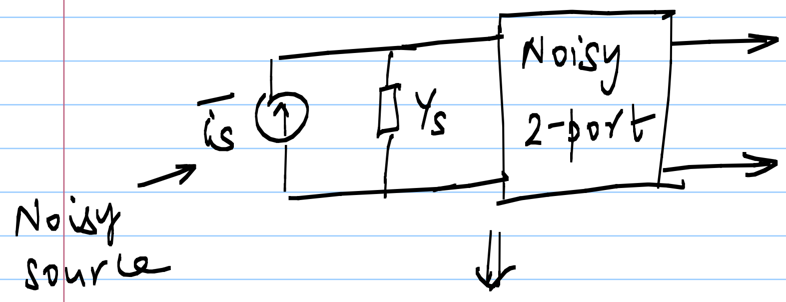
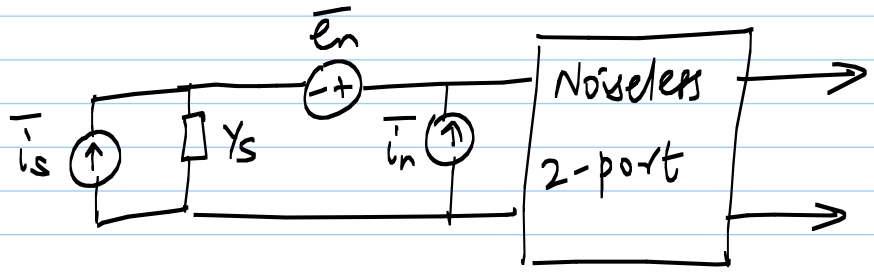
Lecture 14: Classical 2-port Noise Theory



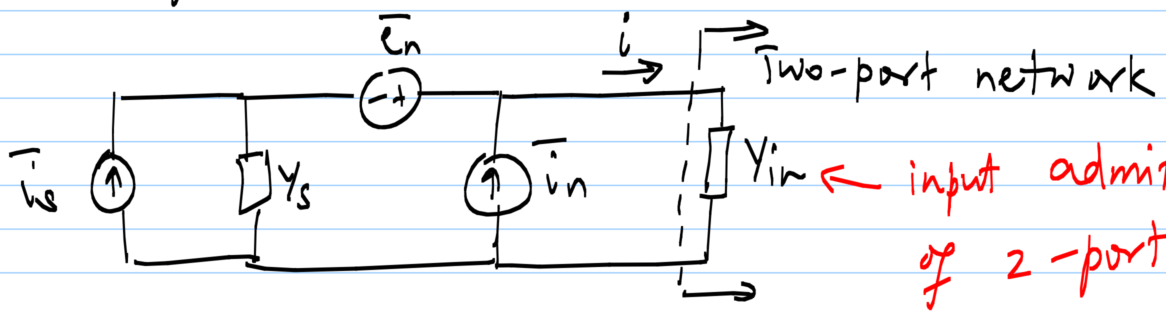


Equivalent ckt with noiseless 2-port

* OC & SC cases → you need both $\bar{e}_n$ & $\bar{i}_n$
* In general, $\bar{e}_n$ and $\bar{i}_n$ are correlated
* actual ckt may have no physical input noise current

Noise Factor

* $F = \dfrac{\text{total output noise power}}{\text{output noise due to input source only}}$
* by convention, source is at a temp of 290K.
* F is a measure of degradation in SNR due to a system {degradation in SNR ↑ ⇒ F ↑}
* If system adds no noise, $F = 1$



← input admittance of 2-port

$$i = i_s \cdot \left(\frac{Y_{in}}{Y_s + Y_{in}}\right) + (i_n + e_n Y_s)\cdot\left(\frac{Y_{in}}{Y_{in} + Y_s}\right)$$

