Lecture 13: Noise in RF Circuits

Noise – any random interference unrelated to signal of interest

* represents inherent uncertainties at the physical level – is a "random" or "stochastic" process.
* represented by some kind of probability density function (PDF)
* "mean square" noise power is given by

$$\overline{n^2(t)} = \int_{-\infty}^{\infty} n^2(t) P_n(n)\, dn \qquad \{P_n(n) = \text{PDF}\}$$

* PDF provides no info on how fast $n(t)$ varies in time domain

* many natural phenomena exhibit Gaussian Statistics

* Gaussian PDF:

$$P_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-m)^2}{2\sigma^2}$$

m = mean

σ = std deviation

* For a Gaussian,

68% samples lie between $(m-\sigma)$ & $(m+\sigma)$

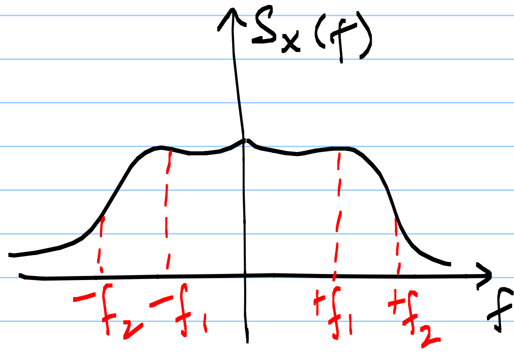
99% samples lie between $(m-3\sigma)$ & $(m+3\sigma)$

Power Spectral Density (PSD)

* Useful to characterize noise in frequency domain for RF Design

$S_x(f) \rightarrow$ provides power carried in a signal in a unit BW around f

* $S_x(f)$ is an even function of f for real $x(t)$



2-sided spectrum

1-sided spectrum

-ve part of spectrum is "folded"
e.g. spectrum Analyzer

2-sided – often used in graphical analysis of freq. domain operations

1-sided – used in noise calculations at a circuit level

e.g. thermal noise voltage across a resistor of value R

$S_x(f) = 2KTR$

K = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K

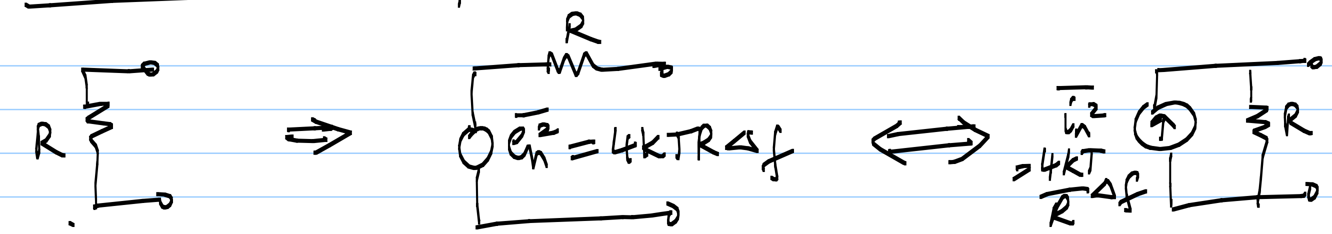
T = absolute temp.

In ckt noise calculations, we often write

$\overline{V_n^2} = 4KTR\Delta f$

mean square noise voltage

1) Thermal noise of resistor



noiseless

Δf = measurement BW (brickwall)

* $\frac{\overline{e_n^2}}{\Delta f} \equiv$ mean square noise voltage density

$= 4kTR = 1.6\times10^{-20}\cdot R \quad V^2/Hz$ at Room Temp.

