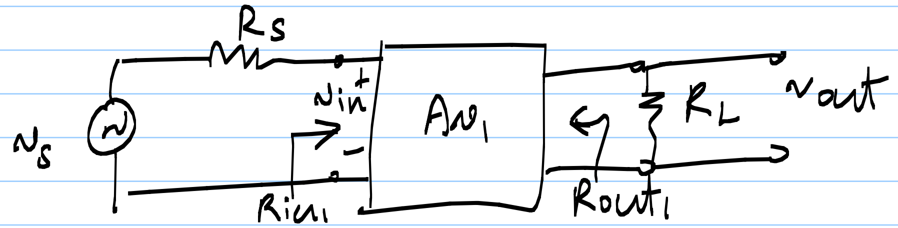
Lecture 15: NF (contd.)

Available Power Gain $= A_p$

$$A_p \equiv \frac{\text{available } P_{out} \text{ (under matched condition)}}{\text{available source power (matched)}}$$



* $R_s = R_{in_1}$; $R_{out_1} = R_L$

$$v_{in} = v_s \cdot \frac{R_{in_1}}{R_s + R_{in_1}} = \frac{v_s}{2}$$

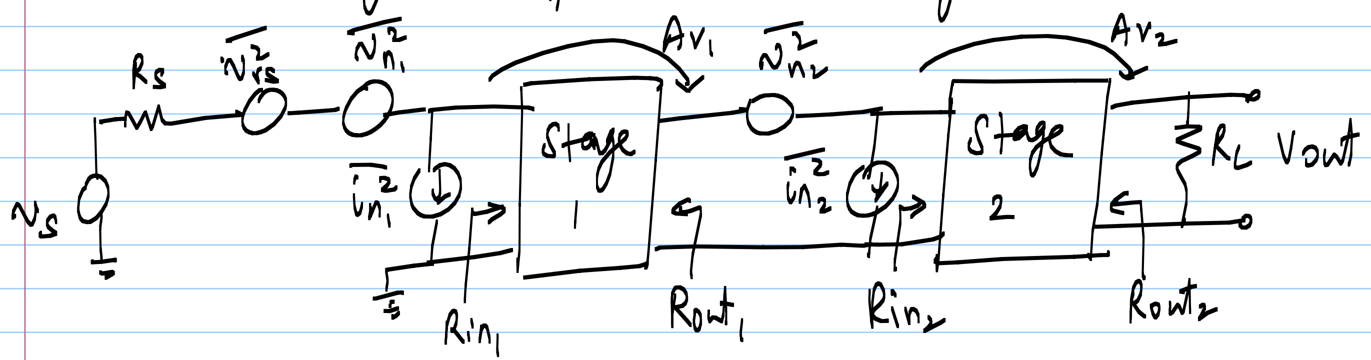
$$P_{av,s} = \frac{(v_s/2)^2}{R_s} = \frac{v_s^2}{4R_s}$$

$$v_{out} = v_{in} \cdot A_{v_1} \cdot \left(\frac{R_L}{R_L + R_{out_1}}\right) = v_s\left(\frac{R_{in_1}}{R_s + R_{in_1}}\right) \cdot A_{v_1} \cdot \frac{1}{2}$$

$$P_{out,av.} = v_s^2 \left(\frac{R_{in_1}}{R_s + R_{in_1}}\right)^2 \cdot A_{v_1}^2 \cdot \frac{1}{4R_{out_1}}$$

$$\Rightarrow A_p = \left(\frac{R_{in_1}}{R_s + R_{in_1}}\right)^2 \cdot A_{v_1}^2 \cdot \frac{R_s}{R_{out_1}}$$

Noise Figure of Cascaded Systems



$A_{v_1}, A_{v_2} \Rightarrow$ <u>unloaded</u> voltage gains of stage 1 & 2

* Noise power at input to stage 1:

$$\overline{v^2_{n,in_1}} = \overline{\left[i_{n_1} (R_s \| R_{in_1}) + v_{n_1} \left(\frac{R_{in_1}}{R_s + R_{in_1}} \right) \right]^2}$$

accounts for correlation between $\overline{i_{n_1}}$ & $\overline{v_{n_1}}$

$$+ \overline{v^2_{R_s}} \left(\frac{R_{in_1}}{R_s + R_{in_1}} \right)^2$$

* Noise power at input to stage 2:

$$\overline{v^2_{n,in_2}} = \overline{v^2_{n,in_1}} \cdot A^2_{v_1} \cdot \left(\frac{R_{in_2}}{R_{out_1} + R_{in_2}} \right)^2$$

← contribution from 1st stage

$$+ \overline{\left[i_{n_2} (R_{out_1} \| R_{in_2}) + v_{n_2} \cdot \left(\frac{R_{in_2}}{R_{in_2} + R_{out_1}} \right) \right]^2}$$

