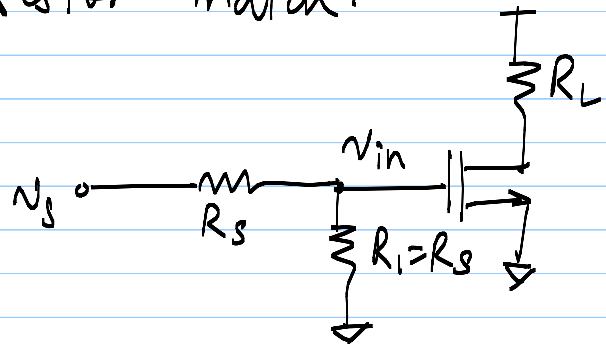
Lecture 17 : LNA design

Input Matching :

1) Resistor match :



* Broadband match
* Attenuates signal before amplifier ($v_{in} = \frac{v_s}{2}$)
* R_1 adds its thermal noise
* F can be calculated to be

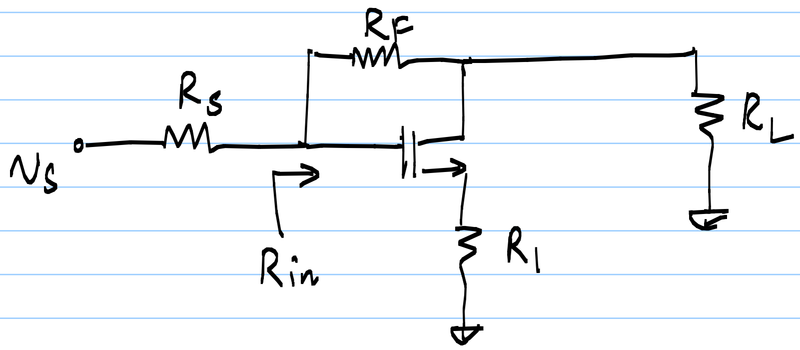
$$F = 2 + \frac{4\gamma}{\alpha} \cdot \frac{1}{g_m R_s}$$

{ derivation will be a HW problem }

→ ignores gate noise

→ $NF = 10 \log_{10} (2 + x) > 3dB$!

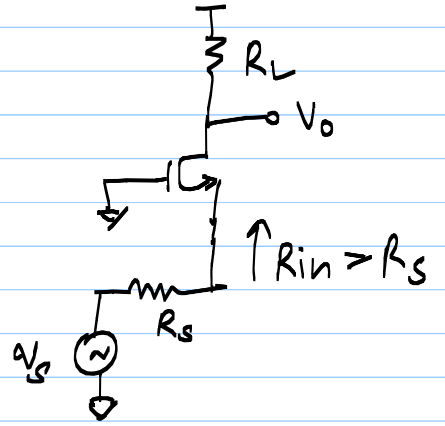
2) Shunt-series amplifier



(biasing not shown)

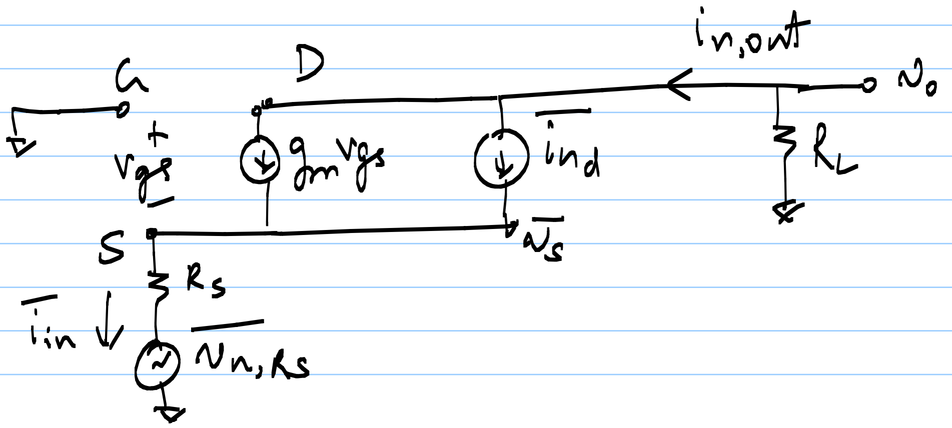
* No attenuation before amplifier
* R_F generates thermal noise (degrades F)
* F still much larger than F_{min}.

