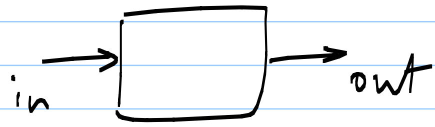
Lecture #11 : Basic Concepts in RF Design

— Non-linearity, time-variance etc.

1) Linearity :



$$x_1(t) \to y_1(t) \quad \& \quad x_2(t) \to y_2(t)$$

$$\Rightarrow a x_1(t) + b x_2(t) \to a y_1(t) + b y_2(t)$$

for all a & b

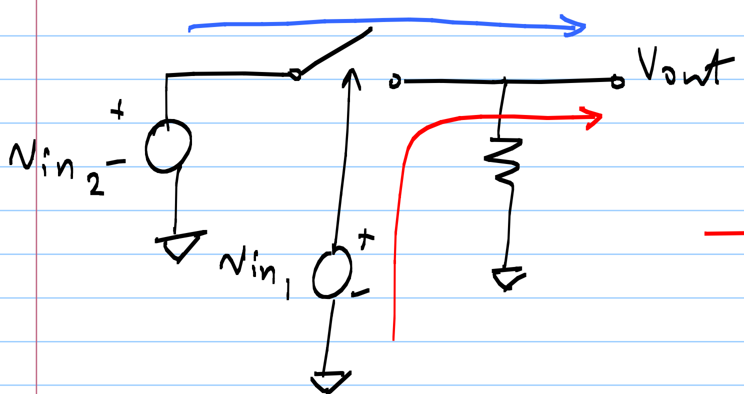
* non-zero initial conditions
* finite offsets

} are non-linear

2) Time Invariance :

$$x(t) \to y(t)$$

$$\Rightarrow x(t-\tau) \to y(t-\tau) \quad \text{for all } \tau$$



$v_{in_1} = V_1 \cos \omega_1 t$

$v_{in_2} = V_2 \cos \omega_2 t$

→ Nonlinear – Vout depends only on v_{in_1} polarity

Time Variant – Vout also depends on v_{in_2}

→ linear – $a x_1(t) + b x_2(t)$ holds

time variant – Vout depends on v_{in_2}

* We will see more of these in the context of mixers

$$f_1, f_2 \cdots f_n \longrightarrow \boxed{\text{LTI}} \longrightarrow f_1, f_2 \cdots f_n$$

$$f_1, f_2 \cdots f_n \longrightarrow \boxed{\begin{matrix}\text{Non-} \\ \text{Linear} \\ \text{or} \\ \text{LTV}\end{matrix}} \longrightarrow f_1, f_2 \cdots f_n ; f_1', f_2' \cdots f_m'$$

$\left.\begin{matrix}\text{Nonlinear} \\ \text{or} \\ \text{Linear time-variant system}\end{matrix}\right\}$ can produce frequency components that are not present in the input signal

3) <u>Memory</u>: output $y(t)$ depends on past inputs

$$y(t) = \alpha\, x(t) \Rightarrow \text{memoryless linear system}$$

LTI $\Rightarrow$ α is a constant

LTV $\Rightarrow$ $\alpha = f(t)$

memoryless nonlinear system:

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

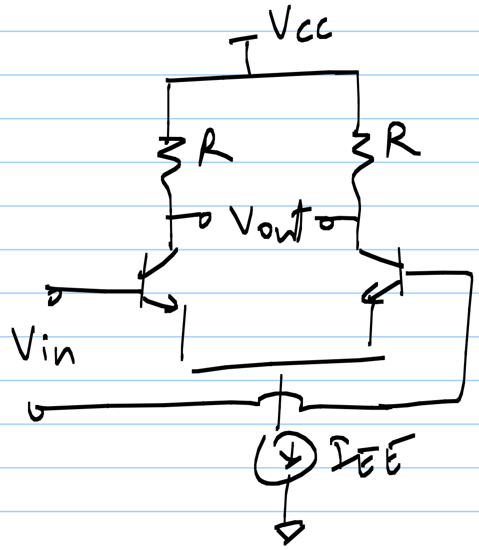
$\alpha_i = f(t)$ if time-variant

4) <u>Symmetry</u>:

odd symmetry: $\left.\begin{matrix} x(t) \to y(t) \\ -x(t) \to -y(t)\end{matrix}\right\}$ $\alpha_i = 0$ for even i

