Lecture 12: Linearity – IIP_3, cascaded systems

Measures of linearity:

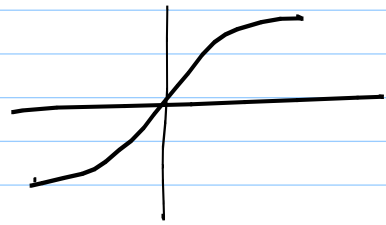
1-dB compression point
third-order intercept – IIP_3
} conventional heterodyne receivers

second-order intercept – IIP_2 – Direct conversion receivers

Recap: general non-linearity is of the form

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

(i.e. "soft" non-linearity)



Let $x(t) = A\cos\omega_1 t + A\cos\omega_2 t$

and ω_1, ω_2 are nearly equal

$y(t)$ has the following components:

1) DC: $\alpha_0 + \alpha_2 A^2$

output bias point

DC offset at output (even powers)

2) fundamental: $\left(\alpha_1 A + \frac{3}{4}\alpha_3 A^3\right)(\cos\omega_1 t + \cos\omega_2 t)$

desired gain term

gain compression or expansion term (odd powers)

3) Harmonic terms:

$$\left(\frac{\alpha_2 A^2}{2}\right)(\cos 2\omega_1 t + \cos 2\omega_2 t) + \frac{\alpha_3 A^3}{4}(\cos 3\omega_1 t + \cos 3\omega_2 t)$$

→ usually attenuated, not very important in RF

4) <u>IM2</u> :

$$(\alpha_2 A^2)\left[\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right]$$

proportional to A^2

problem term for Direct conversion receivers

→ both usually filtered in narrowband systems

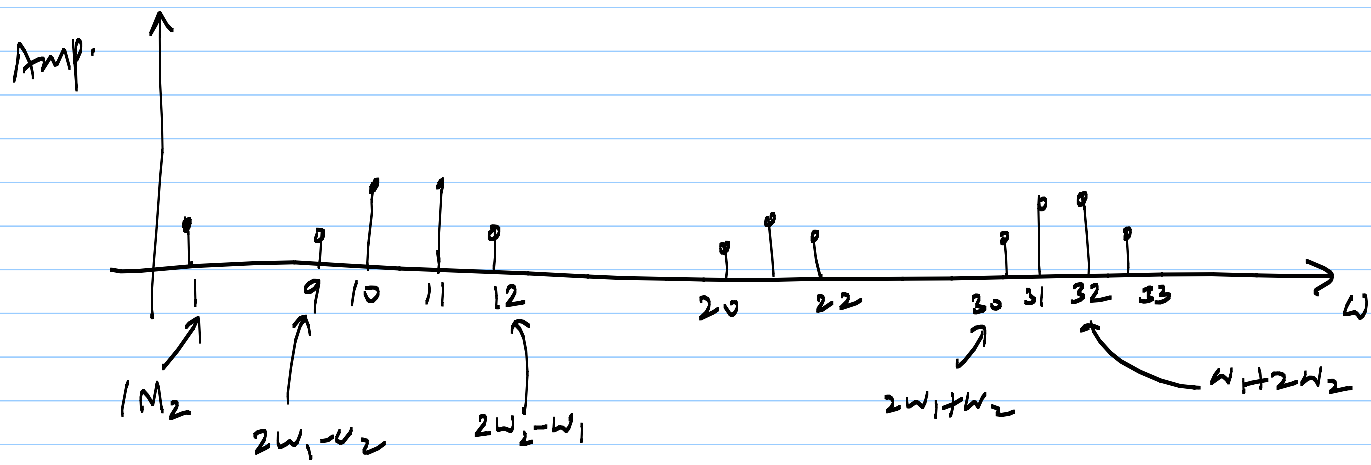
5) <u>IM3</u> :

$$\left(\frac{3}{4}\alpha_3 A^3\right)\left[\cos(\omega_1+2\omega_2)t + \cos(\omega_1-2\omega_2)t + \cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t\right]$$

prop. to A^3

→ $(\omega_1+2\omega_2)$ & $(2\omega_1+\omega_2)$ terms are far away, and are filtered

e.g. $\omega_1 = 10$, $\omega_2 = 11$ (adjacent channel interferer)



