Lecture 8

Note Title 8/11/2008

$Y(t) = X(t) + N(t)$ → white, Gaussian, PSD $N_0/2$, random process

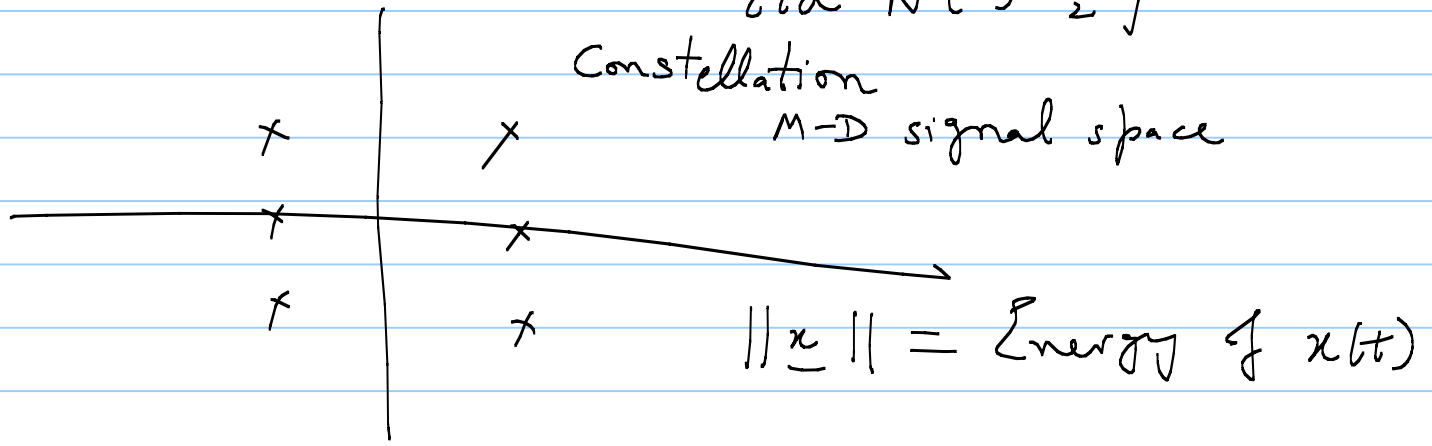
↓ Lin Alg

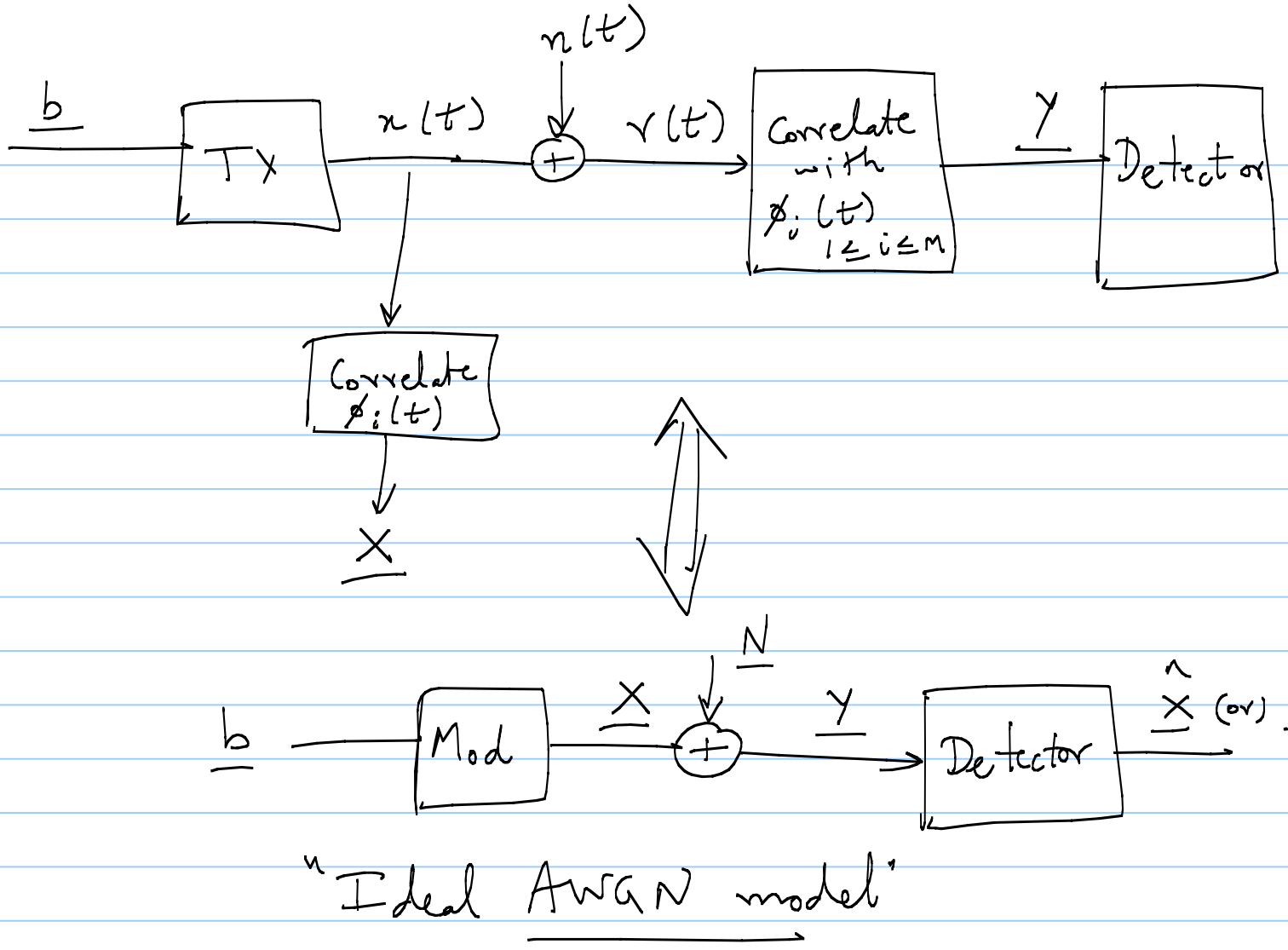
$\underline{Y} = \underline{X} + \underline{N}$