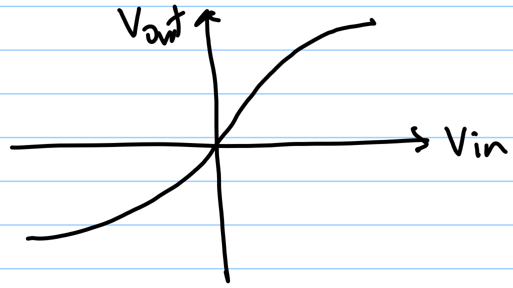
odd symmetry $\leftrightarrow$ differential (or) balanced

e.g.



$$V_{out} = I_{EE} \cdot R \cdot \tanh\left(\frac{V_{in}}{2V_T}\right)$$

tanh is an odd function



5) <u>Dynamic</u> : $y(t)$ depends on $x(t), x(t-\tau_1), x(t-\tau_2) \ldots, y(t-\tau_1'), y(t-\tau_2') \ldots$

ie present output depends on past inputs and outputs

a) LTI dynamic system

$$y(t) = h(t) * x(t)$$

impulse response

b) LTV dynamic system $\rightarrow$ $h(t)$ is a function of time

$$\delta(t) \rightarrow h(t)$$

$$\delta(t-\tau) \rightarrow h(t,\tau)$$

$$\Rightarrow y(t) = h(t,\tau) * x(t)$$

c) Nonlinear, dynamic system:

$h(t)$ can be approximated with a <u>Volterra Series</u>

$$y(t) = k_0 + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} k_n(t_1, t_2 \cdots t_n)\, x(t-t_1)\, x(t-t_2) \cdots x(t-t_n)\, dt_1 dt_2 \cdots dt_n$$

k_n = n^{th} order Volterra Kernel

Effects of Nonlinearity:

* consider only memoryless TV systems
* assume $y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$

a) Harmonics – output usually contains integer multiples of input frequency

$$x(t) = A\cos\omega t \longrightarrow \underset{\text{system}}{\boxed{NL}} \longrightarrow y(t)$$

$$y(t) = \alpha_1 A\cos\omega t + \alpha_2 A^2 \cos^2\omega t + \alpha_3 A^3 \cos^3\omega t$$

$$= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t)$$

$$+ \alpha_3 \frac{A^3}{4}(3\cos\omega t + \cos 3\omega t)$$

$$= \underbrace{\alpha_2 \frac{A^2}{2}}_{DC} + \underbrace{\left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t}_{\text{fundamental (gain)}}$$

$$+ \underbrace{\frac{\alpha_2 A^2}{2}\cos 2\omega t}_{2^{nd}\text{ harm.}} + \underbrace{\frac{\alpha_3 A^3}{4}\cos 3\omega t}_{3^{rd}\text{ harm.}}$$

* even harmonics result from α_i with even i
  - → no even harm. if odd symmetry (differential)
  - → mismatches between differential paths corrupt symmetry leading to finite even harmonics
* amplitude of n^{th} harmonic $\propto$ (A^n + higher powers of A)