3) Common-gate stage



* Broadband match
* $R_{in} = \frac{1}{g_m}$ (neglecting r_{ds}) $= 50\Omega$
  → device size & bias current
* No extra noise sources

Noise factor



* $\overline{i_{nd}}$ and $\overline{v_{n_{Rs}}}$ are uncorrelated
  → analyze $\overline{i_{out}}$ for each separately and add <u>mean-square values</u>

1) $\overline{v_{n,Rs}}$:

$$\overline{i_{n,out}} = g_m v_{gs} = -g_m \overline{v_s}$$

also, $\overline{v_s} = \overline{v_{n,R_s}} + \overline{i_{in}} R_s$

$$= \overline{v_{n_{Rs}}} + \overline{i_{n,out}} \cdot R_s$$

$$\frac{-\overline{i_{n,out}}}{g_m} = \overline{v_{n_{Rs}}} + \overline{i_{n,out}} R_s$$

$$\Rightarrow \overline{i_{n,out}} = \frac{-g_m \overline{v_{n_{Rs}}}}{1+g_m R_s}$$

we know that

$$R_{in} = 1/g_m = R_S$$

$$\Rightarrow \quad \overline{i_{n,out}} = \frac{-\overline{v_{n,R_S}}}{2R_S} \Rightarrow \overline{i^2_{n,out}} = \frac{\overline{v^2_{n,R_S}}}{4R_S^2}$$

2) $\overline{i_{n,d}}$:

$$\overline{v_s} = \overline{i_{n,out}} \cdot R_S$$

$$\overline{i_{n,out}} = -g_m \overline{v_s} + \overline{i_{nd}}$$

$$= -g_m R_S \overline{i_{n,out}} + \overline{i_{nd}}$$

$$\Rightarrow \overline{i_{n,out}} = \frac{\overline{i_{nd}}}{1+g_m R_S} = \frac{\overline{i_{nd}}}{2}$$

$$\Rightarrow \overline{i^2_{n,out}} = \frac{\overline{i^2_{nd}}}{4}$$

$$\Rightarrow \overline{i^2_{n\,out,\,tot.}} = \frac{\overline{v^2_{n,R_S}}}{4R_S^2} + \frac{\overline{i^2_{nd}}}{4}$$

$$\therefore \quad F = \frac{\frac{\overline{v^2_{R_S}}}{4R_S^2} + \frac{\overline{i^2_{nd}}}{4}}{\frac{\overline{v^2_{R_S}}}{4R_S^2}} = 1 + \frac{R_S^2 \cdot \overline{i^2_{nd}}}{\overline{v^2_{n,R_S}}}$$

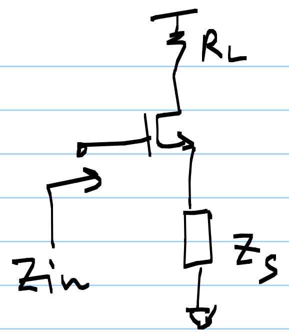
$$= 1 + R_S^2 \cdot \frac{4kT\gamma g_{d0}\Delta f}{4kTR_S\Delta f}$$

$$= 1 + \gamma g_{d0} R_S = 1 + \gamma \frac{g_{d0}}{g_m}$$

$$\Rightarrow \boxed{F = 1 + \frac{\gamma}{\alpha}}$$

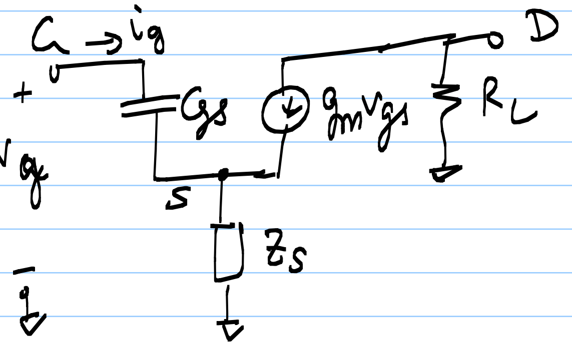
for long-channel case, $F = 1.67 \Rightarrow NF_{min.} = 2.2\,dB$