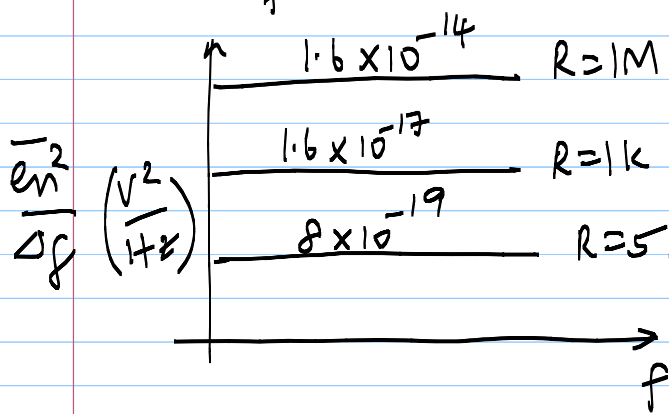
* $\frac{\overline{i_n^2}}{\Delta f} \equiv$ mean square noise current density

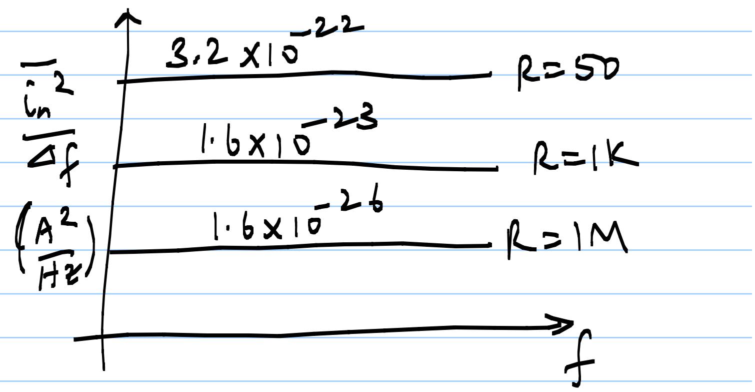
$= \frac{4kT}{R} = \frac{1.6\times10^{-20}}{R} \quad A^2/Hz$ at Room temp

$\overline{e_n^2} = \overline{i_n^2}\cdot R^2$ {note the R^2}

$\frac{\overline{e_n}}{\sqrt{\Delta f}} \equiv$ r.m.s. noise voltage density $\{V/\sqrt{Hz}\}$

$\frac{\overline{i_n}}{\sqrt{\Delta f}} \equiv$ r.m.s. noise current density $\{A/\sqrt{Hz}\}$