$\underline{X}$ → discrete

$\underline{N}$ → random vectors, iid $N(0, \frac{N_0}{2})$

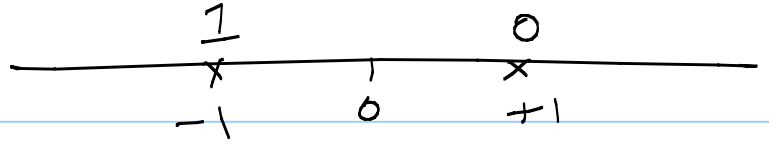
Constellation





"Ideal AWGN model"

BPSK



$$f_{Y/X}(y|+1) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(y-1)^2}{2\sigma^2}}$$

$$\sigma = \sqrt{\frac{N_0}{2}}$$

(2) FSK

$N=1$ $b = 0$ or 1

$$x_0(t) = \sqrt{\frac{2E_s}{T}} \cos \frac{2\pi m_0 t}{T}, \quad 0 \le t \le T$$

$$f_0 = \frac{m_0}{T}$$

$$x_1(t) = \sqrt{\frac{2E_s}{T}} \cos \frac{2\pi m_1 t}{T}, \quad 0 \le t \le T$$

$$f_1 = \frac{m_1}{T}.$$

$$\tilde{x}_0(t) = \sqrt{\frac{2E_s}{T}}\, \text{rect}_T\left(t - T/2\right) ?$$

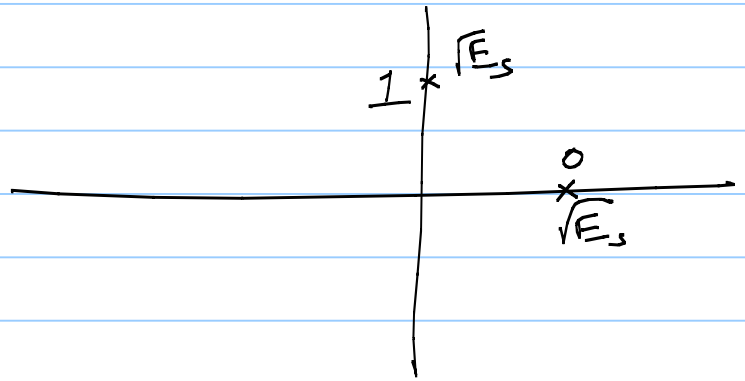
$f_0 \longrightarrow$

$$\tilde{x}_1(t) = \sqrt{\frac{2E_s}{T}} e^{j2\pi(f_1 - f_0)t}, \quad 0 \le t \le T$$

$$\phi_0(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_0 t \quad , \quad 0 \le t \le T$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_1 t \quad , \quad 0 \le t \le T$$

$$\underline{x}_0 = \begin{bmatrix} \sqrt{E_s} \\ 0 \end{bmatrix} \qquad \underline{x}_1 = \begin{bmatrix} 0 \\ \sqrt{E_s} \end{bmatrix}$$