* Noise power at output of stage 2:

$$\overline{v_{out}^2} = Av_2^2 \cdot \left(\frac{R_L}{R_{out_2} + R_L}\right)^2 \cdot \overline{v_{n,in_2}^2}$$

* Total voltage gain:

$$Av_{,tot.} = \frac{R_{in_1}}{R_s + R_{in_1}} \cdot Av_1 \cdot \frac{R_{in_2}}{R_{out_1} + R_{in_2}} \cdot Av_2 \cdot \frac{R_L}{R_L + R_{out_2}}$$

* Overall Noise factor:

$$F = \frac{\text{total noise power at output}}{\text{output noise due to } R_s \text{ only}} = \frac{\overline{v_{out}^2}}{Av_{,tot}^2 \cdot 4kTR_s}$$

After a bunch of algebra:

$$F = 1 + \frac{\overline{|v_{n_1} + i_{n_1} R_s|^2}}{4kTR_s}$$

F_{1,R_s} = noise factor of stage 1 w.r.t. R_s

$$+ \frac{\overline{|v_{n_2} + i_{n_2} R_{out_1}|^2}}{Av_1^2} \cdot \frac{1}{\left(\frac{R_{in_1}}{R_s + R_{in_1}}\right)^2} \cdot \frac{1}{4kTR_s}$$

analyse further

* Noise factor of stage 2 w.r.t. source impedance R_{out_1} is

$$F_{2,R_{out_1}} = 1 + \frac{\overline{|i_{n_2} R_{out_1} + v_{n_2}|^2}}{4kTR_{out_1}}$$

* the second term in expression for overall F is

$$\frac{\overline{|v_{n2}+i_{n2}R_{out,1}|^2}}{4kTR_s}\cdot \underbrace{\frac{1}{\left(\frac{R_{in,1}}{R_s+R_{in,1}}\right)^2}\cdot\frac{1}{A_{v_1}^2}\cdot\frac{R_s/R_{out_1}}{R_s/R_{out_1}}}_{A_p}$$

$$=\frac{\overline{|v_{n2}+i_{n2}R_{out_1}|^2}}{4kTR_{out_1}}\cdot\frac{1}{A_p}=\left(F_{2,R_{out_1}}-1\right)\cdot\frac{1}{A_p}$$

$$\Rightarrow \boxed{F=F_{1,R_s}+\frac{F_{2,R_{out_1}}-1}{A_p}}$$

In general for m stages,

$$\boxed{F_{tot.}=1+(F_1-1)+\frac{F_2-1}{A_{p_1}}+\cdots+\frac{F_m-1}{A_{p_1}\cdots A_{p(m-1)}}}$$

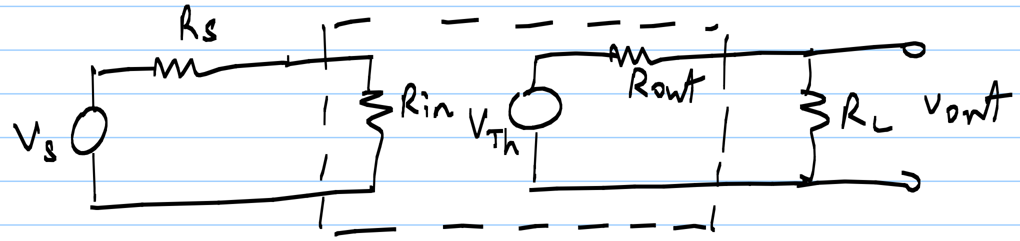
where F_i = F of i^{th} stage w.r.t. source impedance driving that stage

FRIIS EQUATION

* Noise of front-end stages is most significant

Noise Factor of Lossy Circuits:

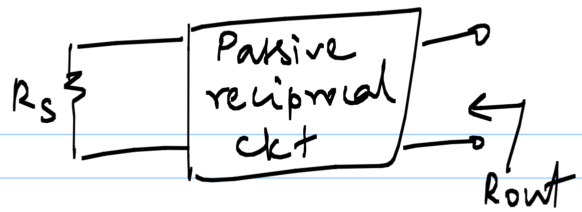
* off-chip passive filters → finite in-band loss
  → matched to 50Ω



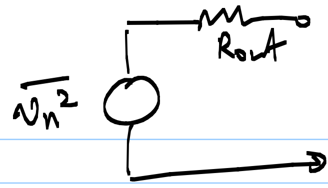
Available power Loss $(L) = \dfrac{P_{in}}{P_{out}}$

assume $R_{in} = R_S$ & $R_{out} = R_L$

$$\Rightarrow L = \frac{(V_S^2/4R_S)}{(V_{Th}^2/4R_{out})} = \frac{V_S^2}{V_{Th}^2} \cdot \frac{R_{out}}{R_S}$$