<u>IIP3</u>

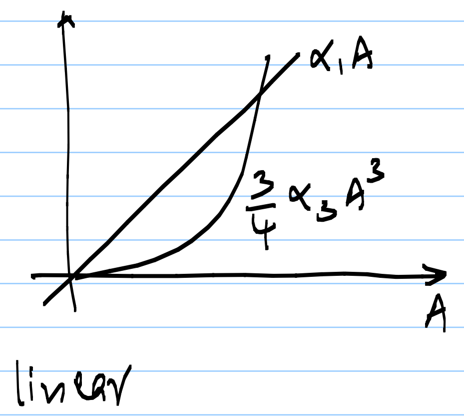
* Measured using a two-tone test
* A is chosen to be small enough so that
  a) higher order non linear terms are negligible
  b) gain is constant $(=\alpha_1)$

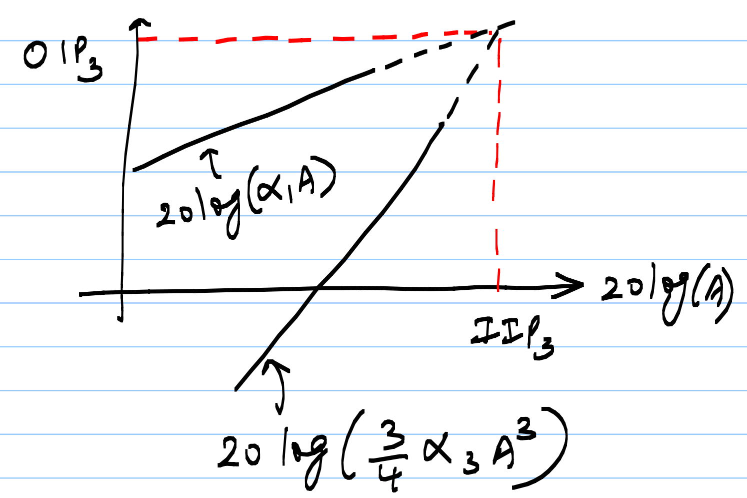
* As $A \uparrow$, fundamentals $\propto A$

$IM_3 \propto A^3$

* Plot on a log scale vs A

$\rightarrow$ slope of IM3 $= 3 \times$ slope of fund.





IIP_3 = input IP_3
OIP_3 = output IP_3 } these are actually extrapolated points

because IP_3 characterises only 3rd order NLs

In reality, both fund. & IM_3 show compression at high A.

<u>Expression for IP_3:</u>

let $x(t) = A\cos\omega_1 t + A\cos\omega_2 t$

$$y(t) = \left(\alpha_1 + \frac{9}{4}\alpha_3 A^2\right) A\cos\omega_1 t + (\quad " \quad) A\cos\omega_2 t$$

$$+ \frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 - \omega_2)t + \frac{3}{4}\alpha_3 A^3 \cos(2\omega_2 - \omega_1)t$$

$$+ \cdots$$

assume $\alpha_1 >> \frac{9}{4}\alpha_3 A^2$

We know that at IP_3, fund and IM_3 have same amplitude/power, and this happens at A_{IP_3}

$$|\alpha_1 \cdot A_{IP_3}| = |\frac{3}{4} \cdot \alpha_3 \cdot A_{IP_3}^3|$$

$$\Rightarrow \quad A_{IP_3}^2 = \frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|$$

since we work with power levels,

$$IIP_3 = \frac{A_{IP_3}^2}{2R_s} = \frac{2}{3}\left|\frac{\alpha_1}{\alpha_3}\right| \cdot \frac{1}{R_s}$$

<u>Method to measure IIP_3:</u>

input amplitude $= A_{in}$

output ampl. $(\omega_1, \omega_2) = A_{\omega_1,\omega_2}$

IM_3 ampl. $= A_{IM_3}$

$$\frac{A_{\omega_1,\omega_2}}{A_{IM_3}} \approx \frac{\alpha_1 A_{in}}{\frac{3}{4}\alpha_3 A_{in}^3} = \frac{4}{3} \cdot \frac{|\alpha_1|}{|\alpha_3|} \cdot \frac{1}{A_{in}^2}$$