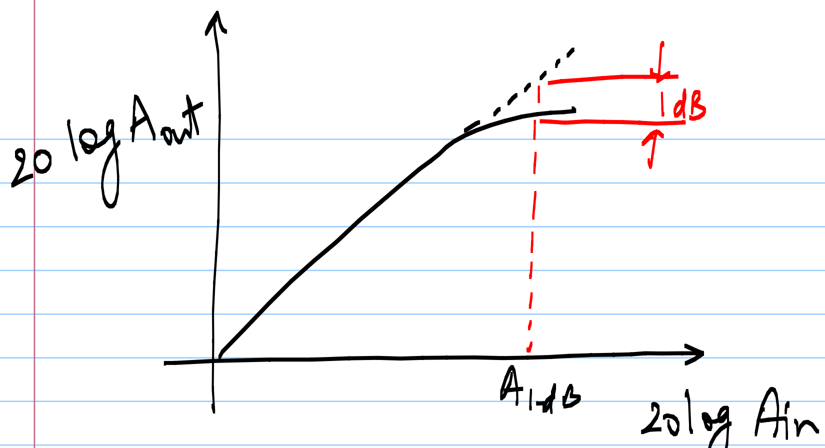
b) Gain Compression

* small signal gain assumes no harmonics
  → i.e. linearised operation, gain $\approx \alpha_1$

e.g. diff. pair gain

$$\frac{v_{out}}{v_{in}} \approx \frac{I_{EE} R}{2 V_T}$$

* extra terms due to nonlinearity such as $\frac{3}{4}\alpha_3 A^3$ cause variation of gain with input level (A).
* In general $\alpha_3 < 0 \Rightarrow \alpha_1 + \frac{3}{4}\alpha_3 A^2 \downarrow$ with A
  → output is "compressive" (i.e. gain → 0 as A ↑)
* quantified by "1 dB compression point"
  → input signal level that causes the small-signal gain to drop by 1 dB



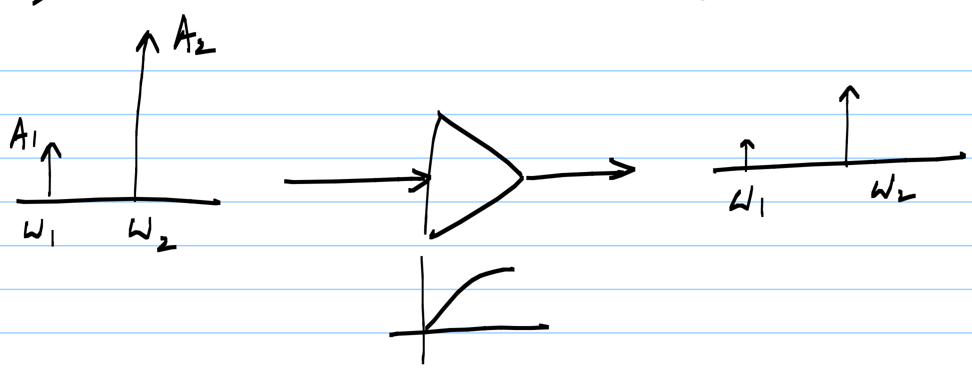
$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{1-dB}^2\right| = 20\log|\alpha_1| - 1\,dB$$

$$\Rightarrow A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

typical LNAs → $P_{1dB} \approx -20$ to -25 dBm

(63.2 to 35.6 mVpp in a 50Ω system)

c) Desensitisation & Blocking:



$$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$$

$$y(t) = \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \cdots$$

for $A_1 << A_2$,

$$y(t) \approx \underbrace{\left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right)}_{\text{gain}} A_1 \cos\omega_1 t + \cdots$$

gain – decreasing function of A_2 since $\alpha_3 < 0$

* for large A_2, gain $\rightarrow 0$
  $\Rightarrow$ desired signal is "blocked"
* $A_2 \cos\omega_2 t$ is called a "blocker"
  $\rightarrow$ large interferer that desensitise a ckt
* typical RF receiver blocker spec would be 60–70dB above the desired signal

d) Cross-Modulation:

$\rightarrow$ transfer of modulation from blocker to desired (weak) signal

from (c), we know that gain$(\omega_1) = f(A_2)$

e.g. say interferer has AM – $A_2(1 + m\cos\omega_m t)\cdot\cos\omega_2 t$

$$y(t) = \left[\alpha_1 A_1 + \frac{3}{2}\alpha_3 A_1 A_2^2 \left(1 + \frac{m^2}{2} + \frac{m^2}{2}\cos 2\omega_m t + 2m\cos\omega_m t\right)\right]\cdot\cos\omega_1 t + \dots$$

$\Rightarrow$ desired signal (@ ω_1) contains AM at ω_m & $2\omega_m$

e) <u>Intermodulation</u>:

Suppose the non-linearity of an active LPF is to be characterised.