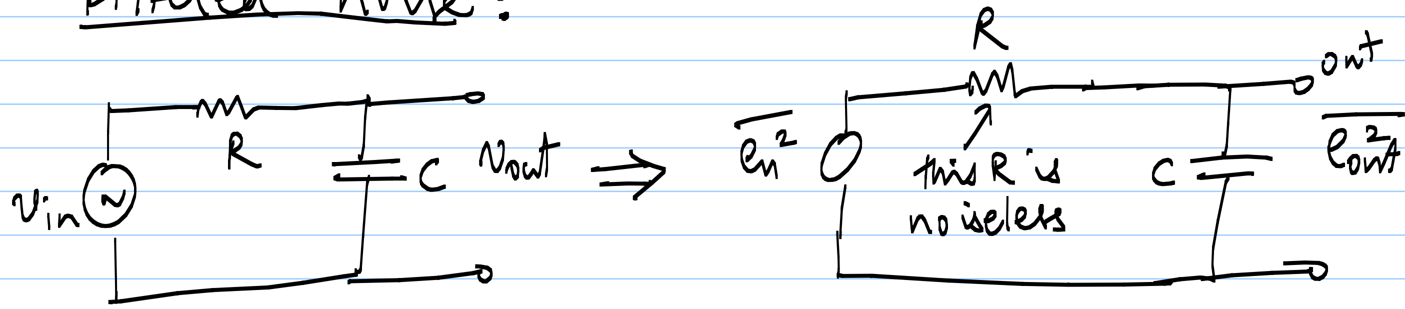


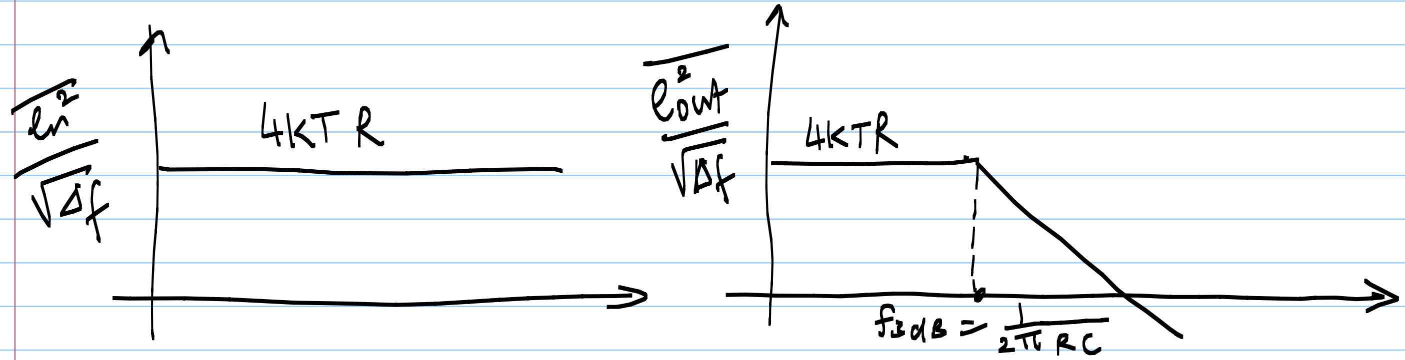
<u>Spot Noise</u> = mean square noise density integrated over 1 Hz BW @ freq. of interest

* <u>White Noise</u> $\equiv$ Spot noise is independent of frequency (ie. flat PSD)

$1\,k\Omega \rightarrow 4\,nV/\sqrt{Hz}$; $50\,\Omega \rightarrow 0.9\,nV/\sqrt{Hz}$

Filtered noise:





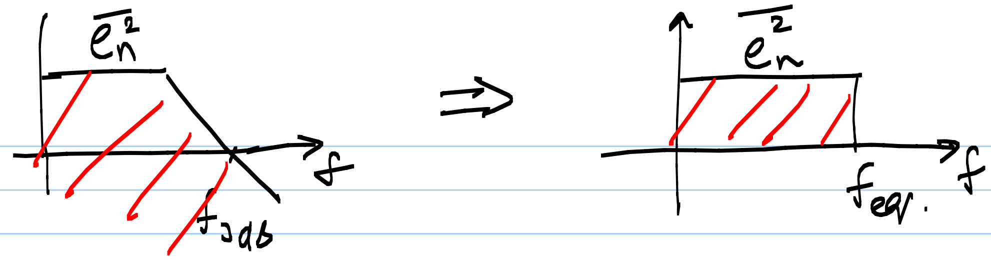
* Noise is low-pass filtered
* Integrated mean-square noise:

$$\overline{e_{out}^2} = \int_0^{\infty} \frac{\overline{e_n^2}}{|1+j2\pi RC|^2}\,df$$

$$= \frac{\overline{e_n^2}}{2\pi RC} \cdot \frac{\pi}{2}$$

$$\boxed{\overline{e_{out}^2} = \frac{kT}{C} \quad V^2}$$

* Equivalent Noise BW $\equiv$ BW of brick-wall filter whose output integrated noise power is same as above 1-pole filter

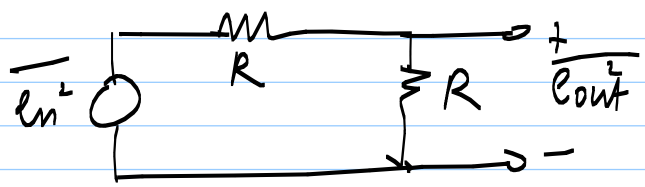


$$\frac{kT}{C} = \overline{e_n^2} \cdot f_{eq}$$

$$= 4kTR \cdot f_{eq}$$

$$\Rightarrow \boxed{f_{eq.} = \frac{1}{4RC} = \frac{\pi}{2} f_{3dB}}$$

* Available Noise Power :



Noise power delivered to load resistor in a 1Hz BW $\Big\} = \frac{(\overline{e_n}/2)^2}{R} = \frac{\overline{e_n^2}}{4R}$ (W/Hz)

$\Rightarrow$ available noise Power $= \frac{4kTR}{4R} \cdot B$

where $B =$ bandwidth of interest

$\boxed{P_{ave.} = KTB \text{ Watts}}$ independent of resistance R!