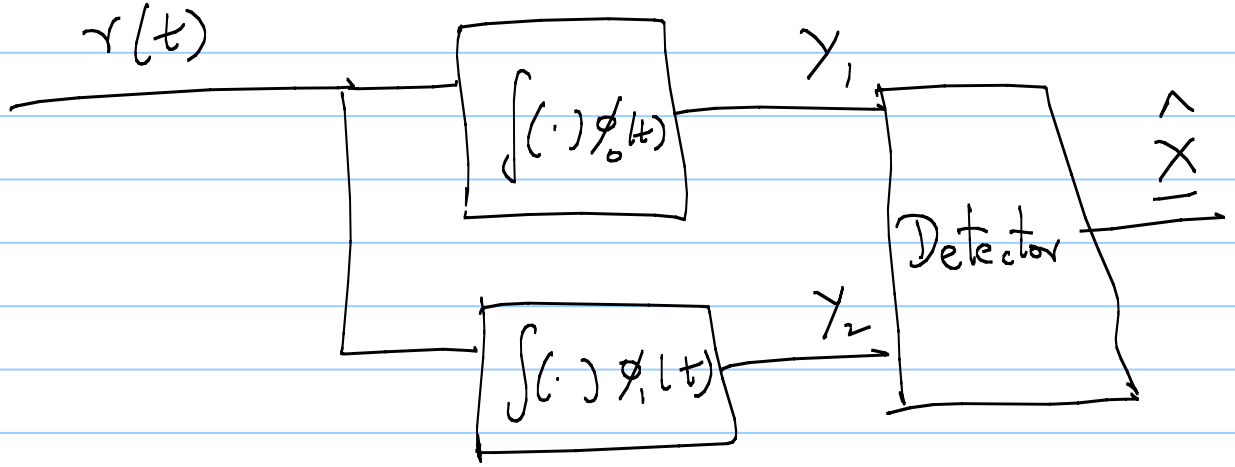


$$\underline{X} = \begin{cases} \underline{x}_0 & \text{w.p. } 1/2 \\ \underline{x}_1 & \text{w.p. } 1/2 \end{cases}$$

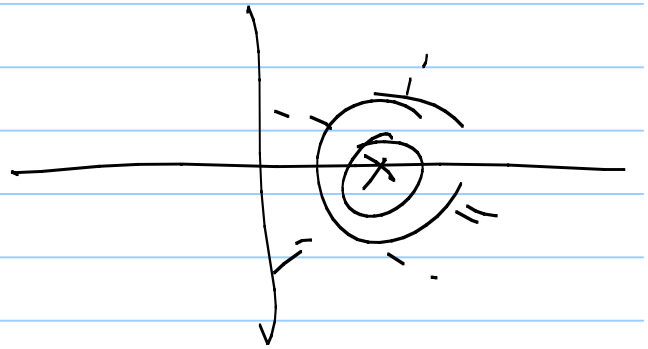
$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$E\left[|\underline{X}|^2\right] = E_s$$

R_x :

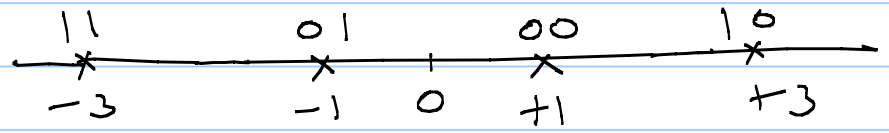


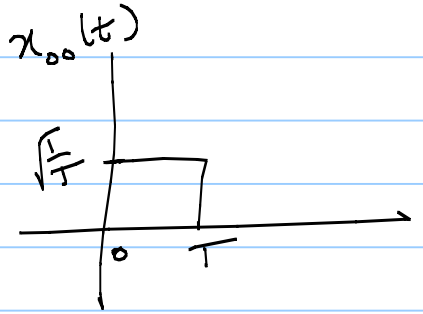
$$f_{\underline{y}|\underline{x}}\left(\underline{y} \mid \begin{bmatrix}\sqrt{E_s}\\ 0\end{bmatrix}\right) = ?$$

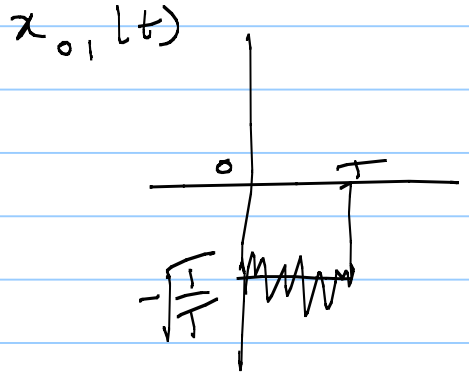


③ 4-PAM

$N=2 \quad \underline{b} = [b_1 \; b_2]$ $\qquad \phi_1(t) = \sqrt{\frac{1}{T}}, \quad 0 \le t \le T$







$$X = \begin{cases} -3 & \text{w.p. } 1/4 \\ -1 & \text{“} \\ +1 & \text{“} \\ +3 & \text{“} \end{cases}$$

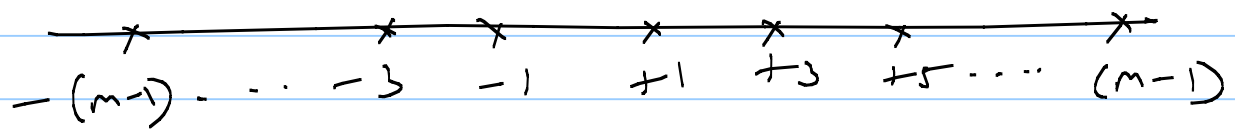
$Y = X + N$

$N(0, \frac{N_0}{2})$

$E[X^2] = 5$

④ M-PAM m: power of 2

$N = \log_2 m$ $\underline{b} = [b_1\ b_2 \cdots b_{\log_2 m}]$ $\phi_1(t) = \sqrt{\frac{1}{T}}$, $0 \le t \le T$



$$E[X^2] = \frac{m^2-1}{3}$$

$$Y = X + N$$