4) Resistive input without resistors



* assume transistor model with only C_{gs} & g_m

$\Downarrow$



$$\frac{v_g - v_{gs}}{Z_s} = g_m v_{gs} + i_g$$

$$\Rightarrow v_g = i_g Z_s + (1 + g_m Z_s) v_{gs}$$

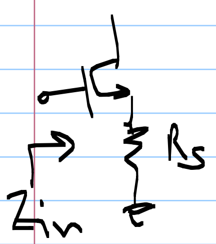
also $v_{gs} = i_g / sC_{gs}$

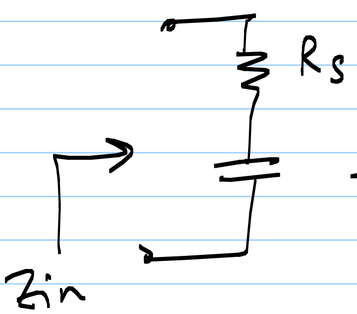
$$\Rightarrow v_g = i_g Z_s + (1 + g_m Z_s) \cdot \frac{i_g}{sC_{gs}}$$

$$\Rightarrow Z_{in} = \frac{v_g}{i_g} = Z_s + \frac{1}{sC_{gs}} + \frac{g_m Z_s}{sC_{gs}}$$

a) $Z_s = R_s$

$$\Rightarrow Z_{in} = R_s + \frac{1 + g_m R_s}{sC_{gs}}$$



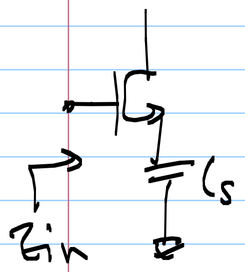


$$\frac{C_{gs}}{1 + g_m R_s} \approx \frac{1}{\omega_T R_s}$$

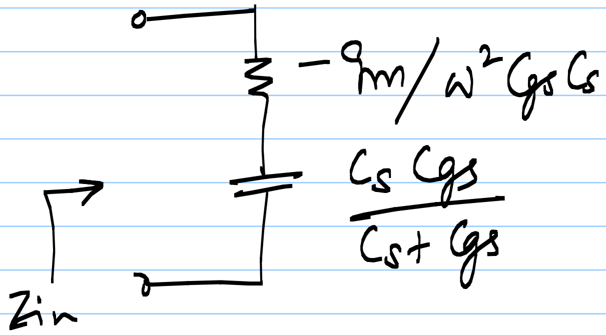
b) $Z_s = \dfrac{1}{sC_s}$

$$\Rightarrow Z_{in} = \frac{1}{sC_s} + \frac{1}{sC_{gs}} + \frac{g_m}{s^2 C_{gs} C_s}$$

$$\Rightarrow Z_{in}(j\omega) = \frac{1}{j\omega}\left(\frac{C_s + C_{gs}}{C_s \cdot C_{gs}}\right) - \frac{g_m}{\omega^2 C_{gs} C_s}$$



$\Rightarrow$



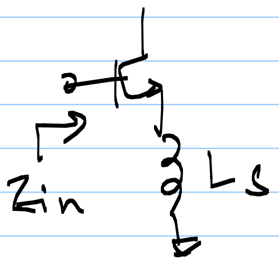
← negative resistance!

* -ve resistance useful in oscillators

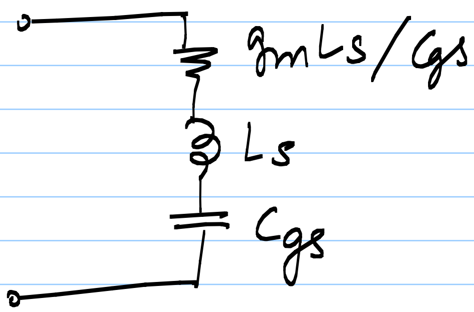
* beware of parasitic source cap. in amplifiers – can cause instability/oscillations

c) $Z_{in} = sL_s$

$$\Rightarrow Z_{in} = sL_s + \frac{1}{sC_{gs}} + \frac{g_m L_s}{C_{gs}}$$



$\Rightarrow$



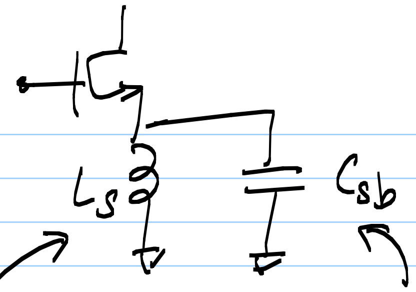
← equivalent resistance with no physical R

* Low-noise (no physical resistor)

* power match : set $\frac{g_m L_s}{C_{gs}} = 50\,\Omega$

* $\frac{g_m L_s}{C_{gs}} = \omega_T L_s$

* **Beware**:



positive resistance @ gate

negative resistance @ gate

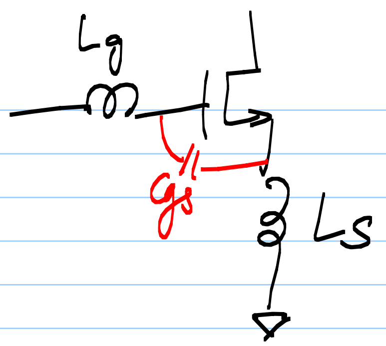
* Set $\omega_T L_s = 50\,\Omega$

$$f_o = \frac{1}{2\pi\sqrt{L_s C_{gs}}}$$

{ Z_{in} is purely real around resonant freq. f_o }

* We want another degree of freedom - to set f_o to be equal to operating freq. (lower)