Keys to low-noise design

→ minimise temp

→ use minimum BW necessary (don't overdesign!)

$$P_{ave.}/\text{Hz} = \frac{kTB}{B} = kT \quad W/\text{Hz}$$

$$= 4.14 \times 10^{-21} \quad W/\text{Hz}$$

$$= -174 \text{ dBm/Hz}$$

* Total RMS noise over BW defines minimal signal
* Total distortion defines maximum signal

2) Noise in MOSFETS

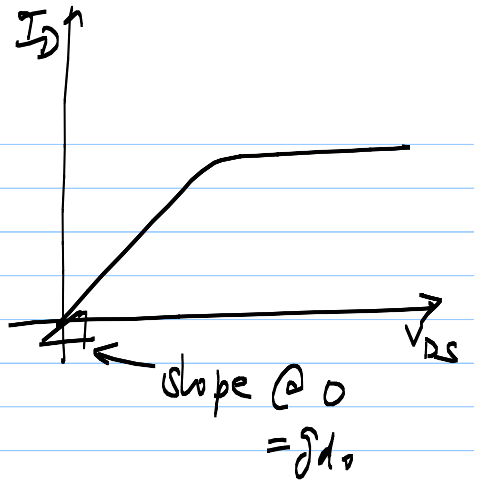
A) Thermal Noise:

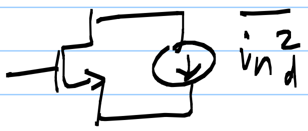
I. In triode region:

$$I_d = k'\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$g_d = \frac{\partial I_D}{\partial V_{DS}} = k'\left(\frac{W}{L}\right)(V_{GS}-V_T-V_{DS})$$

$$\Rightarrow g_{d0} = k'\left(\frac{W}{L}\right)(V_{GS}-V_T)$$

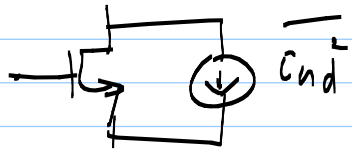




$$\boxed{\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f}$$

γ is an empirical parameter (=1 in triode region)

II. Long channel device in sat:



$$\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f$$

$\gamma = \frac{2}{3}$ for a long-channel device

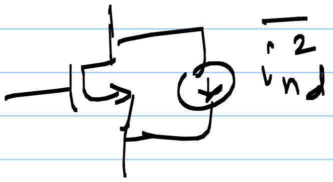
$$I_D = \frac{k'}{2}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$

$$\Rightarrow \quad g_m = \frac{\partial I_D}{\partial V_{GS}} = k'\left(\frac{W}{L}\right)(V_{GS}-V_T) = g_{d0}$$

$$\therefore \quad \boxed{\overline{i_{nd}^2} = 4kT \cdot \frac{2}{3} g_m \, \Delta f}$$

common form

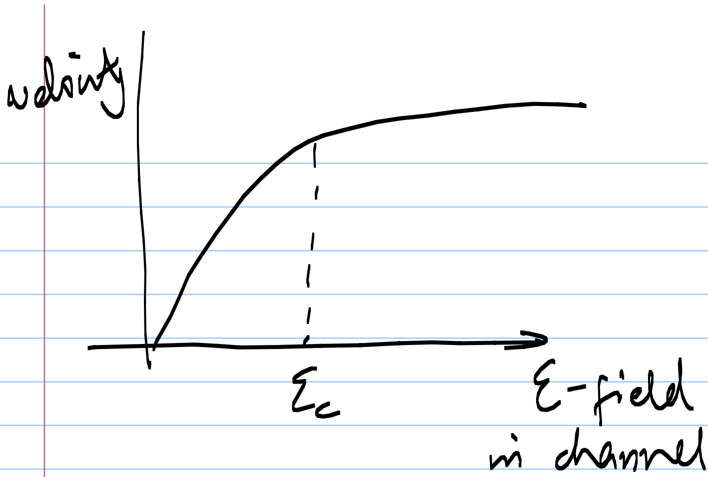
III. Short-channel device in sat:



