Lecture 16 : Sensitivity, SFDR, MOSFET 2-port noise parameters

* **Sensitivity** ≡ minimum signal power level that can be detected with desired SNR

Recall that available noise power = KTB watts

$$\Rightarrow SNR_{in} = \frac{P_{sig}}{KTB}, \text{ where } P_{sig} = \text{received signal power}$$

We want to determine $P_{sig,min}$ for a desired $SNR_{out,min}$.

$$F = \frac{SNR_{in}}{SNR_{out}}$$

$$= \frac{P_{sig}/KTB}{SNR_{out}}$$

$$\Rightarrow P_{sig} = (KTB)\cdot(SNR_{out})\cdot(F)$$

expressed in decibels (dBm or dB)

$$P_{in,min}\big|_{dBm} = 10\log(kT) + NF\big|_{dB} + SNR_{min}\big|_{dB} + 10\log B$$

@ $T = 300K$, $10\log kT = -174\ dBm/Hz$

$$P_{in,min} = \underbrace{-174\,\text{dBm/Hz} + NF + 10\log B}_{=\text{noise floor}} + SNR_{min}.$$

* $P_{in,min}$ is a function of BW
  → a receiver with narrowband channel may appear to be sensitive compared to another with a wider channel

Dynamic Range:

$$DR \equiv \frac{\text{max input level tolerated}}{\text{min. input level meeting SNR requirements}}$$

However in RF design
* Distortion is defined differently (IIP_3)
* input signal must provide SNR_{min}.

Spurious Free Dynamic Range = SFDR
→ upper end defined by IM behaviour
→ lower end defined by sensitivity

* upper end = max input level for which IM3 products in a two-tone test do not exceed the noise floor.

Recall that

$$P_{IIP_3} = P_{in} + \frac{P_{out} - P_{IM_{out}}}{2}$$

$$= P_{in} + \frac{P_{in} - P_{IM,in}}{2} \quad \left\{\begin{aligned} P_{out} &= P_{in} + G|_{dB} \\ P_{IM,out} &= P_{IM,in} + G|_{dB} \end{aligned}\right\}$$

$$= \frac{3P_{in} - P_{IMin}}{2}$$

$$\Rightarrow P_{in} = \frac{2P_{IIP_3} + P_{IM,in}}{3}$$

$$NFL = -174 + NF + 10\log B$$

$$\therefore \quad P_{in,max} = \frac{2P_{IIP_3} + NFL}{3}$$

$$P_{in,min.} = NFL + SNR_{min.}$$

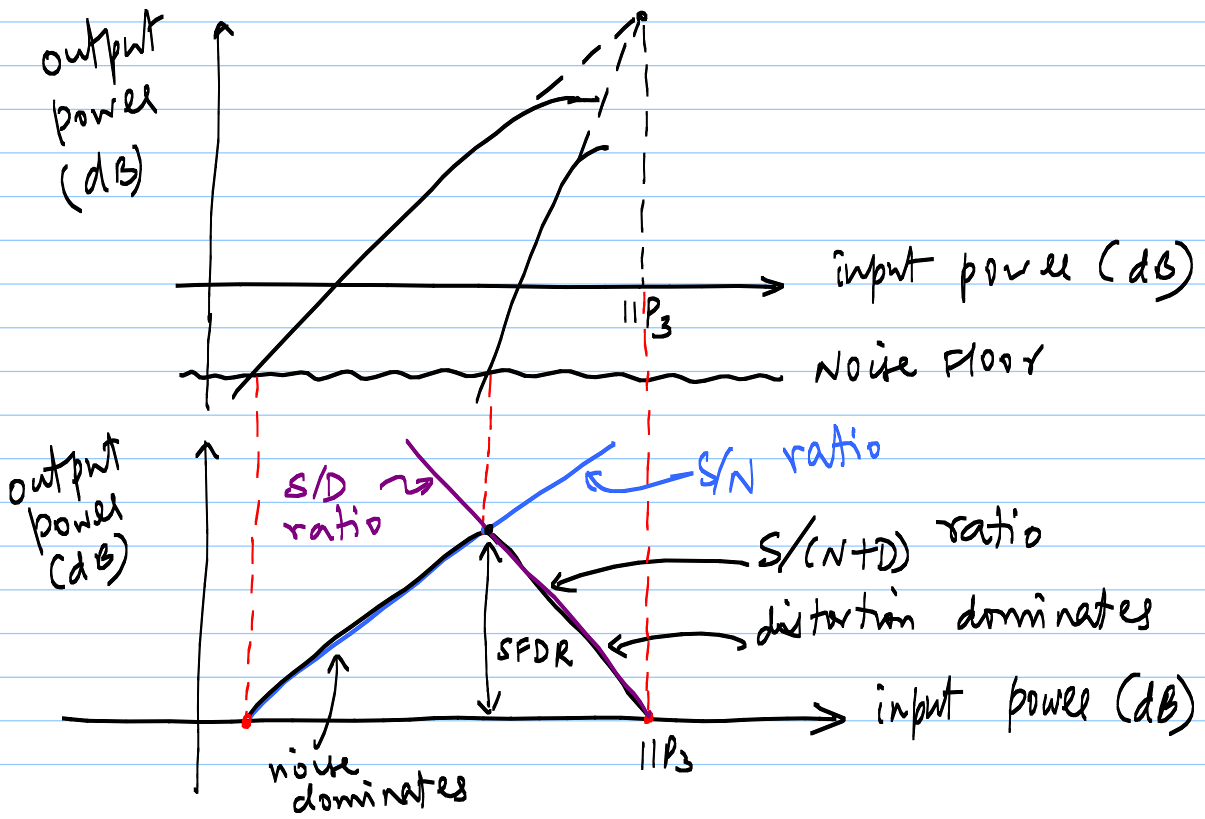
$$\Rightarrow SFDR = P_{in,max} - P_{in,min.}$$