→ add a series inductor L_g at the gate



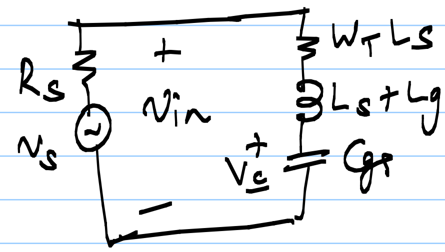
$$Z_{in} = \omega_T L_s + j\omega(L_s + L_g) + \frac{1}{j\omega C_{gs}}$$

$$\Rightarrow f_o = \frac{1}{\sqrt{C_{gs}(L_g + L_s)}}$$

* Q of resonant circuit:

<u>recall</u>: $Q = \frac{\omega_o L}{R}$ for a series RLC

$$\Rightarrow Q_{in} = \frac{\omega_o(L_g + L_s)}{R_s + \omega_T L_s}$$



* effective g_m @ f_o:

<u>recall</u>: $|V_L| = |V_C| = Q|V_{in}|$ for a series RLC

$$\Rightarrow V_{gs} = V_c = Q_{in} \cdot v_s$$

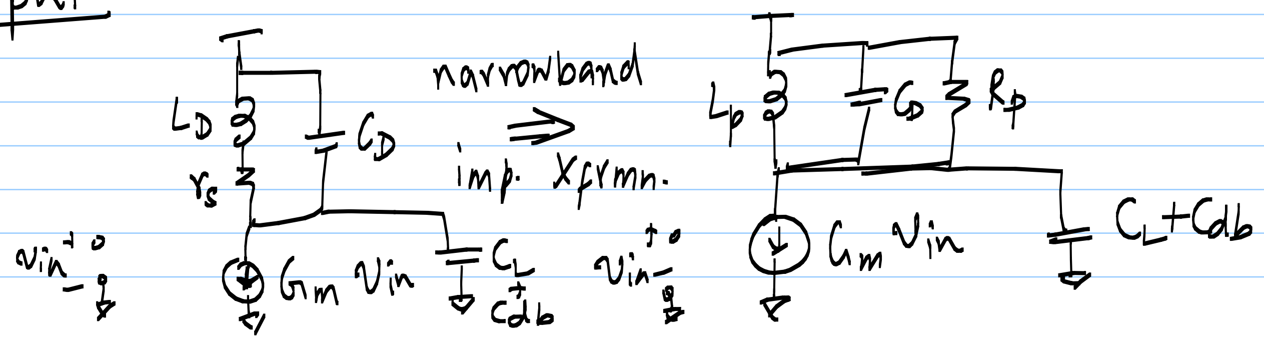
also, here $v_{in} = \dfrac{\omega_T L_s}{R_s + \omega_T L_s} \cdot v_s = \dfrac{v_s}{2}$ {available voltage @ gate}

$$\Rightarrow V_{gs} = 2\, Q_{in}\, v_{in}$$

$$\Rightarrow i_d = g_m V_{gs} = (g_m \cdot 2 Q_{in}) \cdot v_{in}$$

$$\Rightarrow \boxed{G_m = 2\, Q_{in} \cdot g_m}$$

Output



* series to parallel transformation
* tuned parallel RLC Load
  → r_s = parasitic series resistance of inductor
  → $f_0 = \dfrac{1}{2\pi\sqrt{L_D(C_D + C_{db} + C_L)}}$ = desired operating freq.
  → $C_D \gg C_{db}, C_L$ (input cap. of mixer) so that f_0 does not vary with parasitics
* IC LNAs – output does not drive $50\,\Omega$
  → input of mixer is capacitive

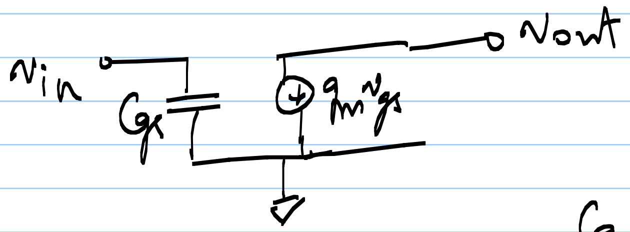
Stability: quantified using Stern Stability Factor

