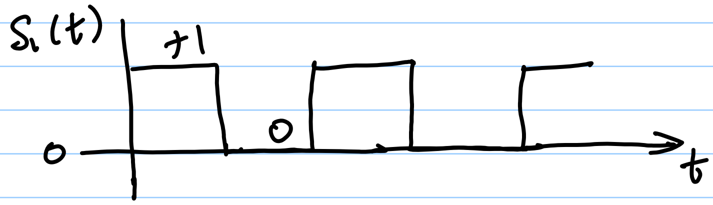


$$\text{sgn}(x) = \text{"signum" or sign function}$$

$$= \begin{cases} -1 & \forall\ x < 0 \\ 0 & x = 0 \\ +1 & \forall\ x > 0 \end{cases}$$

Note: Single-ended o/p current $\{I_{out}^{+} \text{ or } I_{out}^{-}\}$ is given by $I_{IN} \times S_1(t)$ where

$$S_1(t) = 0.5 + 0.5\,s(t)$$

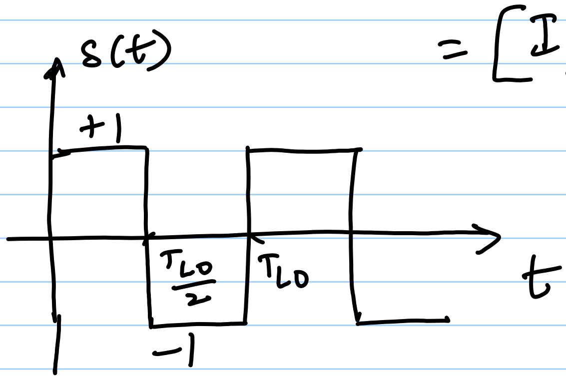


$$I_{out}^{+} = (I_{DC} + i_{RF}) \times \{0.5 + 0.5\,s(t)\}$$

direct feed through

Differential current $I_{out}(t)$

$$= \left[I_{DC} + I_{RF} \cos \omega_{RF} t\right] \cdot \text{sgn}\left[\sin \omega_{LO} t\right]$$



$$\omega_{LO} = \frac{2\pi}{T_{LO}} = 2\pi f_{LO}$$

Fourier series of $s(t)$:

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_{LO}t) + b_n \sin(n\omega_{LO}t)\right]$$

$$\frac{a_0}{2} = 0 \quad \text{(DC average)}$$

$$a_n = \frac{2}{T_{LO}} \int_0^{T_{LO}} \mathrm{sgn}(t) \cos(n\omega_{LO}t)\, dt = 0 \;\forall n \quad \left\{ \begin{matrix} \mathrm{sgn}(t) \\ \text{is odd} \end{matrix} \right\}$$

$$b_n = \frac{2}{T_{LO}} \int_0^{T_{LO}} \mathrm{sgn}(t) \sin(n\omega_{LO}t)\, dt$$

$$= 2 \cdot \frac{2}{T_{LO}} \int_0^{T_{LO}/2} \sin(n\omega_{LO}t)\, dt$$

$$b_n = \frac{4}{T_{LO}} \cdot \frac{1}{n\omega_{LO}} \left[-\cos(n\omega_{LO}t)\right]\Big|_0^{T_{LO}/2}$$

$$= \frac{2}{n\pi}\left[1 - \cos n\pi\right] \mathbin{/\!/}$$

$$\therefore \; b_n = \begin{cases} 0 & \forall \text{ even } n \\ \frac{4}{n\pi} & \text{for odd } n \end{cases}$$

$$\therefore \; s(t) = \frac{4}{\pi}\left[\sin \omega_{LO}t + \frac{1}{3}\sin 3\omega_{LO}t + \frac{1}{5}\sin 5\omega_{LO}t + \cdots\right]$$

$$i_{out}(t) = \left[I_{DC} + I_{RF}\cos\omega_{RF}t\right] \cdot s(t)$$

$$= \underbrace{I_{DC} \cdot s(t)}_{\text{I}} + \underbrace{I_{RF}\cos\omega_{RF}t \cdot s(t)}_{\text{II}}$$