$$= \frac{2P_{IIP_3} + NFL}{3} - (NFL + SNR_{min.})$$

$$= \frac{2(P_{IIP_3} - NFL)}{3} - SNR_{min.}$$

e.g. Consider a receiver with $NF = 9\,dB$, $P_{IIP_3} = -15\,dBm$, $B = 200\,kHz$, $SNR_{min} = 12\,dB$

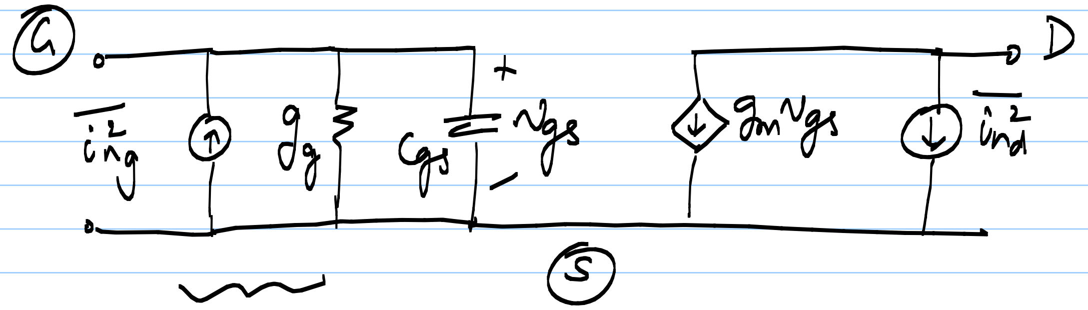
$$\Rightarrow SFDR \approx 53\,dB$$



MOSFET 2-port noise parameters:

→ brief description only

→ for complete treatment, please refer to Section 11.2 in Thomas Lee's textbook



from Van der Ziel

* Recall, $\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f$

$$\overline{i_{ng}^2} = 4kT\delta g_g \Delta f$$

$$g_g = \omega^2 C_{gs}^2 / 5 g_{d0}$$

* also recall that gate & drain noise are correlated

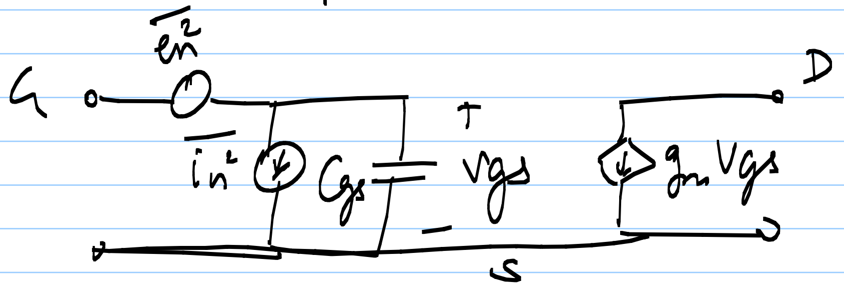
correlation coefficient $C \equiv \dfrac{\overline{i_{ng} \cdot i_{nd}^*}}{\sqrt{\overline{i_{ng}^2}}\sqrt{\overline{i_{nd}^2}}}$

* for a long channel MOSFET, $C = j0.395$

→ assume true for short channel also

* 2-port noise parameters are

$$R_N = \frac{\overline{e_n^2}}{4kT\Delta f}; \quad G_u = \frac{\overline{i_u^2}}{4kT\Delta f}; \quad Y_c = G_c + jB_c = \frac{i_c}{e_n}$$



1) input is shorted $\Rightarrow$ gate noise does not go to o/p

ckt $\Rightarrow V_{gs} = 0$, $\overline{i_{out}^2} = \overline{i_{nd}^2}$

2-port $\Rightarrow V_{gs} = \overline{e_n} \Rightarrow \overline{i_{out}^2} = g_m^2\, \overline{e_n^2}$

equating :

$$\overline{e_n^2} = \frac{\overline{i_{nd}^2}}{g_m^2}$$

$$\Rightarrow R_N = \frac{\overline{e_n^2}}{4kT\Delta f} = \frac{\gamma g_{do}}{g_m^2} = \frac{\gamma}{\alpha g_m} \;//$$

$$\alpha \equiv \frac{g_m}{g_{d_0}}$$

2) Input is open ckt :

a) $\overline{i_d^2}$ only : $\overline{i_n^2} = \frac{\overline{i_{nd}^2} \cdot (j\omega C_{gs})^2}{g_m^2} = \overline{e_n^2} \cdot (j\omega C_{gs})^2$