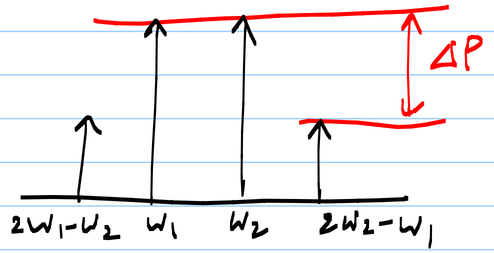
we also know

$$A_{IP_3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$\therefore \quad \frac{A_{\omega_1,\omega_2}}{A_{IM_3}} = \frac{A_{IP_3}^2}{A_{in}^2}$$

$$\therefore\ 20\log A_{\omega_1,\omega_2} - 20\log A_{IM_3} = 20\log A_{IP_3}^2 - 20\log A_{in}^2$$

$$\Rightarrow\ 20\log A_{IP_3} = 20\log A_{in} + \frac{1}{2}\left(20\log A_{\omega_1,\omega_2} - 20\log A_{IM_3}\right)$$



$$\boxed{IIP_3\big|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}\big|_{dBm}}$$

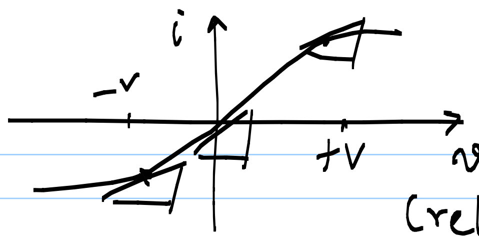
* IP_3 is measured with only one input level $\Rightarrow$ no extrapolation

Estimate using the three-point method:

$\rightarrow$ find gains at bias point, +ve extreme and −ve extreme

e.g. $\quad i(V_{DC}+v) = C_0 + C_1 v + C_2 v^2 + C_3 v^3 + \ldots$

we want to calculate C_i from $g(v)$ (ie g_m)



(relative to bias point)

$$g(v) = \frac{\partial i}{\partial v}$$

$$= C_1 + 2C_2 v + 3C_3 v^2$$

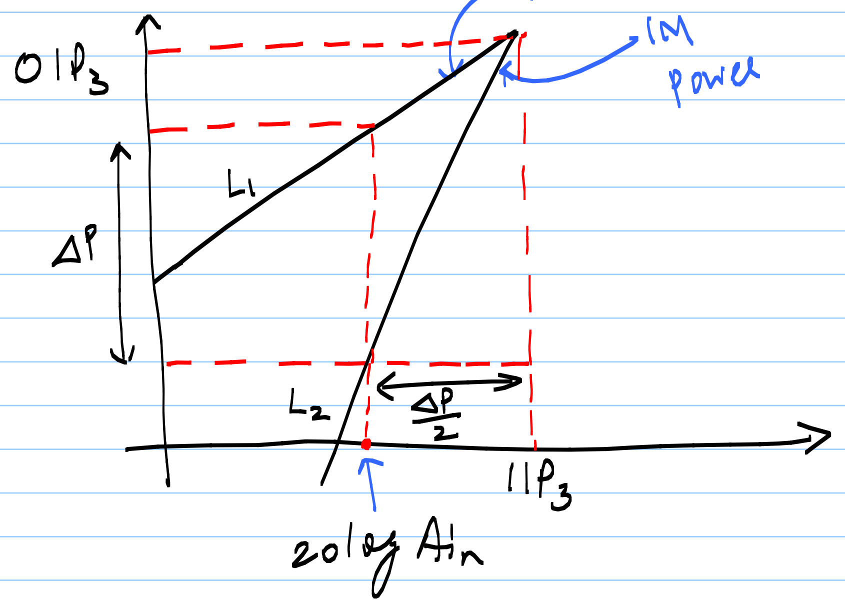
$$\left.\begin{aligned} g(0) &= C_1 \\ g(+V) &= C_1 + 2C_2 V + 3C_3 V^2 \\ g(-V) &= C_1 - 2C_2 V + 3C_3 V^2 \end{aligned}\right\} g(v) \text{ at 3 points}$$

$$\Rightarrow C_1 = g(0); \qquad C_3 = \frac{g(V)+g(-V)-2g(0)}{6V^2}$$

$$C_2 = \frac{g(V)-g(-V)}{4};$$

$$IIP_3 = \frac{2}{3}\left|\frac{C_1}{C_3}\right| \cdot \frac{1}{R_s} = \frac{4V^2}{R_s}\left|\frac{g(0)}{g(V)+g(-V)-2g(0)}\right|$$