$\Rightarrow$



It can be shown that $v_n^2 = 4KT R_{out} \Delta f$

Noise power at output:

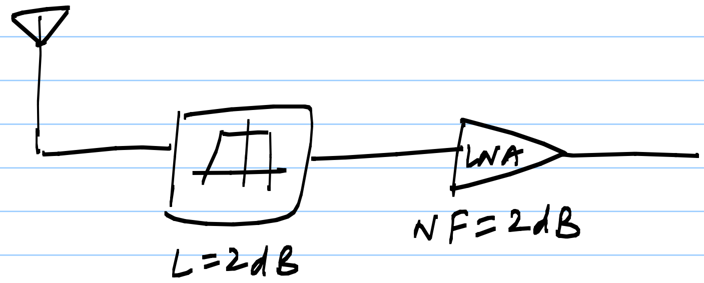
$$\overline{v_{n,out}^2} = 4KT R_{out} \left(\frac{R_L}{R_L + R_{out}}\right)^2$$

$$A_V = \frac{V_{Th}}{V_S} \cdot \frac{R_L}{R_L + R_{out}}$$

$$F = \frac{\overline{v_{n,out}^2}}{\overline{v_{n,R_S}^2} \cdot A_V^2} = \frac{\cancel{4KT} R_{out} \cancel{\left(\frac{R_L}{R_L + R_{out}}\right)^2}}{\cancel{4KT} R_S \cdot \frac{V_{Th}^2}{V_S^2} \cdot \cancel{\left(\frac{R_L}{R_L + R_{out}}\right)^2}}$$

$$\Rightarrow F = \frac{v_s^2}{v_{th}^2} \cdot \frac{R_{out}}{R_s} = \underline{\underline{L}}$$

e.g.



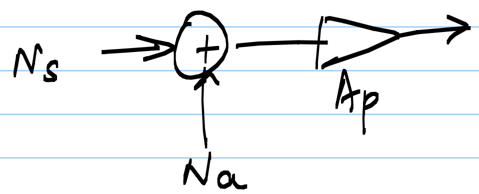
$$F_{tot.} = F_{filt.} + \frac{F_{LNA} - 1}{L^{-1}} = L + L\cdot(F_{LNA} - 1)$$

$$= L \cdot F_{LNA}$$

$$\Rightarrow NF_{tot.} = L_{dB} + NF_{LNA}$$

overall NF @ antenna = NF(LNA) + L = 4dB !

* Alternative definition of F:



consider an amplifier of available power gain A_p, and input-referred noise power N_a

$$F = \frac{A_p(N_a + N_s)}{A_p \cdot N_s} = 1 + \frac{N_a}{N_s}$$

let signal power be S

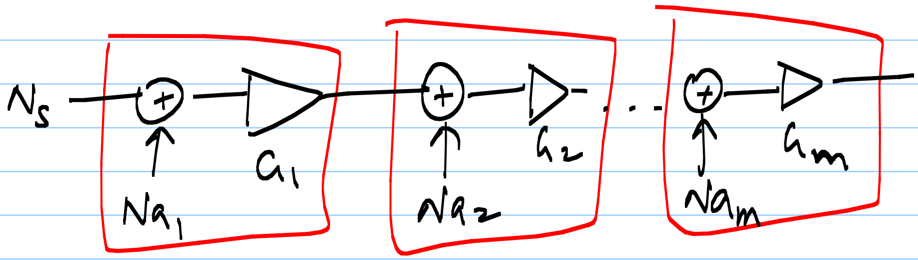
$$SNR \text{ @ input} = SNR_{in} = \frac{S}{N_s}$$

$$SNR \text{ @ output} = SNR_{out} = \frac{A_p \cdot S}{A_p \cdot (N_a + N_s)}$$

$$\frac{SNR_{in}}{SNR_{out}} = \frac{S/N_S}{A_p S / A_p (N_a + N_S)} = \frac{N_a + N_S}{N_a} = F$$

$$\Rightarrow \boxed{F = \frac{SNR_{in}}{SNR_{out}}}$$

* Alternative Derivation of Friis Equation :



* assume all impedances are equal (& matched)

* $G_i = A_{p_i}$

$$\text{Total output noise power} = (N_s + N_a) \cdot G_1 G_2 \cdots G_m + (Na_2) \cdot G_2 \cdots G_m + \cdots + Na_m \cdot G_m$$

noise power due to source alone

$$= (N_s) \cdot G_1 \cdot G_2 \cdots G_m$$

$$\Rightarrow F = 1 + \frac{Na_1}{N_s} + \frac{Na_2}{N_s \cdot G_1} + \cdots + \frac{Na_m}{N_s \cdot G_1 G_2 \cdots G_{m-1}}$$

$$\boxed{F = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_m - 1}{G_1 G_2 \cdots G_{m-1}}}$$