$\Rightarrow \overline{i_n}$ is completely correlated with $\overline{e_n}$

b) $\overline{i_{ng}} + \overline{i_{nd}}$

let $\overline{i_{ng}} = \underbrace{\overline{i_{ng_c}}}_{\text{correlated with } \overline{e_n}} + \underbrace{\overline{i_{ng_u}}}_{\text{uncorrelated with } \overline{e_n}}$

$$\Rightarrow Y_c = \frac{\overline{i_{n1}} + \overline{i_{ng_c}}}{\overline{e_n}} = j\omega C_{gs} + \frac{\overline{i_{ng_c}}}{\overline{e_n}}$$

$$= j\omega C_{gs} + g_m \frac{\overline{i_{ng_c}}}{\overline{i_{nd}}}$$

$$= j\omega C_{gs} + g_m \frac{\overline{i_{ng_c} \cdot i_{nd}^*}}{\overline{i_{nd} \cdot i_{nd}^*}} \qquad \left\{ \begin{array}{l} \text{multiply and} \\ \text{divide by } \overline{i_{nd}^*} \end{array} \right\}$$

$$= j\omega C_{gs} + g_m \frac{\overline{i_{ng_c} \cdot i_{nd}^*}}{\sqrt{\overline{i_{nd}^2}} \cdot \sqrt{\overline{i_{nd}^2}}}$$

$$= j\omega C_{gs} + g_m \cdot \frac{\overline{i_{ng_c} \cdot i_{nd}^*}}{\sqrt{\overline{i_{nd}^2}}\sqrt{\overline{i_{nd}^2}}} \cdot \frac{\sqrt{\overline{i_{ng}^2}}}{\sqrt{\overline{i_{ng}^2}}}$$

$\rightarrow = c$ correlation coefft.

$$= j\omega C_{gs} + g_m \cdot c \cdot \sqrt{\frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}}} \quad \left\{ \begin{array}{l} c = \text{correlation} \\ \text{coefficient} \end{array} \right\}$$

$$= j\omega C_{gs} + j\, g_m |c| \sqrt{\frac{\delta g_g}{\gamma g_{do}}} \quad \left\{ \begin{array}{l} c = j\, 0.395 \\ \text{ideally} \end{array} \right\}$$

$$Y_c = j\omega C_{gs} \left\{ 1 + \frac{g_m}{g_{do}} |c| \sqrt{\frac{\delta}{5\gamma}} \right\}$$

$$\Rightarrow \boxed{Y_c = j\omega C_{gs} \left\{ 1 + \alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right\}}$$

$$Y_c = G_c + jB_c$$

$$\Rightarrow \boxed{\begin{array}{l} G_c = 0 \\ B_c = \omega C_{gs} \left(1 + \alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right) \end{array}}$$

$\rightarrow$ $G_c = 0$ $\because$ we have neglected gate resistance

$\rightarrow$ $Y_c \neq j\omega C_{gs}$ (condition for max power transfer)

i.e. min. noise condition is different from max power

3) G_u:

$$\overline{i_{ng}^2} = \overline{(i_{ng_c} + i_{ng_u})^2} = \overline{i_{ng_c}^2} + \overline{i_{ng_u}^2}$$

$$= \underbrace{4kT\delta g_g \Delta f |c|^2}_{\overline{i_{ng_c}^2}} + \underbrace{4kT\delta g_g \Delta f (1-|c|^2)}_{\overline{i_{ng_u}^2}}$$

$$G_u = \frac{\overline{i_{ng_u}^2}}{4kT\Delta f}$$

$$\Rightarrow \boxed{G_u = \frac{\delta \omega^2 C_{gs}^2 (1-|c|^2)}{5 g_{d0}}}$$

* $B_{opt} = -B_c = -\omega C_{gs}\left(1 + \alpha |c| \sqrt{\frac{\delta}{5\gamma}}\right)$

→ inductive in character

→ wrong frequence behaviour ($\propto \omega$ instead of $\propto \frac{1}{\omega}$) $\Rightarrow$ broadband noise match is fundamentally difficult

* $G_{opt} = \sqrt{\frac{G_u}{R_N} - G_c^2} = \alpha \omega C_{gs} \sqrt{\frac{\delta}{5\gamma}(1-|c|^2)}$

* $F_{min} = 1 + 2R_N [G_{opt} + G_c]$

$\Rightarrow F_{min} \approx 1 + \frac{2}{\sqrt{5}} \frac{\omega}{\omega_T} \sqrt{\gamma\delta(1-|c|^2)}$

→ Faster devices yield lower noise at given ω

→ e.g. $\gamma = 2$, $\delta = 4$

ω_T/ω	F_{min} (dB)
20	0.5
15	0.6
10	0.9
5	1.6

→ F_{min} assumes no constraint on power