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2 \cdot |S_{21}| \cdot |S_{12}|}; \quad \Delta = S_{11} S_{22} - S_{21} S_{12}$$

* If $K>1$ and $\Delta<1$, circuit is unconditionally stable

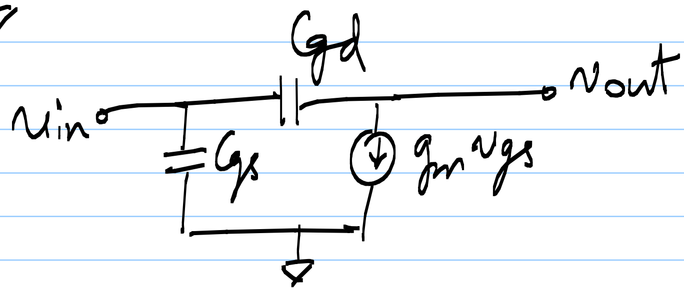
* S_{12} is a measure of Reverse Isolation
  → as reverse isolation increases (i.e. S_{12} decreases), circuit becomes more stable

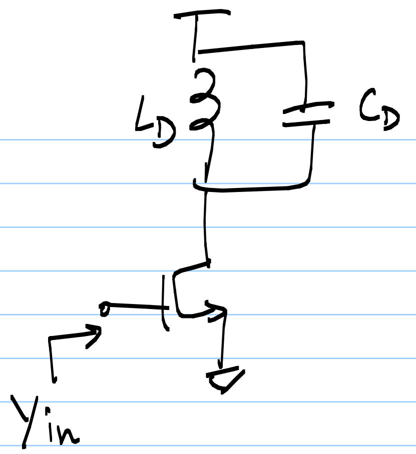
* Simple MOS dct model does not capture this



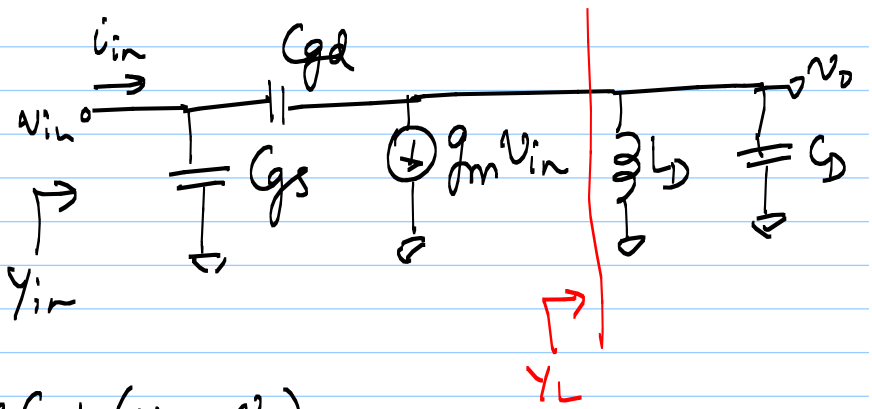
$\Rightarrow S_{12} = 0 \; (-\infty \, dB)$

realistic model }:





$\Rightarrow$



$$\begin{cases} i_{in} = sC_{gs}v_{in} + sC_{gd}(v_{in}-v_o) \\ sC_{gd}(v_{in}-v_o) = g_m v_{in} + v_o \cdot Y_L \end{cases}$$

$$\rightarrow Y_{in} = \frac{i_{in}}{v_{in}} = sC_{gs} + sC_{gd}\,\frac{(Y_L+g_m)}{Y_L+sC_{gd}}$$

usually, $Y_L \gg sC_{gd}$,

$$\Rightarrow Y_{in} = sC_{gs} + sC_{gd} + \frac{g_m sC_{gd}}{Y_L}$$

at low frequencies, $Z_L(j\omega) \approx j\omega L_D$ {inductor dominates}

$$\Rightarrow Y_L(j\omega) = \frac{1}{Z_L} = -\frac{j}{\omega L_D}$$

$$\Rightarrow Y_{in}(j\omega) = j\omega(C_{gs} + C_{gd}) - g_m \omega^2 L_D C_{gd}$$

$$Re\left(Y_{in}(j\omega)\right) = \underbrace{-g_m \omega^2 L_D C_{gd}}_{\text{negative resistance!}}$$

→ can cause instability

→ Neg. res. magnitude $\propto C_{gd}$

→ $C_{gd} = f$ (device width, layout)

→ simulate LNA over a wide frequency range to make sure it is stable at all freq.