(I) $\Rightarrow \dfrac{4 I_{DC}}{\pi} \cdot \left[\sin \omega_{LO} t + \underbrace{\dfrac{1}{3} \sin 3\omega_{LO} t + \cdots}_{\text{feed through of LO harmonics}} \right]$

"LO feed through"

(II) $\Rightarrow \dfrac{4 I_{RF}}{\pi} \cdot \left[\cos \omega_{RF} t \sin \omega_{LO} t + \dfrac{1}{3} \cos \omega_{RF} t \sin 3\omega_{LO} t + \cdots \right]$

$$= \frac{4 I_{RF}}{\pi} \cdot \frac{1}{2} \left[\sin(\omega_{LO} - \omega_{RF}) t + \sin(\omega_{LO} + \omega_{RF}) t + \frac{1}{3} \sin(3\omega_{LO} - \omega_{RF}) t + \frac{1}{3} \sin(3\omega_{LO} + \omega_{RF}) t + \cdots \right]$$

$$= \frac{2}{\pi} I_{RF} \left[\underbrace{\sin(\omega_{LO} - \omega_{RF}) t}_{\text{lower sideband}} + \underbrace{\sin(\omega_{LO} + \omega_{RF}) t}_{\text{upper sideband}} + \text{higher order terms} \right]$$

desired term

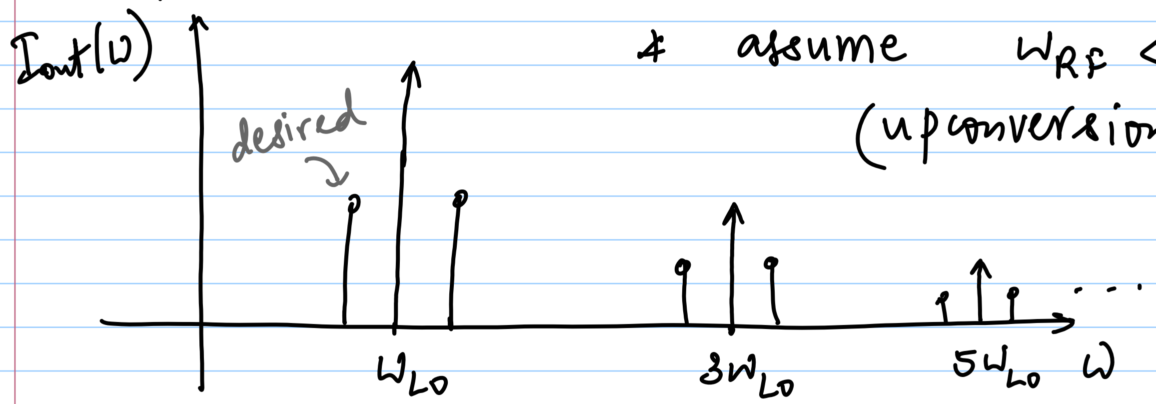
$$\therefore \; i_{out}(t) = \frac{2}{\pi} I_{RF} \left[\sin(\omega_{LO} - \omega_{RF}) t + \sin(\omega_{LO} + \omega_{RF}) t \right]$$
$$+ \left[\text{higher order mixing terms} \right]$$
$$+ \left[\text{LO \& LO harmonic feed through terms} \right]$$

can be filtered out

close to desired term in case of Tx
- cannot be filtered out

* square wave has only odd harmonics of fundamental [ω_{LO}, $3\omega_{LO}$, $5\omega_{LO}$ ...]

* assume $\omega_{RF} << \omega_{LO}$ (upconversion mixer)



* poor isolation between LO & output ports
* input RF is required to be a current
  → V-I converters are usually imperfect
  → more of a problem for down-converters

* no RF feedthrough assumes perfect M_2-M_3 matching
* Immediate switching requires v_{LO} waveform zero crossings to coincide, otherwise
  → LO diff. pair simultaneously ON
  → RF current is "wasted" as common-mode signal ($G_c \downarrow$, $NF \uparrow$)