$$\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f$$

typical $\gamma = 2\text{–}3$, but can even be 4–5

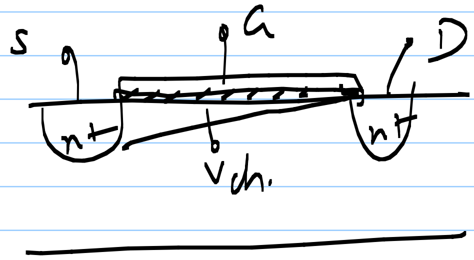
* excess noise (larger γ) due to "hot carriers"



as $\mathcal{E}$ ↑ beyond $\mathcal{E}_c$, v does not ↑

⇒ excess energy goes into heat (ie. thermal energy) ie into increased noise

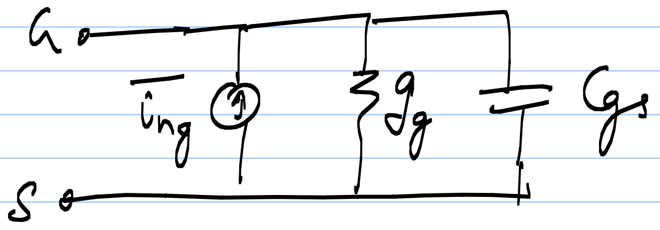
B. Gate-induced noise:



* thermal noise in i_d causes channel voltage $V_{ch.}$ to fluctuate
* V_{GC} gate-channel voltage varies through gate oxide cap.

⇒ noise current flows through the gate

<u>Van der Ziel's model:</u>



$$\boxed{\overline{i_{ng}^2} = 4kT\delta g_g \Delta f}$$

* $g_g = \dfrac{\omega^2 C_{gs}^2}{5 g_{do}}$

* $\delta = 2\gamma$ for long-channel mosfets

* $\overline{i_{ng}} \propto \omega \Rightarrow$ "blue noise"

* Use series-parallel transformations to use a voltage noise source:

* $C_{gs}' \simeq C_{gs}$ (assume high Q) $\left\{ C_{gs}' = C_{gs} \cdot \dfrac{1+Q^2}{Q^2} \right\}$

* r_{gate}:

recall: $R_p = R_s (Q^2+1)$

$$Q = \frac{\omega C_{gs}}{g_g}$$

$$\Rightarrow r_{gate} = \frac{1}{g_g} \cdot \frac{1}{(1+Q^2)} \simeq \frac{1}{g_g Q^2}$$

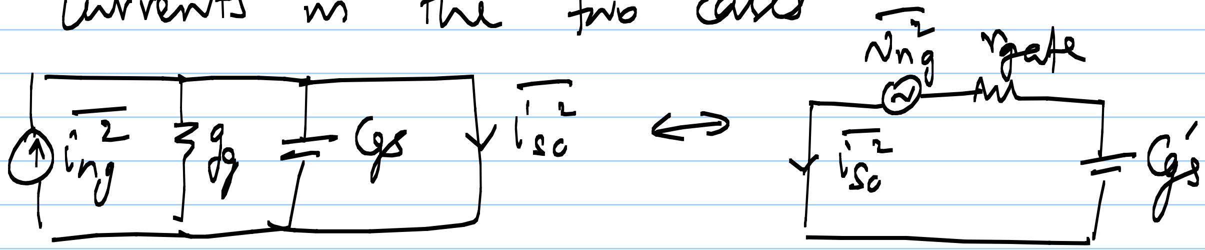
$$= \frac{1}{g_g} \cdot \frac{g_g^2}{\omega^2 C_{gs}^2} = \frac{1}{5 g_{do}}$$

* This model is narrowband only!