Geometrically:



L_1 – slope = 1

L_2 – slope = 3

$\Rightarrow$ input increment of $\frac{\Delta P}{2}$ leads to

$\frac{\Delta P}{2}$ in L_1

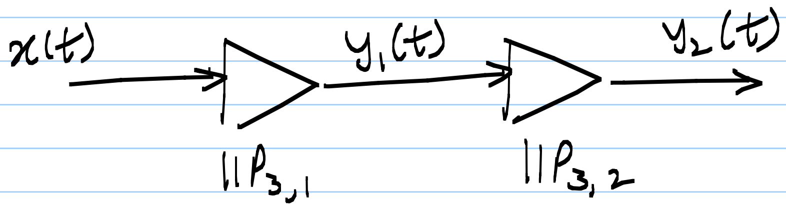
$\frac{3\Delta P}{2}$ in L_2

Note:

$$\frac{A_{1\text{-}dB}}{A_{IP_3}} = \frac{\sqrt{0.145}}{\sqrt{4/3}}$$

i.e. $$\boxed{P_{1\text{-}dB} \approx IIP_3 - 9.6\,dB}$$

Cascaded NL stages:



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$$= \beta_1(\alpha_1 x_1(t) + \dots) + \beta_2 \left(\qquad \right)^2$$

$$+ \beta_3 \left(\qquad \right)^3$$

consider only the first & third order terms,

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \ldots$$

Blindly use formula:

$$\Rightarrow A_{IP_3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}$$

worst case estimate: use $|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|$

$$\frac{1}{A_{IP_3}^2} = \frac{3}{4}\,\frac{|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|}{|\alpha_1\beta_1|}$$

$$= \frac{1}{A_{IP_{3,1}}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP_{3,2}}^2}$$

here, $A_{IP_{3,1}}$ & $A_{IP_{3,2}}$ are voltage quantities

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suppose $x(t) = A\cos\omega_1 t + A\cos\omega_2 t$ is applied

→ output fundamental $= \alpha_1\beta_1 A(\cos\omega_1 t + \cos\omega_2 t)$

→ IM_3 products of 1st stage (ampl. $= \frac{3\alpha_3}{4}A^3$) are amplified by β_1

→ $\alpha_1 A(\omega_1 + \omega_2)$ generates IM_3 products in 2nd stage; IM_3 ampl. $= \frac{3\beta_3}{4}\cdot(\alpha_1 A)^3$

→ $\alpha_2 x^2(t)$ in 1st stage produces $\omega_1 - \omega_2$, $2\omega_1$ & $2\omega_2$ components

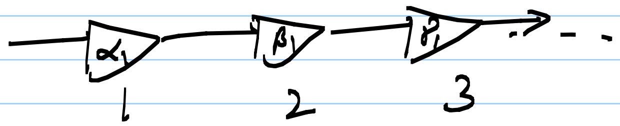
→ $\beta_2 x^2(t)$ in 2nd stage produces $2\omega_1 - \omega_2$ & $2\omega_2 - \omega_1$

→ $\omega_1 - \omega_2$, $2\omega_1$ & $2\omega_2$ are heavily attenuated in narrow band LNAs!

$\Rightarrow \dfrac{3\alpha_2\beta_2}{2\beta_1}$ term is negligible

$$\Rightarrow \frac{1}{A^2_{IP3}} \approx \frac{1}{A^2_{IP3,1}} + \frac{\alpha_1^2}{A^2_{IP3,2}}$$

general expression for a cascade:



$$\frac{1}{A^2_{IP3}} \approx \frac{1}{A^2_{IP3,1}} + \frac{\alpha_1^2}{A^2_{IP3,2}} + \frac{\alpha_1^2\beta_1^2}{A^2_{IP3,3}} + \cdots$$

$\Rightarrow$ If $\alpha_1, \beta_1 \ldots > 1$ (usually true), nonlinearity of latter stages becomes increasingly more imp.!