$\Rightarrow$ Harmonic distortion alone is not a good indicator since harmonics fall OOB (distortion appears small)

<u>two-tone test</u>: 2 signals of different freq. are applied $\rightarrow$ output has components that are <u>not</u> harmonics of input (the signals get "mixed" due to non-linearity)

$\Rightarrow$ Intermodulation

$$x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$$

$$y(t) = \alpha_1 (A_1\cos\omega t + A_2\cos\omega_2 t) + \alpha_2 (\quad)^2 + \alpha_3 (\quad)^3$$

Ignore dc terms & harmonics; you are left with fundamental and intermodulation products at

fund $\omega = \omega_1 \Rightarrow \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t$

fund $\omega = \omega_2 \Rightarrow \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$

IM
$\omega = \omega_1 \pm \omega_2 \Rightarrow \alpha_2 A_1 A_2 \cos(\omega_1+\omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1-\omega_2)t$

$\omega = 2\omega_1 \pm \omega_2 \Rightarrow \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1+\omega_2)t$

$+ \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1-\omega_2)t$

$\omega = 2\omega_2 \pm \omega_1 \Rightarrow \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2+\omega_1)t$

$+ \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2-\omega_1)t$

$\omega = \omega_1 \pm \omega_2 \Rightarrow IM_2$ or 2nd order IM products

$\omega = 2\omega_1 \pm \omega_2$, $2\omega_2 \pm \omega_1 \Rightarrow IM_3$ or 3rd order IM products

$IM_3 \Rightarrow$ key metric because if ω_1 is close to ω_2, $2\omega_1 - \omega_2$ & $2\omega_2 - \omega_1$ are also close to ω_1 & ω_2 !

e.g. $\omega_1 = 1\,MHz$, $\omega_2 = 1.01\,MHz$

$2\omega_1 - \omega_2 = 0.99\,MHz$

$2\omega_2 - \omega_1 = 1.02\,MHz$

$\Rightarrow$ reveals non-linearities in cases like LPF...

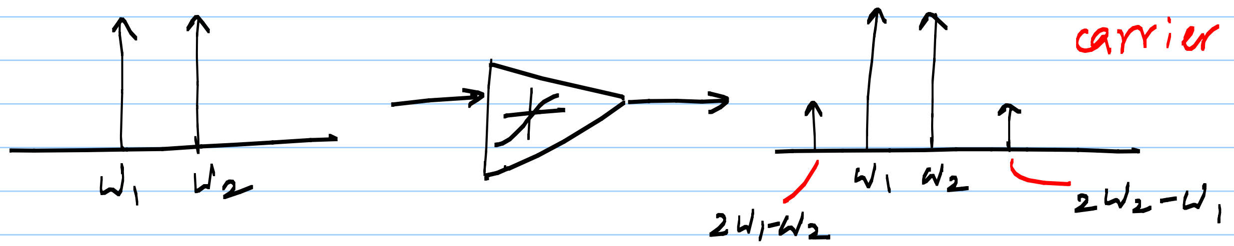
In a typical "two-tone test", $A_1 = A_2 = A$

$$\text{IM distortion} = \frac{IM_3}{\text{fund component}} = \frac{IM_3}{\alpha_1 A}$$

e.g. $\alpha_1 A = 1V_{pp}$; $\frac{3}{4}\alpha_3 A^3 = 10mV_{pp}$

$\Rightarrow$ IM components are at $\underline{\underline{-40dBc}}$

dB w.r.t. carrier



→ In general, signal amplitude or phase could get corrupted due to intermodulation