* $\overline{i_s}$ is assumed to be uncorrelated with $\overline{e_n}$ & $\overline{i_n}$

$$F = \frac{\overline{i_s^2}\left(\frac{Y_{in}}{Y_s+Y_{in}}\right)^2 + \left|\overline{i_n}+\overline{e_n}Y_s\right|^2 \cdot \left(\frac{Y_{in}}{Y_s+Y_{in}}\right)^2}{\overline{i_s^2}\cdot\left(\frac{Y_{in}}{Y_s+Y_{in}}\right)^2}$$

$$= 1 + \frac{\left|i_n + \overline{e_n}Y_s\right|^2}{\overline{i_s^2}}$$

Let $\overline{i_n} = \overline{i_c} + \overline{i_u}$

$\overline{i_c}$ is correlated with $\overline{e_n}$

$\Rightarrow \overline{i_c} = Y_c \cdot \overline{e_n}$; Y_c = correlation admittance

i_u is uncorrelated with $\overline{e_n}$

$$\Rightarrow F = 1 + \frac{\left|\overline{i_u} + (Y_c+Y_s)\overline{e_n}\right|^2}{\overline{i_s^2}}$$

because $\overline{i_u}$ is un-correlated with $\overline{e_n}$

$$= 1 + \frac{\overline{i_u^2} \quad |Y_c+Y_s|^2\, \overline{e_n^2}}{\overline{i_s^2}}$$

* Next, we define each of these 3 independent noise sources as an equivalent resistance or conductance and their associated thermal noise sources:

$$R_N = \frac{\overline{e_n^2}}{4kT\Delta f}$$

$$G_u = \frac{\overline{i_u^2}}{4kT\Delta f}$$

$$G_s = \frac{\overline{i_s^2}}{4kT\Delta f}$$

$$\Rightarrow F = 1 + \frac{G_u + |Y_c + Y_s|^2 \cdot R_N}{G_s}$$

Now, let $Y_c = G_c + jB_c$ and $Y_s = G_s + jB_s$

$$\Rightarrow F = 1 + \frac{G_u + \left[(G_c+G_s)^2 + (B_c+B_s)^2\right] \cdot R_N}{G_s}$$

* Noise of any 2-port can be characterized by 4 parameters: $\{R_N, G_u, G_c, B_c\}$

Conditions that minimize F (ie <u>optimum source admittance</u>):

1) $\frac{\partial F}{\partial B_s} = 0 \Rightarrow 2(B_c+B_s) \cdot R_N = 0$

$$\Rightarrow \boxed{B_s = -B_c = B_{opt.}}$$ ← Design Condition for $F_{min.}$

2) $\frac{\partial F}{\partial G_s} = 0$

apply (1) here

$$\Rightarrow \frac{2(G_c+G_s)R_N}{G_s} - \frac{G_u + \left[(G_c+G_s)^2 + (B_c+B_s)^2\right] R_N}{G_s^2} = 0$$

$$\Rightarrow G_s^2 = \frac{G_u}{R_N} + G_c^2$$

$$\Rightarrow \boxed{G_s = \sqrt{\frac{G_u}{R_N} + G_c^2} = G_{opt.}}$$ ← Design condition for $F_{min.}$

$F_{min.}$ is given by:

$$F_{min.} = 1 + \frac{G_u + \left[(G_c+G_{opt})^2 + (B_c + \cancel{B_{opt}})^2\right] R_N}{G_{opt}}$$

also, $G_n = (G_{opt}^2 - G_c^2)\cdot R_N$

$$\therefore F_{min} = 1 + \frac{G_{opt}^2 R_N - \cancel{G_c^2 R_N} + \cancel{G_c^2 R_N} + 2G_cG_{opt}R_N + G_{opt}^2 R_N}{G_{opt}}$$

$$\boxed{F_{min} = 1 + 2R_N\left(G_{opt} + G_c\right)}$$

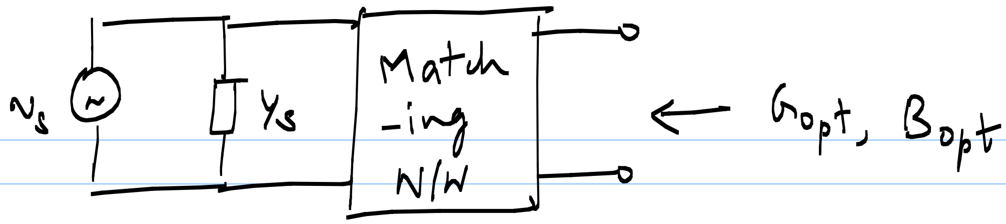
or $$\boxed{F_{min} = 1 + 2R_N\left[\sqrt{\frac{G_n}{R_N} + G_c^2} + G_c\right]}$$