→ converts blue noise to white at freq. of interest

* Gate noise is typically important at high freq. only $\left\{ \overline{i_{ng}^2} \propto \omega^2 \right\}$

* to determine $\overline{v_{ng}^2}$, equate short-ckt noise currents in the two cases



$$\overline{i_{sc}^2} = \overline{i_{ng}^2}$$

$$\overline{i_{sc}} = \frac{\overline{v_{ng}}}{r_{gate} + \frac{1}{sC_{gs}'}} = \frac{\overline{v_{ng}}}{r_{gate}} \cdot \frac{1}{1 + \frac{1}{sC_{gs}' r_{gate}}}$$

$$\overline{i_{ng}^2} = \frac{\overline{v_{ng}^2}}{r_{gate}^2} \cdot \frac{1}{1 + \frac{1}{r_{gate}^2 \omega^2 C_{gs}'^2}}$$

$$\approx \overline{v_{ng}^2} \cdot \omega^2 C_{gs}'^2$$

$$\Rightarrow \overline{v_{ng}^2} = \frac{1}{\omega^2 C_{gs}'^2} \cdot 4kT\delta g_g \cdot \Delta f$$

recall : $r_{gate} = \dfrac{g_g}{\omega^2 C_{gs}'^2}$

$$\Rightarrow \overline{v_{ng}^2} = 4kT\delta\, r_{gate}\, \Delta f$$

Notes :

→ BSIM3 uses fixed $\gamma = \frac{2}{3}$

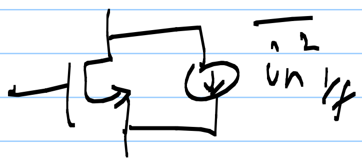
→ BSIM3 does not include $\overline{i_{ng}^2}$ model

→ Excess noise is to be included separately in design simulations (also for projects)

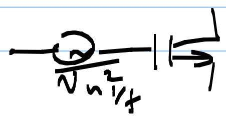
C. Flicker (1/f) Noise :

In general $\overline{N^2} = \frac{K}{f^n} \Delta f$ (K, n – empirical constants)

In mosfets :



$$\overline{i_{n_{1/f}}^2} = \frac{K}{f} \frac{g_m^2}{WLC_{ox}^2} \cdot \Delta f$$



$$\overline{v_{n_{1/f}}^2} = \frac{K}{f} \cdot \frac{1}{WLC_{ox}^2} \cdot \Delta f$$

* Recall ; $\omega_T = \frac{g_m}{C_{gs}}$

we can also write

$$\overline{i_{n_{1/f}}^2} = \frac{K}{f} \cdot \omega_T^2 A \cdot \Delta f$$

$A = W \cdot L$ = gate area

→ for the same area, faster process has worse 1/f noise

* Use large area for low flicker noise

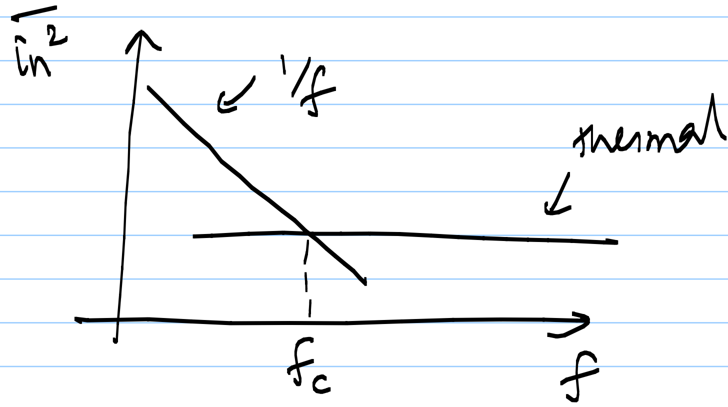
* $K_p \sim 10^{-28}$ c^2/m^2 (PMOS)

$K_n \sim 50 \cdot K_p$ (NMOS)

→ PMOS often have lower 1/f noise, but this is process dependent

→ PMOS need more current for given g_m

* flicker noise is important at low freq (for LTI circuits)



$$f_c \approx \frac{K_{1/f}}{4kT\gamma} \cdot f_T$$

* In many RF circuits, 1/f noise can get "up-converted" to desired frequency band

→ especially important in oscillator & freq. synthesizer design (N.L. or T.V. circuits)