Noise Circles

Recall:

$$F = 1 + \frac{G_n + \left[(G_c + G_s)^2 + (B_c + B_s)^2\right]R_N}{G_s}$$

$$G_{opt}^2 = \frac{G_n}{R_N} + G_c^2 \; ; \quad B_c = -B_{opt}$$



$$G_n = R_N\left(G_{opt}^2 - G_c^2\right)$$

$$\Rightarrow F = 1 + \frac{\left[G_{opt}^2 - G_c^2 + (G_c + G_s)^2\right]R_N + (B_s - B_{opt})^2 R_N}{G_s}$$

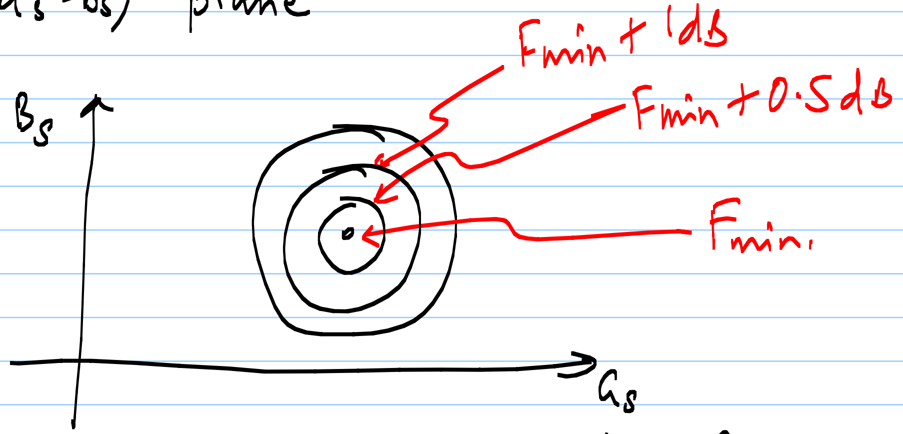
$$= 1 + \frac{\left[G_{opt}^2 - \cancel{G_c^2} + \cancel{G_c^2} + 2G_cG_s + G_s^2\right]R_N + (B_s - B_{opt})^2 R_N}{G_s}$$

add & subtract $2G_sG_{opt}$ to 1^{st} term of numerator

$$\Rightarrow F = 1 + \frac{2(G_c + G_{opt})\cdot G_s \cdot R_N + \left[(G_s - G_{opt})^2 + (B_s - B_{opt})^2\right]R_N}{G_s}$$

$$\Rightarrow F = F_{min.} + \left[(G_s - G_{opt})^2 + (B_s - B_{opt})^2 \right] \cdot \frac{R_N}{G_s}$$

* these are circles in the source admittance (G_s-B_s) plane



* Circles of constant F in G_s-B_s plane
* Also circles on a Smith chart (mapping is a bilinear transformation)

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* Conditions for $F_{min.}$ are slightly different from those for maximum power transfer!
  → Tradeoff between max gain & min. noise
* <u>Noise Figure</u> ≡ noise factor in dB

$$NF = 10\log_{10} F$$

* <u>Noise Temperature</u> T_N
  ≡ increase in temperature required of Y_s for it to account for all of the output noise at the ref. temp. (=290K)

$$F = 1 + \frac{T_N}{T_{ref}} \Rightarrow T_N = T_{ref} \cdot (F-1)$$

→ a 2-port that adds no noise has $T_N = 0K$

→ T_N is useful for cascaded amplifiers and those whose F is close to 1 (ie $NF \sim 0\,dB$)

→ T_N offers a higher resolution description of noise performance